



Introduction to ordinary differential equations (4th ed.) by shepley l. ross

Differential Equations Class Notes Introduction to Ordinary Differential Equations, 4th Edition by Shepley L. Ross, John Wiley and Sons (1989). Copies of the classnotes are on the internet in PDF format as given below. These notes and supplements have not been classroom tested (and so may have some typographical errors). They are based on a sophomore differential equations class I taught at Louisiana State University in Shreveport (MATH 355) in spring 1992. A copy of the book can be browsed online here (accessed March 4, 2019). Announcement for the Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the Class. PDF. Syllabus for the LSUS Class. PDF. Syllabus for the Class. Differential Equations; Their Origin and Application. PDF. Section 1.2. Solutions. PDF. Section 1.3. Initial-Value Problems, Boundary-Value Problems, Boundary-Va Separable Equations and Equations Reducible to This Form. PDF. Section 2.3. Linear Equations and Bernoulli Equations. PDF. Section 2.4. Special Integrating Factors and Transformations. Test 1 (1.1-1.3, 2.1-2.4). PDF. Chapter 3. Applications of First-Order Equations. Section 3.1. Orthogonal and Oblique Trajectories. Section 3.2. Problems in Mechanics. PDF. (This section contains a computation of escape velocity from a gravitational object.) Section 3.3. Rate Problems. PDF. Chapter 4. Explicit Methods of Solving Higher-Order Linear Differential Equations. PDF. Section 4.1. Basic Theory of Linear Differential Equations. PDF. Chapter 4. Explicit Methods of Solving Higher-Order Linear Differential Equations. Coefficients. PDF. Section 4.3. The Method of Undetermined Coefficients. PDF. Section 4.4. Variation of Parameters. PDF. Section 4.4. Variation of Parameters. PDF. Section 4.5. The Cauchy-Euler Equation. PDF. Section 4.4. 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The Fourth Edition of the best-selling text on the basic concepts, theory, methods, and applications of ordinary differential equations retains the clear, detailed style of the first three editions. Includes new material on matrix methods, numerical methods, the Laplace transform, and an appendix on polynomial equations. Stresses fundamental methods, and features traditional applications to the underlying theory. The requested URL was not found on this server. Additionally, a 404 Not Found error was encountered while trying to use an ErrorDocument to handle the request. Apache/2.4.41 (Ubuntu) Server at yvc.moeys.gov.kh Port 443 Student Solutions Manual to accompany INTRODUCTION TO ORDINARY DIFFERENTIAL EQUATIONS Fourth Edition Shepley L. Ross University of New Hampshire With the assistance of Shepley L. Ross, II Bates College JOHN WILEY & SONS New York Chichester Brisbane Toronto Singapore Copyright @ 1989 by John Wiley & Sons, Inc. All rights reserved. This material may be reproduced for testing- or instructional purposes by people using the text. ISBN 0 471 63438 7 Printed in the United states of America 10 9 8 7 6 5 4 3 2 1 Preface This manual is a supplement to the author's text, Introduction to Ordinary Differential Equations, Fourth Edition. It contains the answers to the even-numbered exercises and detailed solutions to approximately one half of both the even- and odd-numbered exercises sets. The following abbreviations have been used: . D.E. differential equation . G.S. general solution . I.F. integrating factor . I.C. initial condition . I.V.P. initial value problem. The author expresses his thanks to his son Shepley L. Ross, II, of Bates College for his many contributions, especially the solutions and graphs of Chapter 8. The author also thanks his colleague Ellen O'Keefe and his graduate students Rita Fairbrother and Chris McDevitt for providing solutions. Shepley L. loss Contents Chapter 1: Chapter 2: Chapter 3: Chapter 4: Chapter 5: Chapter 6: Chapter 7: Chapter 6: Chapter 6: Chapter 7: Chapter 6: Chapter 6 . 262 Series Solutions of Linear Differential Equa t ion s 131 Applications of Second-Order Linear Differential Equations with Constant Coefficients . .. 301 Systems of Linear Differential Equations 373 Approximate Methods of Solving First-Order Equa tions 556 The Laplace Transform 753 Chapter 1 Section 1.2, Page 11 1. (d) We must show that f(x) = (1 + x 2) - 1 satisfies the D.E. (1 + X 2)yH + 4xy' + 2y = 0. Differentiating f(x), we find f'(x) = -(1 + x 2) - 2(2x) and fH(x) = (6x 2 - 2)(1 + x 2) - 3. We now . 669 Answers to Even-Numbered Exercises substitute f(x) for y, f'(x) for y', and fH(x) for yH in the stated D.E. We obtain (1 + x 2)(6x 2 - 2)(1 + x 2) - 3 - 4x(1 + x 2) - 2(2x) 2 - 1 + 2(1 + x 2) - 1 = 0, and hence to 2 2 2 - 2 [(-2x - 2) + (2 + 2x)](1 + x) = 0, 2 - 2 that is, 0(1 + x) is satisfied by f(x) = 0 or 0 = 0. 2 - 1 (1 + x). Hence the given D.E. 2. 3 2 (a) We must show that the relation x + 3xy = 1 defines at least one real function which is an explicit solution of the given D.E. on 0 < x < 1. Solving the given D.E. $1 + x^2 + 2x + 3x^2 + 2x + 4x^2 + 2x + 3x^2 + 2x^2 + 2$ 3. a "e must s ow t at x = x + c e satis les t e 2 - 3x D.E. y' + 3y = 3x e. Differentiating f(x), we find 3 - 3x - 3x 2 f'(x) = (x + c)(-3e) + e (3x). We not substitute f(x) for y and f'(x) for y' ln the stated D.E. We obtain 3 - 3x - 3x 2 3 - 3x 2 - 3x (x + c) (-3e) + e (3x) + 3(x + c)e = 3x e . 2 - 3x The left member reduces to 3x e , so we have 2 - 3x 2 - 3x 3x e = 3x e ; and thus the D.E. is satisfied by $3 - 3x f(x) = (x + c)e \cdot 4$. (b) We must show that g(x) 2x = c 1 e 2x + c 2 x e - 2x + c 3 e satisfies the D.E. ym 2yH - 4y' + 8y = 0. Differentiating g(x), we find g'(x) = 2c 1 e 2x + c 2 x e - 2x + c 3 e, g(x) = 4c 1 e + 4c 2 x e + 4c 2 e + 4 - 2x and gm() 8 2x 2x 12 2x c 3 e, x = c 1 e + 8c 2 x e + c 2 e - 2x 8c 3 e, x = c 1 e + 8c 2 x e + c 2 e - 2x 8c 3 e, y = 4c 1 e + 4c 2 x e + 4c 2 e + 4 - 2x and gm() 8 2x 2x 12 2x c 3 e, x = c 1 e + 8c 2 x e + c 2 e - 2x 8c 3 e, y = 2x + 2c 2 x e + 2x 2x + 8c 2 x e + 2x 2x + 2c 2 x e + 2x 2x + 2c 2 e - 2x e + 2x 2x + 2c 2 e - 2x e + 2x 2x + 2c 2 e - 2x + 28c 3 e 2x 8c 2 xe - 8c 2 e - 2x 2x 8c 2 xe - 4c 2 e + 2x - 2x 8c 2 xe + 8c 3 e 2x + 8c 3 e 2x + 8c 2 xe 2x - 8c e - 1 2x + 8c 1 e + = 0 - 2x 8c 3 e - 2x 8c 3 e and hence to (8c 1 + 12c 2 - 8c 1 - 4c 2 2x + 8c 1)e 2 (8c 2 - 8c 2 - 8c + 8c 2) xe 2x + 2 (-8c 3 - 8c 3 + 8c 3 + -2x + 8c 3)e = 0, h. 0 2x 2x - 2x 0 H t at IS, e + 0.xe + 0.e =, or 0 = 0. ence 2x 2x the given D.E. is satisfied by $g(x) = c \ 1 \ e + c \ 2 \ xe + -2x \ c \ 3 \ e \ 5. \ mx$ (a) To determine the values of m for which f(x) = e satisfies the given D.E., we differentiate f(x) the required number of times and substitute into the D.E. We have $f(x) = e \ InX$, $f'(x) = m \ mx$, $f''(x) = m \$ observe that m = 2 is a root, and so Differential Equations and Their Solutions 5 (m - 2) is a factor of the left member. We now use synthetic division to find the other factor. We have 2 1 -3 -4 12 2 -2 -12 1 -1 -6 0 From this we see that the reduced quadratic factor is m 2 - m - 6. Hence the cubic equation may be written (m 2)(m 2 - m - 6) = 0 and so (m - 2) is a factor of the left member. -2)(m - 3)(m + 2) = 0. Thus we see that its roots are m = 2, 3, -2. These then are the values of m for which $f(x) = e \operatorname{rnx} IS$ a solution of the given D.E. 6. (b) From $f(x) 2x 2x - \cos 2x$, we find $f'(x) = 3e - 2xe = 4e 2x_4 + 2 \sin 2x$, $f''(x) 2x 2x + 4 = 4e - 8xe \cos 2x$. We substitute f(x) for y, f'(x) for y ', fIt (x) for y '' the . D.E. , obtaining ln glven (4e 2x - 2)(m - 3)(m + 2) = 0. Thus we see that its roots are m = 2, 3, -2. These then are the values of m for which $f(x) = e \operatorname{rnx} IS$ a solution of the given D.E. 6. (b) From $f(x) 2x 2x - \cos 2x$, we find $f'(x) = 3e - 2xe = 4e 2x_4 + 2 \sin 2x$, $f''(x) 2x 2x + 4 = 4e - 8xe \cos 2x$. We substitute f(x) for y, f'(x) for y '', fIt (x) for y '' the . D.E. , obtaining ln glven (4e 2x - 2x) (4e - 2x $8xe 2x + 4(4e 2x + 4(3e 2x 4 \cos 2x) 2x - 4xe + 2 \sin 2x) 2x - 4xe + 2 \sin 2x)
2x - 2xe - \cos 2x) = -8 \sin 2x$. Collecting like terms in the left member, we find $2x 2x (4 - 16 + 12)e + (-8 + 16 - 8)xe + (4 - 4) \cos 2x - 8 \sin 2x = -8 \sin 2x$. Collecting like terms in the left member, we find $2x 2x (4 - 16 + 12)e + (-8 + 16 - 8)xe + (4 - 4) \cos 2x - 8 \sin 2x = -8 \sin 2x$. Collecting like terms in the left member, we find $2x 2x (4 - 16 + 12)e + (-8 + 16 - 8)xe + (4 - 4) \cos 2x - 8 \sin 2x = -8 \sin 2x$. f(O) = 1 = 2 and f'(O) = 4e O - = 4. Hence f(x) also satisfies the stated conditions. 7. (a) We must show that the D.E. Iy'l + Iyl + 1 = 0 has no (real) solutions. Assume, to the contrary, that this D.E. has a real differentiable function f as a solution on some interval I. Since f is a solution on I, we must have If/(x)1 + If(x)1 + 1 = 0 for all x E I. But for a real obtain $y' = 4c \ 1 \ e \ 3c \ 2 \ e \ 3x$. We apply the I.C. y'(0) = 6 to this derived Differential Equations and Their Solutions 7 relation. That is, we let x = 0, $y' = 6 \ 1n \ y' = 4x \ -3x \ 4c \ 1 \ e \ -3c \ 2 \ = 6$. The two equations $c \ 1 \ + c \ 2 \ = 5$, $4c \ 1 \ - 3c \ 2 \ - 6$ - determine $c \ 1$ and $c \ 2$ uniquely. Solving this system, we find $c \ 1 \ = 3$, $c \ 2 \ = 2$. Substi- $4x \ -3x$ tuting these values back into y = c 1 e + c 2 e we obtain th t. 1 1. 3 4x 2 -3x. f. h e par 1CU ar so ut10n y = e + e sat1s Ylng t e stated I.V.P. 4. (a) We first apply the B.C. y(O) = 0 to the given family of solutions. That 1S, we let x = 0, $y = c 1 \sin x + c 2 \cos x$. We obtain c 2 = o. We next apply the B.C. y(7r/2) = 1 to the given family of solutions. That 1S, we let x = 7r/2, $Y - 1 \ln - y = c 1 S \ln x + c 2 \cos x$. We obtain c 1 = 1. Substi-tuting the values c 1 = 1, c 2 = 0 back into $y = c 1 S \ln x + c 2 \cos x$, we obtain the particular solution $y = c 1 S \ln x + c 2 \cos x$. We obtain the particular solutions. That 1S, we let x = 0, $y = c 1 S \ln x + c 2 \cos x$. We obtain c 1 = 1. Substi-tuting the values c 1 = 1, c 2 = 0 back into $y = c 1 S \ln x + c 2 \cos x$. We obtain the particular solution $y = c 1 S \ln x + c 2 \cos x$. We obtain the particular solutions. That 1S, we let x = 0, $y = o \ln y = c 1$. Sh x + c 2 co s x. We obtain c 2 = o. We next apply the B.C. Y(7r) = 1 to the given family of 8 Chapter 1 solutions. That is, we let x = , y = 1 in y = c 1 sin x + c 2 cosx. We obtain -c 2 = 1, so c 2 = -1. At this point we have both c 2 = 0 and, at the same time, c 2 = -1. This is impossible! So the given boundary-value problem has no solution. 5. We are given that every solution of the stated D.E. may be written in the form 2Y = c 1 x + c 2 x 3 + c 3 x (1) for some choice of the constants so that (1) will satisfy the three stated conditions. We differentiate (1) twice to obtain y' + 2c 2 x + 3c 3 x 2 = c 1 and "2c 2 x + 6c 3 x y - (2) (3) We now apply the condition y(2) = 0 to (1), letting x = 2, y = 0 in (1). We obtain 2c 1 + 4c 2 + 8c 3 = 0. Similarly, we apply the condition y'(2) = 2 to (2), thereby obtaining c 1 + 4c 2 + 12c 3 = 2. Finally, we apply the condition y'(2) = 2 to (2), thereby obtaining 2c 2 + 12c 3 = 6. Thus we have the three equations and Their Solutions 9 2c 1 + 4c 2 + 8c 3 = 0. = 0, c 1 + 4c 2 + 12c 3 = 2, 2c 2 + 12c 3 = 6, In the three unknowns. These can be solved 1n var10US way s. One easy way 1S to eliminate c 1 from the first two equations, obtaining the equivalent of c 2 + 4c 3 = 1. Combining this last with c 2 + 6c 3 = 3, which 1S equivalent to the third equation of the system, we readily find c 2 = -3, c 3 = 1. Then from the second equation one finds c 1 = 2. Thus c 1 = 2, c 2 = -3, c 3 = 1. Substituting these values back into (1), we find the solution of the solution of the solution of the stated I.V.P. 1S 2 3 y = 2x - 3x + x. (a) Let us apply Theorem 1.1. hypothesis. Here f(x,y) = a f(x y) 2 a y, = x cos y · Both f and We first check the 2. d x Sln y an af . . ay are cont1nuous 1n every domain D in the xy plane. The initial condition y(l) = -2 means that Xo = 1 and Yo = -2, and the point (1, -2) certainly lies in some such domain D. Thus all hypothesis are satisfied and the conclusion holds. That is, there is a unique solution; of the D.E. $y' = x 2 \sin y$, defined on some interval 1 x - 11 h about Xo = 1, which satisfies the initial condition, that 18, which is such that ;(1) = -2. Chapter 2 Section 2.1, Page 36 In these solutions of Exercises 3, 5, 6, 7, and 9 follow the pattern of Example 2.5 on page 31. 3. Here M(x,y) = 2xy + 1, N(x,y) = 2xy + 1 and F(x,y) = M(x,y) = x + 4y. From the first of these, we find F(x,y) = x + 4y. From the first of these, we find F(x,y) = x + 4y. From the first of these, we find F(x,y) = x + 4y. From the first of these, we find F(x,y) = x + 4y. Therefore 2.2 x + ;'(y) = x + 4y or dd) = 4y. x + 2y2 + c . o Then (y) = 2y2 + cO' 2 Thus F(x,y) = x y + The one-parameter family of solutions 2 F(x,y) = c 1 1S x y + x + 2y 2 h = c, were c = c 1 - cO. 10 First-Order Equations 11 Alternatively, by the method of group1ng, we first 2 write the D.E. in the form 2xy dx + x dy + dx + 4y dy = o. We recognize this as d(x 2 y) + d(x) + d(2y 2) = d(c) or d(x 2 y + x + 2y 2) = d(c). Hence we obtain the 1 t . 2 2y 2 so u 10n x y + x + - c. 4. 2 3 Here M(x,y) = 3x y + 2, N(x,y) = -(x + y). From these we 2 2 find M (x,y) = 3x t - 3x = N (x,y). Since M (x,y) = 3x 2 + 4xy - 6. From these we find M (x,y) = 6x + 4y = N (x,y), so the D.E. is y x exact. We seek F(x,y) such that F (x,y) = M(x,y) = 6x y + 2y 2 x - 5x + (y). Since M (x,y) = 3x 2 + 4xy - 6. From these we find M (x,y) = 3x + 4xy + (y) the D. E. in the form $(0\ 2\ \cos r\ dr + 20\ \sin r\ dO) + \cos r\ dr = 0$. We recognize this as $d(02\ \sin r) + d(\sin r) = d(c)$ or $d(0\ 2\ \sin r + \sin r) = d(c)$. Hence we obtain the solution $0\ 2\ \sin r + \sin r = c$. First-Order Equations $13\ 2\ 7$. Here $M(x,y) = y\ \sec x + \sec x$ and $N(x,y) = \tan x + 2y$. 2 From these we find $M(x,y) = \sec x = N(x,y)$, of 2 s solutions $F(s,t) = c \ 1 \ 1S - s \ t \ 2 + Co = c \ 1 \ or \ s - s = ct$, where $c = c - c \ . \ 1 \ 0$ The solutions of Exercises 12, 13, 14, and 16 follow the pattern of Example 2.6 on page 32. 12. Here $M(x,y) \ Y \ F(x,y)$ such that $F(x,y) - x \ 3x \ 2 \ y^2 - y^3 + 2x$ and $F(x,y) = 2x \ 3 \ y \ y^3 + 2x$, $N(x,y) = 2x \ 3 \ y \$ + 1. 2 2 = 6x y 3y = N (x,y), so D.E. x M(x,y) = 2 3xy + 1. From the first of these, we have F(x,y) = J(3x2y2y3 + 2x)8x + ;(y) = x 3y2 - xy3 + x 2 + ;(y) First-Order Equations 15 3 2 From this, F(x,y) = 2x y - 3xy + ;'(y). But we must y 323 have F(x,y) = 2x y - 3xy + ;'(y) = 2x 3y 3xy2 + 1, or ;'(y) = 2x 3y 3xy2 + 1, or ;'(y) = 2x y - 3xy + ;'(y) = 2x y - 3xy + ;'(y) = 2x 3y 3xy2 + 1. Therefore, 2x y y 3xy2 + 1, or ;'(y) = 2x 3y 3xy2 + 1. 1. Then ;(y) $3 \ 2 \ 3 \ 2 = y + cO$. Thus $F(x,y) = x \ y - xy + x + y + cO$. The one-parameter family of solutions $F(x,y) = c \ 1 \ 1 \ 3 \ 2 \ 3 \ 2 \ x \ y \ xy + x + y = c$, (*) where $c - c \ 1 - cO$. Applying the I.C. y(-2) = 1, we let x = -2, y = 1 In (*), obtaining -8 + 2 + 4 + 1 = c, from which c = -1. Thus the particular solution of the stated I.V. problem $1 \ 3 \ 2 \ 3 \ 2 \ x \ y \ xy + x + y = c$, (*) where $c - c \ 1 - cO$. Applying the I.C. y(-2) = 1, we let x = -2, y = 1 In (*), obtaining -8 + 2 + 4 + 1 = c, from which c = -1. Thus the particular solution of the stated I.V. problem $1 \ S \ 3 \ 2 \ 3 \ 2 \ x \ y \ xy + x + y = c$, (*) where $c - c \ 1 - cO$. Applying the I.C. y(-2) = 1, we let x = -2, y = 1 In (*), obtaining -8 + 2 + 4 + 1 = c, from which c = -1. Thus the particular solution of the stated I.V. problem $1 \ S \ 3 \ 2 \ 3 \ 2 \ -1 \ x \ Y \ xy + x + y = c$. y = or 3232 + y + 1O. (**) xyxy + x = Alternately, the one-parameter family of solutions (*) could also be found by the method of grouping. We write .22332 the D.E. 1n the form (3x y dx + 2x ydy) - (y dx + 3xy dy) + 2x dx + dy = o. We recognize this as d(x 3y2) - d(xy3) + d(x 2) + d(y) = d(c), and so again obtain the one-parameter family of solutions (*) could also be found by the method of grouping. We write .22332 the D.E. 1n the form (3x y dx + 2x ydy) - (y dx + 3xy dy) +
2x dx + dy = o. We recognize this as d(x 3y2) - d(xy3) + d(x 2) + d(y) = d(c), and so again obtain the one-parameter family of solutions (*) could also be found by the method of grouping. We write .22332 the D.E. 1n the form (3x y dx + 2x ydy) - (y dx + 3xy dy) + 2x dx + dy = o. We recognize this as d(x 3y2) - d(xy3) + d(x 2) + d(y) = d(c), and so again obtain the one-parameter family of solutions (*) could also be found by the method of grouping. solutions $3\ 2\ x\ y\ 3\ 2\ x\ y\ +\ x\ +\ y\ =\ c.\ (*)$ The I.C. aga1n yields the particular solution (**). 16 Chapter 2 13. Here $M(x,y) = 2y\ sinx\cos x\ +\ y\ 2y\ sin\ x\ =\ N\ (x,y)$, so D.E. 1S exact. We first seek $F(x,y)\ x\ such$ that $F\ (x,y) = M(x,y) = 2y\ sin\ x\ cos\ x\ +\ y\ 2y\ sin\ x\ =\ N\ (x,y)$ = sin $2\ x\ -\ 2y\ cos\ x$. From these we find $M\ (x,y) = 2y\ sin\ x\ cos\ x\ +\ y\ 2y\ sin\ x\ =\ N\ (x,y)$ = sin $2\ x\ -\ 2y\ cos\ x$. From the first of y these, we have $F(x,y) = M(x,y) = x\ y\ cos\ x\ +\ y\ 2y\ sin\ x\ =\ N\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. From this, $F\ (x,y) = x\ -\ 2y\ cos\ x\ +\ y\ (y)$. cO. The one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = c, (*) where c = c 1 - cO. Applying the I.C. y(O) = 3, we let x = 0, y = 3 in (*), obtaining 3 sin 2 0 - 9 cos 0 = c, from which c = -9. Thus the particular solution of the stated I.V. problem is .22 9 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively, the one-parameter family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln x = . (**) Alternatively family of solutions F(x,y) = c is 1.22 Y Sln x - y cos x = - or 2 . 2 9 y cos x - y Sln parameter family of solutions (*) could also be found by the method of grouping. To do First-Order Equations 17 so, we first write the D.E. in the form (2y sinxcosxdx + sin 2 x dy) + (y2 sinxdx - 2y cosxdy) = o. We recognize this as d(y sin 2 x) + d(_y2 cosx) = d(c) and hence again obtain the solution (*) in the form y sin 2 x - y2 cos x = c. The I.C. aga1n yields the particular solution (**). 14. x x 2 x Here M(x,y) = ye + 2e + y and F(x,y) = we first seek F(x,y) such that F(x,y) = M(x,y) = x x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = x x 2 x ye + 2e + y and F(x,y) = M(x,y) = 0)ax + ;(y) = ye + 2e + xy + ;(y). x From this, F (x,y) = e + 2xy + ;'(y). But we must have y F (x,y) = N(x,y) = eX + 2xy + ;'(y) = O. Then ;(y) = cO. Thus F(x,y) = x x 2 ye + 2e + xy + cO. The one-parameter family of solutions F(x,y) = c 1 IS x x 2 (*) ye + 2e + xy = c, where c = c 1 - cO. Applying the I.C. y(O) = 6, we let x = 0, y = 6 in (*), obtaining 6eO + 2e O + 0(6 2) = c, from which c = 8. Thus the particular solution of the stated I.V. problem 1S x x 2 ye + 2e + xy = 8 or x 2 x e y + xy + 2e = 8. (**) 18 Chapter 2 Alternatively, the one-parameter family of solutions (*) could also be found by the method of grouping. To do so, we first write the D.E. in the form (yeXdx + eXdy) + (y 2 dx + 2xydy) + 2e x dx = o. We recognize this as d(eXy) + d(2e x) = d(c) and hence once again obtain the solution (**). 16. -2 / 3 -1 / 3 1 / 3 4 / 3 -2 / 3 Here M(x,y) = x y + 8x y, N(x,y) = 2x Y - x1/3y-4/3. From these we find My(x,y) = x - 2/3y-4/381/3 -2/3 + 3x y = Nx(x,y) so the D.E. is exact. We first -2 / 3 -1 / 3 F(x,y) = M(x,y) = x y + x - N() - 2 4/3 -2/3 1/3 -4/3 - x, y - x y - x y. seek F(x,y) such that 8x 1 / 3y1/3 and F (x,y) y From the first of these, we have F(x,y) = fM(x,y)Ox + f -2 / 3 -1 / 3 1 / Fy(x,y) = x 1 / 3 y - 4/3 + 2x 4 / 3 y - 2/3 + ;'(y). But we must have Fy(x,y) = N(x,y) = 2 4/3 - 2/3 1/3 - 4/3 ? 4/where c 2 = c 1 - cO. We can simplify this slightly by dividing through by 3 and replacing c 2/3 by c, thus obtaining 1/3 - 1/3 2 4/3 1/3 x y + x y = c. (*) First-Order Equations 19 Applying the I.C. y(l) = 8, we let x = 1, $Y = 8 \ln (*)$ to obtain + 4 = c, from which c = 9/2. Thus the particular solution of the stated I.V. problem IS 1/3 - 1/3 2 4/3 1/3 x y + x y = c. (*) First-Order Equations 19 Applying the I.C. y(l) = 8, we let x = 1, $Y = 8 \ln (*)$ to obtain + 4 = c, from which c = 9/2. Thus the particular solution of the stated I.V. problem IS 1/3 - 1/3 2 4/3 1/3 y + x y = c. (*) y = or 2x 1 / 3 y - 1/3 + 4x 4 / 3 y 1/3 = 9. (**) Alternatively, the one-parameter family of solutions (*)
could also be found by the method of grouping. To do so, we first write the D.E. In the form (x- 2 / 3 y - 1/3 d 2 4/3 - 2/3 d) - 0 H e x y y + x y x + x y y - . w 1 / 3 - 1 / 3 4 / 3 1 / 3 recognize this as d(3x y) + d(6x y) = d(c 2) and hence obtain the solution 3x 1 / 3y - 1/3 + 6x 4 / 3y 1/3 = c 2. Once again, this quickly reduces to (*), and the I.C. again yields the particular solutions (**). 18. (a) Here M(x,y) 2 2, N(x,y) 3 + 4xy. From = Ax y + 2y - x - these we find M (x,y) = 2 and N (x,y) 2 Ax + 4y = 3x + y x 4y. The given D.E. IS exact if and only if M (x,y) - Y N (x,y) , 1.e., if and only if Ax 2 + 4y = 3x 2 + 4y. x So the given D.E. lS exact if and only if A = 3. Substituting 3 for A in the glven D.E. lS exact. We (3x y + 2y) dx + (x + now proceed to solve this D.E. 22 3 4xy) dy = 0. We (3x y + 2y) dx + (x + now proceed to solve this D.E. 22 Here M(x,y) = 3x + 4y = N(x,y), so y x 20 Chapter 2 the D.E. lS exact. We seek F(x,y) such that F(x,y) = x M() 3 2 2y 2 d F() N() 3 4 xy = x y + an x, y = x + xy. y From the first of these, we find F(x,y) = x + 4xy + ;'(y). Therefore x + y 3 4xy + ;'(y) = x + 4xy, or ;'(y) = 0. Then ;(y) = 0. Then ;(y) = x + 4xy + ;'(y) =parameter f 0 1 f 1 0 F() 0 3 2 2 aml y 0 so utlons x, y = c 1 lS x y + xy = c, where c = c 1 - cO. Alternatively, by the method of grouplng, we first 232 write the D.E. in the form (3x y dx + x dy) + (2y dx + 4xydy) = o. We recognize this as d(x 3 y) + d(2xy 2) = d(c) or d(x 3 y + 2xy2) = d(c). Hence we obtain the 1 . 3 2 2 so ut10n x y + xy = c. 20. (b) x 2 3x x Here N(x,y) = 2ye + y e, so N(x,y) = 2ye + y e, so N(x,y) = 2ye + x 3y2e3x. For the stated D.E. to be exact ls 2x 3 3x M(x,y) = y e + y e + ;(x). Hence the most general function M(x,y) such that the stated D.E. is exact ls 2x 3 3x M(x,y) = y e + y e + ;(x). (x), where x is an arbitrary function of x. First-Order Equations 21 21. 2 Here M(x,y) = 4x + 3y, N(x,y) = 4x + 3x y, N(x,y) = 6y f 2y = N (x,y), the D.E. IS not y x exact. (b) We multiply the given equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation, n+1 n 2 n+1 we have M(x,y) = 4x + 3x y, N(x,y) = 4x + 3x y, N(x,y) = 4x + 3x y dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation, n+1 n 2 n+1 we have M(x,y) = 4x + 3x y, N(x,y) = 4x + 3x y dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 ... (4x + 3x y) dx + 2x ydy = 0. For thIS equation through by x n to obtain n+1 n 2 n+1 . 2x y. For this equation to be exact, we must have M(x,y) = 6x n y y 6 = 2(n + 1), from n = 2(n + 1). x + xy + cO. The one-parameter family of solutions F(x,y) = 0 in the form 4x dx + (3xy dx + 2x 3y dy) = 0. We recognize this as d(x 4) + d(x 3y 2) = d(c), and hence we obtain the solutions x 4 + x 3y 2 = 0. c. 22 Chapter 2 23. (a) Here M(x,y) = 2 y)-x. 2 f' (x + 222 Y + x f(x + y) and N(x,y) = y f(x + Since M(x,y) = 1 + 2xy f'(x 2 + y2) f 2xy y 2 y) - 1 = N(x,y), the given D.E. is not exact. x 2 2 (b) Ye multiply the glven D.E. is not exact. x 2 2 (b) Ye multiply the glven D.E. is not exact. x 2 2 (c) Ye mul we have 2 2 M(x,y) = y xf(x + y) and 2 2 + 2 2 x + y x + y 2 2 N(x,y) yf(x + y) x = -2 2 2 2 x + y x + y From these we find 2 2 2 2 2 2 2 2 M(x,y) = 2xy(x + Y)f'(x + Y) - 2xyf(x + y) + x - y y(2 2) 2 x + Y = N(x,y) x So the D.E. of part (b) is exact, and hence <math>2 2 l/(x + y) lS an I.F. of the given D.E. First-Order Equations 23 24. Applying Exercise 23(a) with f(x 2 + y2) = (x 2 + y2)2, we see that the given D.E. is not exact. By Exercise 23(b), we know that 1/(x 2 + y2) is an I.F. of the given D.E. 2 Hence we multiply the given D.E. [x 2 : y2 + x(x 2 + y2)]dX + [y(X 2 + y2) x 2 : 1]dY = 0, 2 2 which is therefore exact. Here M(x,y) = y/(x + y) + x(x + y) $2 + y^2$, $N(x,y) = y(x^2 + y^2) - X(x^2 + y^2)$, $M(x,y) = Y(x^2 + y^2) + X(x^2 + y^2)$, $M(x,y) = Y(x^2 + y^2) + X(x^2 + y^2)$, $M(x,y) = y(x^2 + y^2) + X(x^2 + y^2)$, $M(x,y) = X(x^2 + y^2)$, $M(x,y) = X(x^2 + y^2) + X(x^2 + y^2)$, $M(x,y) = X(x^2 + y^2)$, M(x,y) =(y). From this, F(x,y) = -x/ex + y) + y 2 2 3 x y + ;'(y). But we must have F(x,y) = N(x,y) = x y + y y 2 2 3 d(y) 3 - x/(x + y). Therefore, $(J'(y) = y \text{ or } Y'dy = Y \cdot 4 \text{ Then}$, ;(y) = y /4 + cO. Thus $F(x,y) = \arctan(x/y) + 422 4 x/4 + x y/2 + Y/4 + cO'$ or more simply, $F(x,y) = \arccos(x/y) + (x + y)/4 + cO$. The one-parameter family 222 of solutions $F(x,y) = c \ IS \ arc \ tan(x/y) + (x + y)/4 = c, where \ c = c \ 1 - cO. 24 \ Chapter \ 2 \ Section \ 2.2, Page \ 46 \ The equations \ ln \ Exercises \ 1-7 \ and \ 15-17 \ are \ separable, and \ 18-20 \ are \ homogeneous. 2. Since \ xy + 2 = x(y + 2) + l(y + 2) = (x + 1)(y + 2)(x + 2)($ separable. We first separate variables to obtain (x + 1)dx + dy - 0 Next x + 2 + 2x + 2 + 2x + 2 + 2x + 2 + 2x +an In term). W multiply by 2 and simplify to 2 2 2 obtain Inlx + 2xl + In(y + 2) = Inc 1 2. From this, we have Ix 2 + 2xl(y + 2)2 = c. If x 0 (or x -2), this may be 2 2 expressed somewhat more simply as (x + 2x)(y + 2)2 = c. If x 0 (or x -2), this may be 2 2 expressed somewhat more simply as (x + 2x)(y + 2)2 = c. If x 0 (or x -2), this may be 2 2 expressed somewhat more simply as (x + 2x)(y + 2)2 = c. If x 0 (or x -2), this may be 2 2 expressed somewhat more simply as (x + 2x)(y + 2)2 = c. If x 0 (or x -2), this may be 2 2 expressed somewhat more simply as (x + 2x)(y + 2)2 = c.
If x 0 (or x -2), this may be 2 and simplify to 2 a content of the function of the fun 1) + ds/(s2 + 1) = o. Next we integrate. By a well-known formula, f $ds/(s2 + 1) = arc \tan s$. We next apply the same formula with s = r 2, ds = 2r dr. Thus we obtain f2r $dr/(r 4 + 1) = arc \tan r 2$. First-Order Equations 25 Hence we find the one-parameter family of solutions ln the form 2 arc $tan r + arc \tan s = arc \tan r 2$. for the arbitrary constant, Since each term on the left IS an arc tan). We could leave the solutions in this form, but they are unweildy. We take the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side of (*), applying the formula $\tan(A + B) = 1$ the tangent of each side (*), applyi tan r) tan (arc tan s) which reduces to 2r + s 2 1 - r s 2 = c or r + s = c(1 - r 2 s). 6. The D.E. is separate variables to d e V dv bt. co s u u 0 N t. t t o aln. 1 + = . ex we ln egra e: Sln u + e V + 1 f cos u du sin u + 1 v f V e dv = In(sin u + 1) and e + 1 = In(e v + 1). Hence we find the one-parameter family of solutions in the form In(sin u + 1) and e + 1 = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + I = In(e v + 1) and e + In(e v + 1 u + 1 + In(e v + 1) = In c (where we write In c for the arbitrary constant, since each term on the 26 Chapter 2 left is a In term). We simplify to obtain In[(sin u + 1) (e v + 1) = c. 7. This equation is separable. We first separate variables to obtain (x + 4) dx j(x 2 + 3x + 2) + y dy j(y 2 + 1) = O. Next we integrate: f dy = ; In (y2 + 1). To integrate y + 1 the dx term, we use partial fractions. We set x + 4 = xJ x + 1 L: 12) we obtain solutions ln the form In 2 + 2 In (y + 1 (x + 2) = In c 1 (where we write In c 1 for the arbitrary constant, 81nce each term on the left 18 a In term). We multiply by 2 and simplify to obtain In (x + 1)6 + In (y + 1) + (x + 2)6 2 1) (y + 1) = c (x + From this, we have (x + 1)6 + In (y + 1) + (x + 2)6 2 1) (y + 1) = c (x + From this, we have (x + 1)6 + In (y + 1) + (x + 2)6 2 1) (y + 1) = c (x + From this, we have (x + 1)6 + In (y + 2)4. 9. We first write the D.E. in the form 2 = 2xy + 3y and 22xy + x thence dx = 2(y/x) + 3(y/x) 2(y/x) + 1 In this form we recognize First-Order Equations 27 the D.E. 1S homogeneous. We let y = vx. Then = v + x dv and v = y dx x We make these substitutions to obtain dv 2v + 3v 2 dv 2 2v + 3v v + x - = or x - = -v = dx 2v + 1 dx 2v + 1 2v + 3v2 v 2 dv 2 - vv + v We now separate 2v + 1 or $x = 1 \cdot dx 2v + variables$ to obtain (2v + 1) dv = dx 2 x v + v obtain Inlv 2 + vi = Inlcxl. We now resubstitute We integrate to 2v = Y to obtain L + y = I cxl. We simplify to obtain x 2 x x Iy 2 + xyl Icxlx 2 from which we find 2 3 = , y + xy = Inlcxl. From this, we have Iv 2 + vi = Inlcxl. We now resubstitute We integrate to 2v = Y to obtain L + y = I cxl. We simplify to obtain x 2 x x Iy 2 + xyl Icxlx 2 from which we find 2 3 = , y + xy = Inlcxl. $cx \cdot du = 23$ 10. We first write the D.E. the uv - u from ln form dv = 3v which we at once obtain du = -v recognize that the D.E. IS homogeneous in x and y, we introduce a new variable w by du dw u letting u = wv. Then dv - w + v dv and w = v du u Substituting into the D.E. dv = v dw 3 dw 3 v dv = w - w" Hence v dv = -wand separating we obtain $-2 w_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I
+ 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we have $2 v_2 = 1 n I v I + 2c 1$ " Resubstituting w u = v ' we ha v + c. u 28 Chapter 2 11. Ye first write the D.E. in the form = tan() + . In this form we recognize that the D.E. is homogeneous. We let y = vx. Then = v + x and v = x. Then = v + x and v = x and v = x and v = x. Then = v + x and v = x and v = x and v = x. Then = v + x and v = x and v = x and v = x. Then = v + x and v = x and v = x. Then = v + x and v = x and v = x. Then = v + x and v = x and v = x. Then = v + x and v = x. Then v = x. Then v = x and v = x. Then v = x. Then v = x and v = x. Then v = x and v = x. In I x I + In I c I, I cxl. or In I sin v I = In I cx I. From this we have I sin v I = We now resubstitute v = Y to obtain dy = dx We then divide x and y, this may be expressed somewhat more simply as sin y = cx. x 13. We first write the D.E. In the form dy dx = x3 + y2 j x 2 + y2 and denominator by x 3 to obtain dy = dx We then divide numerator $21 + Lj 2 2 3 x + y x j x^2 + y^2 x = ... fj 2 1 Y 2 + .x + y x x y 1 j x 2 + 2 - x x y "... 2 Assuming x > 0$ so that x = J':2, this becomes = dx 1..., 2L, 1 + y J:2 i x 2 2 x + y [] 1 J':2 j x 2 x + y [] 1 J':2 j x 2 xv = 1 obtain (1 + v 2) 3/2 = lnlxl + lnicol (where we have chosen to write Inicol for the arbitrary constant). We multiply by 3 and simplify to obtain (1 + v 2) 3/2 = lncx 3, where c = I c o 1 3. Ve now resubstitute v = to obtain (1 + v 2) 3/2 = lncx 3 and simplify to obtain (1 + v 2) 3/2 = lnc + ()2)3/2 = In c x 3. Ve simplify to obtain [x2:2y2]3/2 = In c x 3 or Ix2 + lr/2 = 3 x x 3 In c x 3. 3 (22) 3/2 In c x, from which we find x + y = 14. This D.E. is homogeneous. Recognizing this, we could let y = vx and substitute in order to separate the variables. However, the resulting separable equation is not readily tractable. This being the case, we assume x > y > 0 and 30 Chapter 2 divide the entire equation through by. Then we solve for dx/dy, putting the D.E. in the form dx dy = V x/y + 1 + V x/y - 1 V x/y + 1 + V x/y - 1 V x/y + 1 + V x/y - 1 V u + 1 + V x/y - 1 V u + 1 + V x/y - 1 V u + 1 + V u +u = y x/y, we can write this more simply as In(u + j u 2 1) = In(c/y), from which we at once have u + j u 2 - 1 = c/y which readily simplifies to $x + j x 2 - y^2 = c$. First-Order Equations 31 17. The D.E. is separable. We separate variables to obtain (3x + 8) 2x + 5x + 6 4y dy = 2 0. y + 4 tegrate the dx term, we use partial fractions. 3x + 8 We write 2x + 5x + 6 B 3' and so 3x + 8 = A(x + 3) + = 3x + 8 A = + (x + 2)(x + 3) x + 2 x + B(x + 2). Then x = -2 gives A = 2; and x = -3 gives B = 1. f 2x Thus we find 3x + 8 dx = + 5x + 6 2 f x d + x 2 + f x d + x 3 = 2 In I x + 21 + In I x + 31 = In(x + 2)(x + 3) x + 2 x + B(x + 2). + 3). Using this, we obtain solutions in the form In (x + 2)2(x + 3) - 2 In $(y^2 + 4) = lnlcl$ or In $\{x + 2, 2\}(x + 3) = c(y^2 + 4)$. We now apply the I.C. y(l) = 2 to this, obtaining 36 = 64c, c = ; 6 'Thus we find the particular solution $16(x + 2)2(x + 3) = 9(y^2 + 4)$. 19. We first write the D.E. in the form dy = 5y - 2x or dx 4x - Y dy = 5(y/x) - 2 We that this D.E. dx 4 - y/x. recognize IS homogeneous. We let Then = v dv and y = vx. + x - dx v = y/x. Making these substitutions, the D.E. dx 4 - y/x. recognize IS homogeneous. We let Then = v dv and y = vx. + x - dx v = y/x. Making
these substitutions, the D.E. dx 4 - y/x. The determinant of the determinant o we integrate: 32 Chapter 2 jI = lnlxl. To integrate the dv term, we use partial fractions. We set 4 - v + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). Then v - 2 1 + v - 2 B so 4 - v = A(v 1) + B(v + 2). solutions in the form (v + 2) In J v - 1 = In I xl + In c or In J v - 1 1 = clxl (v + 2) from which we obtain Iy/x - 11 = clxl (v + 2) We resubstitute v = y/x to obtain Iy/x - 11 = clxl (v + 2)? from which we obtain Iy - xl = clxl (v + 2)? from which we obtain Iy - xl = clxl (v + 2)? 2x)2. Taking y x, this may be expressed somewhat more simply as y - x = c(y + 2x)2. Applying the I.C. y(l) = 4 we let x = 1, Y = 4 and find $3 = c(6 \ 2)$, from which we find c = 1/12. We thus obtain the particular solution y - x = (1/12)(y + 2x)2, or (2x + y)2 = 12(y - x). 20. We first write the D.E. In the form dy $3x \ 2 + 9xy + 5y2$ dx = $6x \ 2 + 4xy$ or First-Integrating, we find 2 Inlv 2 + 3v + 31 = In Ix I + Inlcl or 2 2 2 2 In (v + <math>3v + 3) = Inlcxl. From this, (v + 3v + 3) = Inlcxl. From this, (v + 3v + 3) = Inlcxl. We resubstitute v = y/x to obtain [2 2] 2 Y + 3: + 3x = lex I. We simplify, taking IxI = x > 0, thereby obtaining $(y^2 + 3xy + 3x + 2)^2 = 5 cx$. Applying the I.C. y(2) = -6, we let x = 2, Y = -6 and find 144 = 32c or c = 9/2. e thus obtain the particular . 2 225 solution 2(y + 3xy + 3x) = 9x. 34 Chapter 2 21. (b) Here M(x,y) = 2 Dx 2 + Exy + = Ax + Bxy + Cy , N(x,y) = 2 Dx + Ey. Now the given homogeneous D.E. 18 x exact if and only if M(x,y) = N(x,y), i.e., if and y x only if Bx + 2Cy = 2Dx + Ey. But Bx + 2+ Ey if and only if B = 2D and 2C = E. Therefore, we have it that the given homogeneous D.E. is exact if and only if B = 2D and E = 2C. 23. (a) The given D.E. is of the form (Ax 2 + Bxy + Cy2)dx + 2 2 (Dx + Exy + Fy)dy = 0, where A = 1, B = 0, C = 2, D = 0, E = 4, F = -1. From exercise 21(b), we know the D.E. IS homogeneous. Also, since B = 0 = 0. 2D and E = 4 = 2C, we know from Exercise 21(b) that the given D.E. is also exact. First, let us solve the given D.E. as a homogeneous D.E. We first write the D.E. In this form $y_2 = x + 2y$ or z = 1 + 22(y/x). In this form $y_2 = 4xy(y/x) - 4(y/x)$ we recognize that the D.E. is homogeneous. We let y = vx. Then z = v + x: and v = y/x. We make dv these substitutions to obtain v + x - = dx 2 1 + 2v 2 v - 4v or dv 1 2 3 + 6v - v W e now separate variables to x - = dx 2 v - 4v 2 dv dx obtain (v - 4v) W e integrate to obtain 11 + 6v 2 - v 3 1 = -3 In Co Ixl. We multiply by -3 First-Order Equations 35 and simplify to obtain 11 + 6v 2 - v 3 1 = -3 In Co I x I or Inl1 + 6v 2 - v 3 1 = co-3Ixl-3 or 1 1 + 6v 2 - v 3 1 = cc-3Ixl-3 or 1 1 + 6v 2 - v 3 1 = cc-3Ixl-3 or 1 1 + 6v 2 - v 3 1 = cc-3Ixl-3 or 1 1 + 6v 2 - v 3 1 = cc-3Ixl-3 or 1 1 + 6v 2 - v 3 1 = cc-3Ixl-3 or 1 + 6v 2 - v 3 1glven D.E. as an exact D.E. 222 Here M(x,y) = x + 2y, N(x,y) = 4xy - y. From the first of and F(x,y) = M(x,y) = 4xy - y. From the first of and F(x,y) = M(x,y) = 4xy - y. From the first of and F(x,y) = 4xy - y. But we must have F(x,y) = N(x,y) 4xy - 2 - Y - . Y 4xy - Y 2 or ;'(y) 2 So = -y . 3 2 3 F(x,y) x 2xy Y - + - + c . - 3 3 0 Therefore 4xy + ;'(y) = 3 ;(y) = -Y3 + cO. Thus The one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where We multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2. So y = c, c = the one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where We multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2. So y = c, c = the one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where We multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2. So y = c, c = the one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where We multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2. So y = c, c = the one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where We multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2. So y = c, c = the one-parameter family of solutions F(x,y) = c 1 IS 3 2 3 x 2xy Y - c where W = multiply - + - 2'c 2 = c 1 - cO. 3 by 3 to obtain x - 2 + c - 2 - c family of solutions of the given D E . 3 3 6 2 . . IS X - Y + xy = c. 36 Chapter 2 25. Since the D.E. is homogeneous, it can be expressed in the form = q(). Let x = r cos e dr = r cos de - (t an successively -r sin e + cos e dr/de - g e), dr . dr r cos () + sin () d () = g (t an ()) [-r s 1 n e + cos e d e], dr [sin () - cos () g(tan ())] d() = -[g(tan e) - s 1 n and e. 26. (a) The D.E. of Exercise 8 is (x + y) dx - xdy = O. This can be written = 1 + y/x, and so 1S homogeneous. Using the method of Exercise 25, we let $x = r \cos e$, e Th dy r cos e de + sin e dr -r sin e de + cos e dr = 1 + tan e, r cos e de + sin e dr = (1 + tan e) (-r sin e de + cos e dr), [sin e - (1 + tan e) cos e] $dr = -r[\cos e + (1 + \tan e) \sin e] de$, First-Order Equations 37 $dr \cos () + 1 + \tan () s. in () d() = (s + c) + tan () d$ $n \cos() + 1 n c$, where c > o. Now resubstitute, ac cording to $x = r \cos()$, y = rsin(); that is, let r = j x 2 + y2, tan() = y/x. We obtain successively ln j x 2 2 y/x - In(x/j x 2 + y2) In c, + y = ln j x 2 2 + ln x/j x 2 + y2 + In c = y/x, $+ y \ln c = y/x$, or finally $y = x \ln(cx)$. Section 2.3, Page 56 The equations of Exercises 1 through 14 are linear. In solving, first express the equation in the standard form of equation (2.26) and then follow the procedure of Example 2.14. 38 Chapter 2.1. The D.E. is already ln the standard form (2.26), with P(x) = 3/x, Q(x) = 6x 2. An I.F. is eJP(x)dx = eJ(3/x)dx = 3e3lnlxl = elnlxl = Ixl 3 = *x 3 (+ if x 0, - if x < 0). Ve multiply the D.E. through by this I.F. to obtain x3 3 2 6 $5 d(3) 5 I \cdot b \cdot 3 + x = x \text{ or } dx \ y = 6x$. ntegrating, we 0 taln x y 6 3-3 = x + e or y = x + ex 3. The D.E. is already in the standard form (2.26), wih P(x) =
3, Q(x) = 3x 2 e -3x. An I.F. is e J P(x) dx = e J 3 dx = e 3x Ye multiply the D.E. through by the I.F. to obtain 3x dy 3 3x 3x 2 e dx + e y = d (3X) 2 Integrating we obtain 3x 3 or - e y = 3x . e y = x + e or y = dx y = (x 3 +) -3x or $e e \cdot 6 \cdot 2$ This equation is linear in v. Ye divide through by u + 1 dv 4u 3u to put it in the standard form du + 2v = 2u + 1u + 1 with P(u) = 4u and Q(u) = u 2 + 1 3u u 2 + 1 An I.F. IS eJP(u)du = f 4u exp 2u + du 1 = e 2ln(u 2 + 1) = 22 (u + 1). Ye multiply the standard form equation through by this to . (22 dv 22 obtaln u + 1)du + 4u(u + l)v = 3u(u + 1) or d 2 2 3 du [(u + 1) v] = 3u + 3u. Integrating, we obtain 2 2 3u 4 3u 2 (u + 1) v = + 2 + e. First-Order Equations 39 7. dy 2x + 1 Ye first divide through by x to obtain dx + 2y = x + x x - 1 x h P() 2x 2 + 1 and Q(x) - x X - 1 were x = x + x An I.F. Is $e_1p(x)dx = exp[::: dX] = e_1n(x + 1) v = + 2 + e$. lor x > 0, - if -1 < x < 0). In any case, upon multiplying the standard form equation through by the I. F., we obtain (x 2 + x) + (2x + 1) 2 - 1 that or (x + x) dx + y = x - 1. (x + = 323 x/3 - x + c/3 or 3(x + x)y = x - 3x + c. 8. 2 Ye first divide through by x + x - 2 to put the equation th t d d f dy 3(x + 1) 1 U here ln e s an ar orm dx + (x + 2) (x - 1) Y = x + 2' " 3 (x + 1) d 1 P(x) = (x + 2)(x - 1) x elnlx+21+21nlx-11 = Ix + 21(x - 1)2 = *(x + 2)(x - 1)2, (+ if x -2, - if x -2), where partial fractions have been used to perform the integration. In either case (x > -2 or x -2), upon multiplying the standard form equation through by the 1. F., we obtain (x + 2)(x - 1)2 + 3(x + 1)(x - 1)2y = 3 {x - 1 + c (x - 1) - 2 . 40 Chapter 2 9. Ye first put the equation in the form dy xy + y - 1 = dx + x 0, and then ln the standard form +(1 + ;)y = ;, where P(x) = 1 + 1/x and Q(x) = 1/x. An I.F. 18 eJP(x)dx = e(1+1/x) dx = ex + lnlxl = Ixle x = *xe x, (+ if x 0, - if x < 0). In either case, we multiply the standard form equation through by the I.F. and obtain x dy x x xe dy +(x + 1) e y = e d x x x x or dx [x e y] = e. Integrating, we find x e y = e + c - 1 - x. or y = x (1 + c e). 10. This D.E. is linear ln x (like Example 2.16). We divide through by y and dy to put it in the standard form 2 dx xy dy + + x y - y = 0, dx or - + dy I. F. is (y +)x = 1, where P(y) 1 - y + y' Q(y) = 1. An 2 2 eJP(y)dy = eY/2 + 1nlyl = lyle Y/2 = 2/2 :I: ye y (+ if y 0; - if y < 0). Mult iplying the standard form equation through by this, we obtain First-Order Equations 41 y2/2 dx 2 y2/2 we dy + (y + 1)e x 2 / 2 = ye Y d 2/2 or dy (ye Y x) = 2 / 2 e Y + e or xy = 2 / 2 e Y vide through by eos 0 to put dr 3 the equation ln the standard form dO + (tan 0) r = eos 0, where P(O) = tan 0 and Q(O) = eos 3 0. An I.F. is e J P (0) dO = e In I see 0 I = I see 0 I s multiplying the standard form equation through by the I.F., we obtain see 0 + (see Otan O)r = eos 2 0, that is, d 2 dO [1" see O] = eos O. Upon integrating, we find r see 0 = 0 + sin 20 + cO' where the right-side was integrated as follows: f eos 2 0 d 0 = f 1 + e 2 0s 2 0 d 0 = 0 + sin 20. Substituting 2 sin () eos 0 for sin 2 (), the solutions may be expressed as r see $0 = 0 + \sin 0 \cos 0 + cO'$ Finally, we mult iply through by 2 eos () to obtain $2r = (0 + \sin x) + (\cos x)y = 2 \cos x$, and then ln the standard form dy cos x dx + 1 + sin x 2 cos x Y = 1 + sin x ' where P(x) cox x = 1 + sin x ' Q(x) = 1 2 cos x + Sln x An I.F. IS f cos x dx /P(x)dx 1 + sin x I = 1 e = e = e = + sin x, since 1 + Sln x > o. Multiply the standard form equation through by this, to obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + sin x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + sin x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. $\sin x$) y = 2 + 4 + c 1 or 2(1 + sinx)y = x + sinxcosx + c, where c = 2c 1 and we have employed double-angle formulas. 14. This D.E. IS linear ln y. We first divide through by dy to obtain y sin 2x - cos x + (1 + sin 2 x) = 0 or $(1 \cdot 2) dy (\cdot 2) N d \cdot d + Sln x dx + Sln x y = cos x$. ext we IVI e through by 1 + sin 2 x to put the equation in the standard sin 2x + (sin 2x)y = cos x, that lS, d: [(1 + sin 2 x)y] = cos x. Integrating, we find (1 + sin 2 x)y = sin x + c. 15. This lS a Bernoulli D.E., where n = 2. Ye multiply -2 -2 dv 1 -1 1 through by y to obtain y - x y - x Let l-n -1 dv -2 v = y = y; then dx = -y dx. The preceeding D. E. dO lf 0h 10. dv 11 rea 1 y trans orms into t e lnear equation - d + -v = xx x An I.F. lS e Jdx / x = elnlxl = Ixl = *x. Multiplying through by this, we find x : + v = 1 or d: (xv) = 1 + cx - 1. 18. This is a Bernoulli D.E. in the dependent variable x and independent variable t, where n = -1. Ye multiply through o dx t + 12t + 11 - n by x to obtaln x dt + 2t x = t Let v = x 2 dv dx x; then dt = 2x dt. The preceeding D. E. readily = 44 Chapter 2tf. h 1. dv rans orms lnto t e lnear equation dt + t + 1 tv = t + 1 dt 2(t; 1). An I.F. is ef 1 = et+lntl = It let = *t e t. Multiplying through by this, we find t e t + tt d tt (t + 1) e v = 2 (t + 1) e or dt [t e v] = 2 (t + 1) e . I. b. t 2 t 2 ntegrat lng, we 0 taln t e v = t e + c or v = + c t - 1 e - t. But v = x 2. Thus we obtain the solution ln 2 - 1 - t the form x = 2 + c t e. The equations of Exercises 19 through 24 are linear. 21. x Ye divide thru by (e + 1) - 3(e + 1)] + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the
standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1) + dx = 0 and thence in the standard form dy - + dx + 1 + dx = 0 and thence in the standard form dy - + dx + 1 + dx = 0 and thence in th 1), $y \in X + 1$ where $P(x) = x \in Q(x) - 3e \times (e \times + 1)$. An I.F. IS $JP(x)dx \in x \in + 1 \times e = 1$ n (e + 1) + c or x 2 x - 1 Y = (e + 1) + c (e + 1) + + 1). (*) First-Order Equations 45 Ye apply the I.C. y(0) = 4: Let x = 0, y = 4 in (*) to obtain 4 = 4 + c/2. Thus c = 0, and the particular x 2 solution of the stated I.V.P. IS $Y = (e + 1) \cdot 24$. This equation is linear in the dependent variable, and is already in the standard form, with pet) = -1, $Q(t) = \sin 2t$. An I.F. IS eJP(t)dt = eJ(-1)dt = -t eYe multiply the equation -t dx - t - t through by this to obtain edt - ex = eSln 2t or d - t - t dt (ex) = esin2t. Ye next integrate, using integration by parts twice, or an integrate, using integrate, using integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t + 2 cos 2t. Ye next integrate, using integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t. Ye next integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t + 2 cos 2t. Ye next integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t. Ye next integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t. Ye next integrate eVe (ex) = esin2t + 2 cos 2t + 2 cos 2t. Ye next integrate eVe (ex) = eSin2t + 2 cos 2t + 2 cos 2t. Ye neve (eVeobtain $0 = -\frac{1}{5}(0 + 2) + c$ from which $c = \frac{2}{5}$. Thus the general solution of the stated I.V.P. is $x = 2t \cdot 222 e - Sln t - cos t 5 25$. This is a Bernoulli D.E. with n - 3. Ye first multiply 3 3 d y 4 1-n throug h b y Y to obtain Y + y - x Now let v = Y dx 2x - . = y 4. Then := 4y3 and the preceeding D. E. 46 Chapter 2 transforms into :+ 2v x = x, which is linear In v. In dv 2 the standard form this linear equation is dx + xv = 4x, with P(x) = 2/x, Q(x) = 4x. An I.F. is eJP(x)dx = e J (2/x)dx = e J (2/x solutions of the given Bernoulli Equation in the form 244 x y = x + c. (*) We apply the I.C. y(I) - 2. Let x = 1, $Y = 2 \ln (*) - to obtain 16 = 1 + c$, so c = 15. Thus the particular solution of the stated I.V.P. 2 4 4 + 15. IS x y = x 26. Ve first write the D. E. In the form x + y = x 3 / 2 y 3/2 and multiply through by I/x to obtain dy + 1. y = x 1 / 2 y 3/2. dx $x \neq 0$ recognize this D.E. as a Bernoulli D.E. with n = 3/2. Ve now multiply through by y -3/2 to obtain y $-3/2 + 1 - 1/2 1/2 1 - n - 1/2 x y = x \cdot Now let v = y = y$. Then := -; Y -3/2 and the preceeding D.E. transforms into -2 dv dx 1 + -v x 1/2 h. h. 1... = x, W lC IS lnear In v. In the standard First-Order Equations 47 dv (-2x 1) V = x1/2 form this linear D.E. IS dx + 2' with P(x) = x 1 / 2 An IF. JP(x)dx = 2... IS e - e 1 1 I 1 - 1/2 - 1 / 2 ln x ln x 1 1 - 1/2 - 1 / 2 ln x ln x 1 1 - 1/2 - 1 / 2 ln x ln x 1 1 - 1/2 - 1 / 2 ln x ln x 1 1 - 1/2 - 1 / 2 ln x ln x 1 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x 1 - 1/2 - 1 / 2 ln x ln x - 2 x + c. But v = Y - 1 - 2 x + c. But= Hence we obtain the one-parameter family of solutions of -1/2 - 1/2 the glven Bernoulli equation in the form x y = 1 2 x + c or 1 fXY 1 = -2 + c, so c = -1 + c, so c = -2 + c, so c = For 0 < x < 1, (II) For x > 1, dy y = 2, dy y = 0, - + - + dx dx y(0) = 0, Y (1) = a, where a is the value of lim (x) and denotes the x+1- solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 1. 48 Chapter 2 We first solve (I). The D.E. of this problem 1S linear in standard form with P(x) = 1, Q(x) = 2. An I.F. is eJP(x)dx = eJ(l)dx = eX. Ve multiply the D.E. of (I) h h b h . b . x dy x x d [X] t roug y t lS to 0 taln e dx + ey = 2e or dx e y = 2e x . We integrate to obtain eXy = 2e x + c or y = 2 + -x ce We apply the I.C. of (I), y(O) = 0, to this, obtaining 0 = 2 + c or c = -2. Thus the particular -x solution of problem (I) is y = 2 - 2e . This is valid for 0 < x < 1. Letting denote the solution just obtained, we note that $\lim (x) = \lim (2 - 2e - 1)$. This is the x+1 x+1 a of Problem (II); that IS, the I.C. of (II) is y(l) = 2 - 2e - 1. Now solve (II). The D.E. of this problem is linear IS standard form with P(x) = 1, Q(x) = 0. An I.F., as ln x problem (I), is e. We multiply through by this to obtain x dy x d x e dx + e y = 0 or dx (e y) = o. Ye integrate to obtain eXy = c or y = c e - x Now apply the I. C. of (II), y (1) = 2 - 2e - 1 , to this. Let x = 1, Y = 2 - 2e - 1 , to this valid for x > 1. We write the solution of problem (II) is y = (2e - 2)e - x . This is valid for x > 1. We write the solution of the entire problem, showing intervals where each part is valid, as 2(1 - e - x), 0 < x < 1, y = -x 2(e - 1)e, x = -x 2(e - 1)eproblem will be continuous at x = 2. We first solve (I). The D.E. of this problem is linear in standard form with P(x) = 1, Q(x) = e - x An . $fP(x)dx f(1)dx x \cdot I.F.$ IS e = e = e. We multiply the D.E. of (I) through by this to obtain eX + eXy = 1 or d: $(eXy) \times x = 1$. Integrating we obtain eX + eXy = 1 or d: $(eXy) \times x = 1$. Integrating we obtain eX + eXy = 1 or d: $(eXy) \times x = 1$. Integrating we obtain eX + eXy = 1 or d: $(eXy) \times x = 1$. Integrating we obtain eX + eXy = 1 or d: $(eXy) \times x = 1$. obtaining 1 = 0 + c, or c = 1. Thus the particular solution of problem (I) is $y = e \cdot x(x + 1)$. This is valid for 0 < x < 2. lim x 2 Letting denote the solution just obtained, we find $-x \cdot 2$; (x) = lim e (x + 1) = 3e. This is the a of $x \cdot + 2$ Problem (II); that IS, -2 the I.C. of (II) is y(2) = 3e. Now solve (II). The D.E. of this problem is linear in -2 standard form with P(x) = 1, Q(x) = e. An I.F., as in Problem (I), is eX. We multiply through by this to obtain x dy x x-2 e dx + e y = e or dx [e y] = e · We integrate to 50 Chapter 2 obtain x x-2 e dx + e y = e + c or y = e + Thus the particular solution of problem (II) -x -2 is y = 2e + e This is valid for x > 2. Ye write the solution of the entire problem, showing intervals where each part IS valid, as -x e (x + 1), o < x < 2, y = -x - 2 2e + e, x > 2 30. As in Exercises 27 and 29, we actually have two I.V. problems: (I) For x < 3, (II) For x < 3, (X + 1) dy y < 3, - + = x, - + = dx dx y(0) = 1/2. y(3) = a, where a is the value of lim (x) and denotes the x+3 solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 3. We first express the D.E. In the standard form d d x Y + 1 Y = x l ' where P(x) x + 1 x + = l/(x + 1), Q(x) = x/ex + 1). An I.F. is eIP(x) dx = e Idx/x+1 $- e \ln |x+11| - Ix + 11$. Since x + 1 + 1 = x + 1. Ye multiply the standard First-Order Equations 51 form equation through by this to obtain (x + 1) + Y = x, which turns out to be exactly the equation that we started with. Ve note that this is d [(x + 1)y] = x/2 + c. Applying the I.C. y(O) = x/2 + c. Applying the I.C. y(O) = x/2 +
c. 1/2, we obtain c = 1/2. Thus the particular 1 2 solution of Problem (I) is (x + 1)y = 2(x + 1) or y - 2x + 2(x + 11). This is valid for 0 < x < 3. Letting denote the solution just obtained, we find 2)] 5 lim (x) = 1. [x + This the a of Problem 1m 2(x + = 4.1S x 3 x 3 (II); that is, the I.C. of (II) IS y(3) = 5/4. Now solve (II). We put the D.E. In standard form dy + 1 3 and find, as ln Problem (I), that an dx x + 1 y = x + 1 I.F. lS (x + 1). We multiply the standard form equation through by this to obtain (x + 1) + y = 3, which 18 exactly what we started with here ln Problem (II). We d note that this is dx [(x + 1)y] = 3. Integrating we obtain (x + 1) + y = 3, which 18 exactly what we started with here ln Problem (II). We d note that this is dx [(x + 1)y] = 3. Integrating we obtain (x + 1) + y = 3, which 18 exactly what we started with here ln Problem (II). 5/4, -4. Thus the particular solution of Problem (II) is (x + l)y = 3x - 4 or 3x - 4 Y This is valid for x > 3 = x + 1. The solution of the entire problem IS 52 Chapter 2 x + 1 = 0 a a . fP(x)dx f(b/a)dx(b/a)x Multiplying the lS e = e = e. standard form equation through by this, we obtain $e(b/a)x \, y = k e(b/a)x \, y = k e$ (ii), where = a'(1) becomes dx = -bx/a, ax = y = -bx/a, ax = y = -bx/a + cor a a k - bx/a x e y = -bx/a + cor a a k - bx/a +In case (i), y is given by (2); and since e x 0 and e - bx/a 0 as x CD, we have y 0 as x CD. In case (ii), y is given by (3); and since -bx/a 0 - bx/a x e - bx/a - bx/a xP(x)f(x) = 0 + P(x)O = 0 for all $x \in I$, so f(x) is a solution. (b) By the Theorem 1.1, there is a unique solution of the D.E. and obviously it satisfies the I.C. Thus by the uniqueness of Theorem 1.1, we must have f(x) = 0 for all $x \in I$. (c) Since f and g are solutions of the D.E., so is their difference h = f - g(x) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By this means f(x) - g(x) = 0 and hence f(x) = g(x) for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (c) f(x) = 0 for all $x \in I$. By part (b), hex) = 0 for all $x \in I$. By part (c) f(x) = 0 for all $x \in I$. By part (IS a solution of (A), g'(x) + P(x)g(x) = Q(x). (2) Then, subtracting (2) from (1), we have f'(x) - P(x)g(x) = Q(x) - Q(x) or f'(x) - g(x)J = 0 or, finally, [f(x) - g(x)J = 0 or, finally, [By part (a), since f and g are solutions of (A), their difference f - g is a solution of $\frac{dy}{dx} + P(x)[f(x) - g(x)]' + cP(x)[f(x) - g(x)]' + cP(x)[f($ [f(x) - g(x)] = Q(x). Rearranging terms, this takes the form $\{c[f(x) - g(x)] + f'(x)\} = P(x)$. We see from this that $c(f - g) + f(x)\} = P(x)$. We see from this that $c(f - g) + f(x)\} = P(x)$. We see from this takes the form $\{c[f(x) - g(x)] + f'(x)\} = P(x)$. We see from this takes the form $\{c[f(x) - g(x)] + f'(x)\} = P(x)$. We see from this takes the form $\{c[f(x) - g(x)] + f'(x)\} = P(x)$. form (2.41) with fey = Sln y, P(x) = 1 - and Q(x) = 1. Ye 1 et v = f(y) = sin y, from which $x \, dv$ dy dx - $(cos Y) \, dx$. Substituting this into the stated D E o b dv 1 1 h 0 h 0 1 0 0 \cdot ., lt ecomes - d + -v = , W lC lS lnear ln v. x x An I.F. is eJP(x)dx = eJ(1/x)dx = eJ(1/x)dx = eJ(1/x)dx = eJ(1/x)dx = x or d: (xv) = x. Integrating, we 2 2 find xv = x + c, where c = 2c O. Now replacing v by Sln y, we obtain the solution in 2 the form (A) of Exercise 38, with A(x) = -1, B(x) = x, C(x) = 1. The solution f(x) = x is given. By Exercise 38(b), we make the transformation y = f(x) + 1. $x^2/2$ dv - $x^2/2$ e dx - e x v = 2 - x /2 e or 2 d: (e- X /2 v) accordingly and thus obtain the solution, 2 -x /2 e y - x 2 = f e -x /2 dx + c. Section 2.3, Miscellaneous Review Exercises, Page 59 The solution of each of these review problems 1S given, but most of these solutions are presented 1n abbreviated form with many details omitted. 1. The D.E. is both separable and linear. Upon separating variables and integrating, we obtain 2 In (x 3 + 1) + In I c I = Inlyl, from which we find y = c(x 3 + 1)2. The solution as a linear equation IS almost as easy. 3 - 2 The I.F. is $(x + 1) \cdot 2$. The D.E. IS exact. We seek $F(x,y) = 3x 2y^2 - x y^2 - 3x^2 y^2 - x y^2 - 3x^2 y^2 - x + 1 = 1$ 0 Integrat ion gives In I x 1 - y - 1 - . 11 = In c 1. Upon simplifying and find Ix(y - 1)1 = IC 1 (x + 1)1. find [-1]dX x + lnlx + 11 + lnly - taking antilogs, we Assuming x > 0, y > 1 and simplifying, we can write this as xy = c x + (c - 1), or xy + 1 = c (x + 1), where c = 1 + c 1. Alternately, consider the D.E. as a linear equation. The I.F. [x (x d: 1)] = %x. Integration and 2 simplification at once give y = x 4 + x 5. Writin q the D.E. ln the form dy dx 5 [1 - 3 = , 1 + y x we recognize that it IS homogeneous. We let y = xx. Then the D.E. dv 5v - 3 becomes v + x - d = 1. Simplification yields x + v First-Order Equations 59 (v + 1)dv v 2 - 4v + 3 to the left member, we find -dx x Then upon applying partial fractions = [-v 1 +
v - 2 3]dV = -: Integration glves -lnlv - 11 + 2ln Iv - 31 = -lnlxl + Inlc/. Simplification and taking of antilogarthims 2 i v - 3) _ c y results ln l v _ 11 - x But v - x Upon resubstituting accordingly and simplifying, we find (3x - y)2 = Ic(y - x)l. 6. The D.E. IS exact, separable, and linear. Considered as an exact equation, we seek F(x,y) such that F(x,y) = x M(x,y) 2x 2 and F(x,y) = N(x,y) 2x Y - 2y. From the = e y = e y 2 first of these, we find F(x,y) 2x y (y). From = e + 2 this, $F(x,y) = 2x y^2 + (y)$. But since F $= c_1$, 1 multiplied through by 2, and let $c = 2(c_1 - c_0)$]. Assuming $e_2x - 2_0$, this can also be written as $y = c_2$ ($e_2x - 2$)-1/2 where $c_2 = JC$. We also consider the D.E. as a linear equation. We .. ($2x_2$) ' $2x_0$ d th t. rewrite It as $e_2 + e_3 = a_1 + e_3 + e$ 2)1/2, where we assume e 2x - 2 > 0. Multiplying the standard form through by the I.F., we f o d (2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x 2) 1/2 ln e ave 4 + 23 = 0. Integration yields $In(x + 1) + x + 1y - 12 \ln 12y - 31 = Inl c 1 1$. Simplify, assuming 2y - 30, to obtain In(x + 1)(2y - 3) = c1. Then y = 3/2 + 24 - 2c1c(x + 1), where $c = 2 \cdot Consider$ the D.E. as a linear equation. In standard $338xy_12x$ form the D.E. is y' + 4 - 4. The I.F. is x + 1 x + 1 4 2 (x + 1). Multiplying the standard form equation through by this we find [(x 4 + 1)2y]' = (x 4 + 1) (12x 3). It toddo. °b (41) 2h °3 negra lon an IV1Slon y x + t en glve y = 2 + 4 - 2 c (x + 1). 8. Vriting the D.E. in the form = I 2 + + [P L we 2' see that the D.E. is homogeneous. Let y = vx. Then = dv dv v v 2 v + x dx' and the D.E. becomes $v + x \, dx = -1 - 2 - 2$. First-Order Equations 61 2 S. l. f. t. . ld dv - (v + 3v + 2) and hence lmp 1 lca lon Yle s x dx - - 2 ' 2 dv dx = -- Use of partial fractions glves $v + 2 \times 2 [1 v + 1 v 2] dV = dx \times Integration and immediate simplification 2 ln v + 1 ln c Further glve = -v + 2 \times 1 2 sil] plification v + c But = y glves = -v \cdot v$ + 2 x x Resubstituting accordingly and again simplifying, we 2 obtain [: ++ 2:] = 9. 322 Rewriting the D.E. 1n the form (4x y - 3x y)dx + (2x 4 y - x 3)dy = 0, we find that it is exact. Ve 322 F(x,y) such that F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = x 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = X 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = X 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = X 4 y^2 x 3 y + (y) x 3 + ;'(y)$. But since F (x,y) = $N(x,y) = X 4 y^2 x 3 y + (y) x 3 + ;'(y)$. From the first of these, 4 From this, F(x,y) = 2x Y - Y 4 3 2x y - x, we must have (y) = 0, (y) = cO. Thus F(x,y) + cO' and the solution is $x 4 y^2 - x 3 y = c$. 423 = x Y - x Y 10. The b.E. is linear. The standard form is y' + xy x + 1 = e -x(x + 1), and the I. F. is % eX (x + 1). Multiplying the standard form through by this, we obtain the equivalent of $x' [x e^{-x} - x + 1]$. + Y l] = (x + 1)-2. Integration and multiplication by (x + 1)e-x result in y = e-x[-1 + c(x + 1)]. 62 Chapter 2, ... 11. Vritin g the D.E. ln the form dy = v + dx that it is homogeneous. Ve let y = vx. dv x dx · dv The D.E. becomes v + x- = dx 2 - 7v 3v - 8. Simplification gives (3v - 8)dv = 3v 2 - v - 2 dx xPartial fractions decomposition puts this in the form [3V: 2 - v = 1] dv = -. Integration gives $2\ln (3v + 2) - \ln Ix - 11 = -\ln Ix I + \ln lc$. Simplifying gives (3y + 22X). = x c x 2 and hence (2x + 3y) = Ic(y - x)I. 12. The D.E. is a Bernoulli Equation with n = 3. Ve multiply -3 2 -3 dy -2 through by y to obtain x y dx + xy = x. Let v = l-n -2. dv -3 Y = Y, then dx - 2y dx. The preceeding D. E. readily transforms into the linear equation dv - 2 v dx x 2 An I.F. - J 2 / x dx lS e = 1 2. x Multiplying through by = x this, we obtain d: [:2] = -:3 ' Integrating, we find X = 1 - 2 2 + c. Replacing v by y and simplifying, we find y = 12.1 + cx First-Order Equations 63 13. This is a linear equation. The standard form is 2 $6x^2 y' + x y = 3$ An I.F. is $(x^3 + 1)(6x^2)$. Integration and division by $(x^3 + 1)(2, x + 1)(6x^2)$. Integration and division by $(x^3 + 1)(2, x + 1)(6x^2)$. Integration and division by $(x^3 + 1)(2, x + 1)(6x^2)$. Integration and division by $(x^3 + 1)(2, x + 1)(6x^2)$. and simplifying, we find 2 2 Y - xy - 2x = cx or (y - 2x)(y + x) = cx. 15. The D.E. lS both homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous
and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and Bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli. Ve first consider it as a homogeneous and bernoulli as a homogeneou 1 integration gives Inly 2 - 11 = In c . x and then From this, we 2 readily find v c 1 = -. x simplifying, we find y 2 Resubstituting v 2 - x = cx. = I. and x 64 Chapter 2 Now apply the initial condition y(l) = 2. Letting x = 1, y = 2, we get 3 = c, and he nce obta in the solution in the form y2 = x 2 + 3x or y = j x 2 + 3x. Alternatively, we recognize that the

given D.E. is a Bernoulli equation, with n = -1, by writing in the dy 2 -1 equivalent form 2x dx - Y = x Y We rewrite this as and transform 2 2 dv dv Y = x · let v = Y and -- = 2 y ., dx dx ' the D.E. into the linear equation x - v = standard form of this lS dv - 1:. v = x, and an dx x 2xy dy dx 2 x . The I.F. -1 lS x Multiplying the standard form equation through by this, we obtain (x-1 v)' = 1. Integration then gives x-1 v = x + c. Now resubstituting v = y2 and simplifying slightly, we find y2 = x 2 + 3x. 16. The D.E. is both a separate equation and a Bernoulli Equation. We first solve it as a separable equation. Separating variables, we get 2 dx 2 + dy = 0. We can 1 - x y + 4 use partial fractions or an integrate the dx term. Integrating the equations or an integrate the dx term. Integrating the equations $1(1 + x)(y^2 + 4)1/2 = In(l - x)l$. First-Order Equations 65 Now apply the I.C. y(3) = 0. Letting x = 3, y = 0 in the preceeding gives 8 = 21cl, so Icl = 4. Substituting this value of Icl back into the equation of the family of solutions, we get $1(1 + x)(y^2 + 4)1/21 - 41 - x$. Since the initial x value is 3 > 1, we take x > 1 and 1 - x < 0, so 11 - xl = x-i. Thus we obtain the particular solution $2 \frac{1}{2} 4(x - 1)$. $(x + 1)(y - 4) \frac{1}{2} - 4(x - 1)$. + 4) - Alternately, we now solve the given D.E. as a Bernoulli Equation. We first rewrite it as + 122 Y - x = 82 Y - 1, in which we recogn1ze that it 1S indeed a 1 - x 1-n 2 Bernoulli D.E. with n = -1. We thus let v = y = y, f. d dv 2 dy d f h D E. ln dx - y dx ' an trans orm t e . . lnto dv - + dx 4 16 2 v = 1 - x 2' 1 - x which 18 linear in v, with P(x) = 4 - 16 $2' Q(x) = 2 \cdot 1 - x \cdot 1 - x e JP(x) dx = e J 4 dx/(1-x2) = e 2 \ln I (1+x)/(1-x) I = An I.F. is 2 (1 - x) To integrate the right member, we note that 16 (1 + x) [1 + (1 - x)3 = -16 - (1 - x)2 \cdot 23] \cdot (1 - x) Integrating the D.E., we thus find (+) 2y = -16 [_1 - 1 - x) = -16 \cdot (1 - x) =$ $x + (1 \ 1 \ x) 2 + c$, which readily simplifies into $(1 + x) 2y = -16x + c \cdot 1 - x (1 \ x) 2y = -16x + c \cdot 1 - x (1 \ x) 2y = -16x + 12(1-x)$. Adding 4(1 + x) 2y = -16x + 12(1-x). Adding 4(1 + x) 2y = -16x + 12(1-x). previously found is readily obtained. 17. This D.E., with $M(x,y) = e 2x y^2 - 2x$ and F(x,y) = e 2x y, IS 2x exact; for $M(x,y) = e 2x y^2 - 2x$ and $F(x,y) = e 2x y^2 - 2x$ and $F(x,y) = e 2x y^2 - 2x$ and $F(x,y) = e 2x y^2 - 2x + (y)$. But this must y First-Order Equations 67 2x equal N(x,y) = e y. Thus we must have; (y) = 0 and 2 2x y 2 hence; (y) = c0' Thus F(x,y) = e - x + cO; and from 2 2x y 2 this the solution F(x,y) = c= 2x + 42 finally, -x 2 + 4 /1/2. or, y = e(2x 18. This D.E., with M(x,y) = 3x 2 + 2xy2 and N(x,y) = 4xy = N(x,y) = 2x y + 6y. x 3 + x 2y2 + (y). From this, 2 = 2x y We seek this must equal 2; (y) = 6y and 322 x + x y F(x,y) = 2x y + 6y. hence (y) = 2y3 + cO and F(x,y) = 2x2y + cO; and from this the solutions F(x,y) = 2x2y + cO; and from this the solution in the form x + xy + 2y - c, where c = c + 1 - cO. Applying the I.C. y(1) = 2, we let x = 1, Y = 2, to obtain 322 c = 21. Thus we obtain the solution in the form x + xy + 2y - c, where c = c + 1 - cO. Applying the I.C. y(1) = 2, we let x = 1, Y = 2, to obtain 322 c = 21. Thus we must have Thus F(x,y) = 322 + cO; and from this the solution in the form x + xy + 2y - c. 19. This , D.E. is both a separable equation and a Bernoulli Equation. We first solve it as a separable D.E., writing 4 y dy dx 2 it as 2 - x Integration then gives 2 In (y + 1) = y + 1 Inlxl + Inlcl. Simplifying we find In $(y^2 + 1)^2 = Incxl$ and hence $(y^2 + 1)^2 = Incxl$ and he hence 2 2 Icl = 2. Thus we obtain the solution (y + 1) = 21xl. Since the initial x value IS 2) 0, we take x .) 0 and write the solution as $(y^2 + 1)^2 = 2x$. Alternately, writing the D.E. as 4x - y = y-1 we recognize that it 1-n is a Bernoulli Equation with n = -1. Thus we let $v = y - Y^2 dv = 2y dy$ and substitute into the stated e q uation - , dx dx ' 4 dy 2 1 b o. 2 dv = 1 dApplying the I.C. y(2) = 1 gives c = Ii. Thus we obtain the solution $y^2 = -1 + Ii \times 1/2$, from which we readily obtain the solution $(y^2 + 1)^2 = 2x$ previously found. dy 2 + 7 [B 20. Writing the D.E. In the form - we see that dx - , [] 2 - 2 dy dv it is homogeneous. Thus we let y = vx, dx - v + x dx = 2. $2 \ln (v + 2) = \ln c \ 1 \ 2v + 1 \ r \ x \ 2$ From this we find f;v++2r = But v = resubstituting and simplifying, we find Y. and x' 2 (y + 2x). Jl_u = 2 \cdot f2Y + XT x c x (2x + 2 Ie (2y + x) I. Applying the I.C. y (1) or y) = = 2, we let x = 1, Y = 2, to obtain 16 = Ic15, from which 1 cl = 16 Replacing I c 1 with this value, and taking x + 2y 0, 5. 2 we express the solution as 5(2x + y) = 16(x + 2y). 21. This equation is both separable and linear. Treating it dy x dx as a separable equation, we have -- Integrating y x 2 + 1 then gives $2\ln |y| = \ln(x 2 + 1) + \ln |e|$, which quickly simplifies to $y^2 - c(x 2 + 1)$, with c > o. Applying the I.C. y(JI5) = 2, we let x = JI5, y - 2, to obtain 16c = 4, c = -. Thus we find the solution $y_2 = (x 2; 1)$ or y = (x 2 + 1)1/2 Now treating the equation as a linear equation, we have dy - dx x - x 2 Y = 0. with P(x) = 2 x + 1 x + 1 70 Chapter 2.2 Q(x) = 0. An I.F. is efP(x) $dx = e - f[x/(x + 1)]dx = 2 e - (1/2)\ln(x + 1) - (x 2 - 1/2 + 1)$. Multiplying the D.E. h h b h . b . d [(21) - 1/2 y] 0 t roug y t lS, we 0 taln $dx x + = \cdot$ Integrating, we find (x 2 + 1) - 1/2 y] 0 t roug y t lS = 0. 1)-1/2y = c or y = c(x + 1)1/2. Applying the I.C. $y(J_{15}) = 2$ gives c (x2 + 1) 1/2. Thus we again obtain y - 21 = 2. 22. Here we actually have two I.V. problems (I) For x 2, dy y = 1, dy y = 0, - + - + dx dx y(0) - 0. y(2) = a. - , where a is the value of lim; (x) and ; denotes the x2 solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 2. We first solve (I). The D.E. is linear in standard form, with P(x) = 1, Q(x) = 1. An I.F. is eJP(x)dx = eJdx x = e. Multiplying the I.C. y(O) = 0 gives 0 = 1 + cceO, c = -1. Hence the solution of Problem (I) is y = 1 - e - x, valid for 0 x < 2. First-Order Equations 71 Letting; denote the solution just obtained, we note that lim; (x) = lim (1 - e - x) = 1 - e - 2 This lS the a x2 x2 of problem (II); that is, the I.C. of problem (II) is y(2) - 2 = 1 - e. We now solve problem (II) is y(2) - 2 = 1 - e - 2 This lS the a x2 x2 of problem (II); that is, the I.C. of problem (II) is y(2) - 2 = 1 - e - 2 This lS the a x2 x2 of problem (II); that is, the I.C. of problem (II) is y(2) - 2 = 1 - e - 2 This lS the a x2 x2 of problem (II); that is, the I.C. of problem (II) is y(2) - 2 = 1 - e - 2. 1, Q(x) = 0. An I.F., as ln Problem (I), is eX. Multiplying the D.E. through by this d. b. x dy x d (x) an Integrating, we find e y = c or y = ce Now apply -2 the I.C. of problem (II), namely, y(2) = 1 - e. We have 1 - e - 2 = ce - 2, from which we find c = e 2 - 1. Thus the solution of problem (II) is y = 1 - e. (e 2 - l)e - x, valid for x > 2. We write the solution of the entire problem, showing intervals where each part lS valid, as 1 - x 0 < 2, - e x < y = (e 2 l)e - x 2. - , x > 23. Here, as in problem 22, we actually have two I.V. problems: (I) For x < 2, (II) For x > 2, (x + 2) dy + y - 2x, (x + 2) dy value of lim (x) and denotes the x+2 solution of (I). We first solve (I). The D.E. of this problem is linear. In standard form it dy (x 1) 2x with IS - + 2y = x + 2 dx + P(x) 1 Q(x) = 2x An I.F. JP(x)dx - IS - - + 2' 2. e - x x + e J [1/(x+2)]dx = elnlx+21 = I x + 21. Multiplying the standard form equation through by this, we obtain originally stated D.E., (x + 2) + Y = 2x, or d dx [(x + 2)y] = 2x. Integrating, we find (x + 2)y 2 = x + c. the Thus the particular solution of Problem (I) IS 2 (x + 2)y = x + 2. Denoting this solution by;, we note that lim; (x) = x + 2 2 lim [x + 8] = 3. This lS the a of problem (II); that lS - x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the I.C. of x + 2 x + 2 the
I.C. of x + 2 x + 2 the I.C. of problem (II) IS y(2) = 3. We now solve (II). The D.E. is linear. In standard form it is +(x : 2)y = 4. Just as in (I), an I.F. is Ix + 21. Multiplying the standard form equation through by this, we obtain the originally stated D.E., (X + 2) d dx Y 4 d [(2)] 4 I t t + y = or dx x + Y = . negra lng, we find (x + 2)y = 4x + c. Applying the I.C. of (II), y(2) = 3, we find c = 4. Thus the particular solution of problem (I) is (x + 2)y = 4x + 4, valid for x > 2. First-Order Equations 73 We write the solution of the entire problem as { $(x + 2)y = 2 - x + 8, 0 \times 2, (x + 2)y + 4x + 4, x > 2, 24$. This D.E. is both a Bernoulli D.E. and a homogeneous D.E. We We first solve it as a Bernoulli Equation, with n = 3, -3, y, expressing the D.E. in the thus multiply through by equivalent form x 2 y -3 dy dx 2 1-n = -. We let v = y x - 2 = x - 1 or 2 y - 3 dy - 2 y - 3 dy - 2 y - 2 = x - 1 or 2 y - 3 dy - $2 \ln x = x - 2$ Multiplying the linear equation through $-2 \, dx - 3 - 5 \, d - 2$ by this we get x dx -2x v = -2x + c x + 2 Integrating, we find x v = -2x + c x + 2 Integrating, we find x v = -2x + c x + 2 Integrating. = 2x 2 x 4 + 1 or y - fix (x 4 + 1)1/2. 74 Chapter 2 Now we express the D.E. in the form dy = -y + (y) 3 and dx x x dy solve it as a homogeneous equation + x- v + dx dv 3, which reduces to dv dx We x - = -v + v = dx 2 x v(v - 2) rewrite the left member, uSing partial fractions, and thus have (-1 v 2) dV = 2 dx + 2 v x v I t t 1 I I + .!EJ v 2 2 - 2 L - 2 In I X I negra ion glves - n v + Inl c 1 1. This simplifies to -In v 2 + In I v 2 - 21 4 = In x + In c, 2Jv - 2L 4 In 2 = In cx . v From this, 224 Iv - 21 = cv x y 2 2 2 4 Resubstituting v = -, we find Iy - 2x I = cv x . x Applying the I.C. y(1) = 1, we find c = 1. Thus we find the solution 1 $y_2 - 2x 2 1 = y_2 x 4$. Since $y_2 2x 2 1$ Since $y_2 2x 2 1$ Since $y_2 - 2x 2 1 = 2x 2 - y_2$, and write the .2242 4 222 solution as 2x - y = xy or (x + 1)y = 2x or y = 2x 2 as we obtained before. x 4 + 1' Section 2.4, Page 67 1. This equation is of the form (2.42) of Theorem 2.6 with $M(x,y) = 5xy + 4y^2 + 1$, $N(x,y) = x^2 + 2xy$. To apply that theorem, we first find First-Order Equations 75 M (x,y) - N (x,y) y x N(x,y) - 5x + 8y - 2x - 2y = 2x + 2xy 3(x + 2y) = 3x(x + 2y) = 0. For this equation 432 3 5 4 M(x,y) = 5x y + 4x y + x, N(x,y) = x + 2x y, and M(x,y) = 5x 4 + 8x 3 y = N(x,y). Thus this equation is y x exact. We seek F(x,y) = M(x,y) = M(x,y) and x F(x,y) = M(x,y) = M(x,y). From the first of these, F(x,y) = M(x,y) = M(x,y) = M(x,y). From the first of these, F(x,y) = M(x,y) = M(x,y) = M(x,y). From the first of these, F(x,y) = M(x,y) = M(x,y) = M(x,y) = M(x,y). From the first of these, F(x,y) = M(x,y) = M(x,y) = M(x,y). From the first of these, F(x,y) = M(x,y) = M(x,y). $N(x,y) = c_1 + c_2$ N(x,y) = c 1 + co. The family of solutions is then $F(x,y) = x + x + c_2$. This equation IS of the form (2.42) of Theorem 2.6 with 2 M(x,y) = 2x + tan y, N(x,y) = x + x + c_2. This equation IS of the form (2.42) of Theorem 2.6 with 2 M(x,y) = 2x + tan y, N(x,y) = x + x + c_2. This equation IS of the form (2.42) of Theorem 2.6 with 2 M(x,y) = 2x + tan y, N(x,y) = x + x + c_2. This equation IS of the form (2.42) of Theorem 2.6 with 2 M(x,y) = 2x + tan y, N(x,y) = x + x + c_2. apply that theorem, we use $1 + \tan 2y = \sec 2y$ and find N (x,y) - M (x,y) x Y M(x,y) 2 $1 - 2x \tan y - \sec y 2x + \tan y - \sec y 2x + \tan y - \tan y - 2 = -\tan y + \tan y +$ and simplifying, we find $(2 x \cos y + \sin y) dx + (x \cos y - x 2 \sin y) dy = 0$. For this equation $M(x,y) = 2x \cos y + \sin y$, $N(x,y) = \cos y - x 2 \sin y$, and $M(x,y) = 2x \cos y + \sin y$, $N(x,y) = -2x \sin y + \cos y + \sin y$, $N(x,y) = -2x \sin y + \sin y$, $N(x,y) = -2x \sin y + \sin y$, $N(x,y) = -2x \sin y + \sin y$, $N(x,y) = -2x \cos y + \sin y$, $N(x,y) = -2x \sin y + \sin$ y + 2 = -x Sln y + x cos y + ;(y). From this, F(x,y) = 0, y = 0;(y) = 0;apply M (x,y) - N (x,y) theorem, we first find y x N(x,y) 2.6 with that = 2(2xy + 1). but 3 ' 2y - x First-Order Equations 77 slnce this depends on y as well as x, we cannot proceed N (x,y) - M (x,y) - 2 (1 + 2xy) 2 This depends only, = -- on y so y e-J(2/y)dy = e-21nlyl = elnlyl-2 = lyl-2 Y is an I.F. of the given equation. Multiplying the given equation through by this and simplifying, we find -1 - 2(2x + y)dx + (2y - xy)dy = 0. -1 - 2 For this equation IS exact. y x We seek F(x,y) = M(x,y) = -y = N(x,y). Thus this equation IS exact. y x We seek F(x,y) = -y = N(x,y) = -y = N(x,y). -1 J (2x + y)ax = x + xy + y = c, where c = c 1 - cO. The family of solutions F(x,y) = 2y + cO' Hence F(x,y) = 2y + cO' Hence F(x,y) = 2 + cO' Hence F(x,y) = 2y + cO' Hence F(x,y) = 2 + cO' Hence F(x,y) = 2y + cO' Hence F(x,y) = 2y + cO' Hence F(x,y) = 2 + cO' Hence F(x,y) = 2y + cto obtain (4x p + 1 yq + 2 + 6x P yq + 1)dx + (5x p + 2 yq + 1 + 8x p + 1 yq)dy = 0. For this equation, we have M(x,y) = 4x p + 1 yq + 2 P q + 1 P + 2 q
+ 1 P + 2 q + 1 P(x,y) = N(x,y), and y x hence 6(q + 1) 8(p + 1), that { 5p 8p 4q = -2 6q - -2. - { 4(q + 2) = = 5(p + 2) IS, Solving these, we find p = 2, q = 3. Thus the desired I.F. of the form xPyq is x 2 y3. Now multiplying the original equation through by this I.F. x 2 y3, we obtain the equivalent exact equation 3 5 2 4 4 4 33. (4x y + 6x y) dx + (5x y + 8x y) dy = 0. It 1S easy to check that this is indeed exact. We seek F(x,y) such that $F(x,y) = M(x,y) = 4x \ 3y5 + 6x \ 2y4$ and $F(x,y) = x \ 4x \ 3y5 + 6x \ 2y4$ and $F(x,y) = x \ 4x \ 3y5 + 6x \ 2y4$. From the first of these, $F(x,y) = JfM(x,y)Bx = Jf(4x \ 3y5 + 6x \ 2y4)Bx \ 4y5 \ 3y5 + 6x \ 2y4$. From the first of these, $F(x,y) = 4x \ 3y5 + 6x \ 2y4$. From the first of these, $F(x,y) = JfM(x,y)Bx = Jf(4x \ 3y5 + 6x \ 2y4)Bx \ 4y5 \ 3y5 + 6x \ 2y4$. From the first of these, $F(x,y) = 4x \ 3y5 + 6x \ 2y4$. From the first of these, $F(x,y) = JfM(x,y)Bx = Jf(4x \ 3y5 + 6x \ 2y4)Bx \ 4y5 \ 4y5$ 0, 5xy + 8xy.y; (y) Thus F(x,y) 4 5 3 4 and the = c0. = xy + 2xy + c0' First-Order Equations 79 solution F(x,y) = c 1 1Sx 4y5 + 2x 3y4 = c or x 3y4(xy + 2) = c, where c = c 1 - c0.7. For this equation a 1 = 5, b 1 = 2, a 2 = 2, b 2 = 1, so a 2 2 f 1 b 2 - = = .a 1 5 2 b 1 Therefore this 1S Case 1 of Theorem 2.7. Ye make the transformation t: X + h Y + k, where (h,k) IS the solution of the system { 5h + 2k + 1 = 0 2h + k + 1 = 0 2h + k + 1 = 0. The solution of this system IS h = 1, k = -3 and so the , transformation is t = X + 1 = Y 3. This reduces the glven equation to the homogeneous equation (5X + 2Y)dX + (2X + Y)dY = 0. 80 Chapter 2 Ye write this ln the form dY dX 5 + 2 y'' X 2 + y X. and let Y = vX to obtain $v + X dv = dX 5 + 2v 2 + v \cdot This$ reduces to X dv dX 2 = 5 - v or (v + 2)dv 2 + v 2 v + 4v + 5 dX = - X. Integrating we find Inlv 2 + 4v + 51 = Ic1IX-2. y Now replacing v by X' and simplifying, we obtain 15x2 + 4XY + y21 = I c 1 1. Finally replacing X by x - 1 and Y by y + 3, we obtain the solutions of the original D.E. in the form First-Order Equations 81 15(X - 1)2 + 4(x - 1)(y + 3) + (y + 3)21 = I c 1 1 or 15x 2 + 4xy + y + 2x + 2y = c. 8. a 2 Here a 1 = 3, b i = -1, a 2 = -6, b 2 = 2, and a i b 2 = 2, and a i b 2 = -6, b 2 = 2, and a i b 2 = -6, b 2 = -6 -2 = b. 1 Therefore this is Case 2 of Theorem 2.7. Ye therefore let z = 3x - y. Then dy - 3dx - dz, and the glven D.E. transforms into (z + 1)dx - (2z - 3)(3dx - dz) = 0 or (-5z + 10)dx + (-255(z - 1))dx + (-255(z - 1))d= c 1. or 5x - 2z - In Iz - 21 = 5c 1. 82 Chapter 2 Replacing z by 3x - y and simplifying, we obtain the solution of the given D.E. in the form x - 2y + InI3x - y - 21 = c, where c = -5c 1. 10. For this equation at 1 = 215 b 2 a 1 = 10, b 1 = -4, a 2 = -5, so Therefore this IS Case 1 of Theorem 2.7. Ye make the transformation t: X + 2z - In Iz - 21 = 5c 1. 10. For this equation at 1 = 215 b 2 a 1 = 10, b 1 = -4, a 2 = -5, so Therefore this IS Case 1 of Theorem 2.7. Ye make the transformation t: X + 2z - In Iz - 21 = 5c 1. h, Y + k, where (h,k) is the solution of the system eo: 4k + 12 = 0, +5k + 3 = 0. The solution of this system IS h = 43' k = 13' and so the transformation is t X 4 = -3' Y 1 = -3. This reduces the qIven D.E. to the homogeneous equation (IOX - 4Y)dX - (X + 5Y)dY = 0. We write this ln the form dY = 10 - 4(Y/X) dX 1 + 5(Y/X) First-Order Equations 83 and let Y = vX to obtain dv 10 - 4vv + X dX - 1 + Sv. This reduces to X dv = 5v2 + 5v - 10 dX 5v + 1 or $(Sv + 1)dv_2 + v - 2 = (v + 2)(v - 1)$ and use partial fractions to integrate the left member. Ye find 3 In Iv + 21 + 2 In 1v - 11 = -5 In IXI - 10 dX 5v + 1 or $(Sv + 1)dv_2 + v - 2 = 5 dX X$. 2 We write v + v - 2 = (v + 2)(v - 1) and use partial fractions to integrate the left member. Ye find 3 In Iv + 21 + 2 In 1v - 11 = -5 In IXI - 10 dX 5v + 1 or $(Sv + 1)dv_2 - 12 = 10 dX 5v + 1 dv - 12 = -5 dX X$. 2 We write v + v - 2 = (v + 2)(v - 1) dX 5v + 1 dv - 12 = -5 dX X. 2 We write v + v - 2 = (v + 2)(v - 1) dX 5v + 1 dv - 12 = -5 dX X. 2 We write v + v - 2 = (v + 2)(v - 1) dX 5v + 1 dv - 12 = -5 dX X. 2 We write v + v - 2 = (v + 2)(v - 1) dX 5v + 1 dv - 12 = -5 dX X. 2 We write v + v - 2 = (v + 2)(v - 1) dX 5v + 1 dv - 12 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 2 We write v + v - 2 = -5 dX X. 3 We write v + v - 2 = -5 dX X. 3 We write v + v - 2 = -5 dX X. 3 We write v + v - 2 = -5 dX X. 3 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 4 We write v + v - 2 = -5 dX X. 5 We write v + v - 2 = -5 dX X. 5 We write . Now replacing v by YjX and simplifying, we find 3 2 IY + 2XI(Y - X) = Icl. Finally, replacing X by x + 4/3 and Y by y + 1/3 and simplifying, we obtain 12x + y + 3 > 0, take c > 0, and express this ln the form 3 2 (2x + y + 3)(x - y + 1) = c. 12. The D.E. of this problem IS of the form (2.52) of Theorem a 2 2.7, within a 1 = 3, b 1 = -1, a 2 = 1, b 2 = 1, and hence a 1 1 = -f - 1 3 b 2 = b. 1 Therefore this is Case 1 of the theorem. We let x = X + h, y = Y + k, where (h,k) is the solution of this is h system h + k + 2 = 0. = 1, k = -3, and the transformation . {x = X + 1 so IS = Y - 3. y This reduces the given 13 13 or 2 2 In(v +3)+-arctan 13 v 2 = - In X + c 1 . 13 y Now replacing v by X ' we obtain 222 In [Y IX + 3] + - arc tan 13 y J3X = C 1 . Finally replacing X by x - 1 and Y by y + 3, we obtain the solutions of the original D.E. in the form In[3(x - 1)2 + (y + 3)2] + arc tan 13 y J3X = C 1 . Finally replacing X by x - 1 and Y by y + 3, we obtain the solutions of the original D.E. in the form In[3(x - 1)2 + (y + 3)2] + arc tan 13 y J3X = C 1 . Finally replacing X by x - 1 and Y by y + 3, we obtain the solutions of the original D.E. in the form In[3(x - 1)2 + (y + 3)2] + arctan y + 3 13 /3(x - 1) - c 1. We now apply the I.C.y(2) = -2 to this, obtaining 2 In 4 + - arctan 13 1 13 = c 1 or In 4 + (2/13)(/6) = c 1. \Thus we obtain the solution of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a construction of the stated I.V. problem as 86 Chapter 2 In[3(x - 1)2 + (y + 3)2] + arctan y + 3 (x'-1) = In 4 + 'X 3 13. Here a 1 = 2, b = 3, a = 4, b = 6, and a /a = 2 = 1 2 2 2 1 b 2 /b 1. Thus this is a constructine to the stated I.V IS Case 2 of Theorem 2.7. We let z = 2x + 3y. Then dy = (dz - 2dx)/3, and the given D.E. transforms into (z + 1) dx + (2z + 1) dz = 0, which is separable ln x and z. Separating variables, we obtain dx - 2z + 1 dz = 0 z - 1 or dx - (2 + Z - 1) dz = 0. Integrating, we find x - 2z + 3 In 1 z - 11 = c 1. Replacing z by 2x + 3y and simplifying, we obtain x + 2y - 11 = c 2. First-Order Equations 87 Applying the I.C. y(-2) = 2, we find c 2 = 2 and hence obtain x + 2y - 2 - 11 = c 2. Following the procedure outlined there, we differentiate $y = p x + p^2$ with respect to x, to b. dy dp 2 dp. $dy h \cdot o taln dx = p + x dx + P dx' Slnce p = dx' t 1S lS [x + 2p] = 0$, which is of form (B) of Exercise 20. Assume x + 2p f 0, divide through by
it, and we are $1 f \cdot h \cdot dp 0 f h \cdot h$ ($h \cdot e t WI t Just dx =$, rom w lC p = C were c 1S an arbitrary constant). Then, returning to the given D.E., we find the one-parameter family of solutions y 2 = cx + c. (b) Now assume x + 2p = 0, and eliminate p between this and the given D.E. From this, p = -x/2. Substituting the bit into the given D.E. and simplifying, we obtain y = -x/2. Substituting the bit into the given D.E. of the given D.E. of the given D.E. of the given D.E. From this, p = -x/2. Substituting this into the given D.E. and simplifying, we obtain y = -x/2. Substituting the bit into the given D.E. of the given D.E. and simplifying, we obtain y = -x/2. parameter c between the equation of the given family and its derived equation, we obtain the D.E. of the given family in the D.E. of the orthogonal trajectories by replacing 3y in the D.E. (of Step 1) by x x its negative reciprocal - 3y' thus obtaining dy = dx x - 3y. Step 3. We solve this last D.E. Separating variables, we $3\ 2\ 2$ Integrating, we find = -(x/2) + 2 where $k = -2k\ 1 > 0$. This is the have $3ydy = -x\ dx$. $222\ k\ 1 \ or\ x + 3y = k$, family of orthogonal trajectories of the given family $y = -x\ dx$. $222\ k\ 1 \ or\ x + 3y = k$, family of orthogonal trajectories of the given family $y = -x\ dx$. $222\ k\ 1 \ or\ x + 3y = k$, family of orthogonal trajectories of the given family $y = -x\ dx$. cx. dy cx e Differentiating, we obtain dx = ce. We eliminate the parameter c between the equation of the given family and its derived equation, we Thus c = In y Substituting ln x 88 Applications of First-Order Equations 89 the derived equation, we find dy = y ln y dx x This IS the D.E. of the given family. Step 2. We now find the D.E. of the orthogonal trajectories by replacing y n y by its negative reciprocal - t ' thereby obtaining the D.E. y n y dy _ x dx y 1 n y . Step 3. We solve this last D.E. Separating variables, we have y In y dy = -x dx. Integrating (by parts), we find 2 1 2 2 2 Y2 ny - Y4 = _x 2 + k 1 ory(lny -1/2)=-x + k. This is the family of orthogonal trajectories of the given family of exponentials. 5. Step 1. We first find the D.E. of the given family y = i - x D. ff .. b . dy i -x x + C e . 1 erentlating, we 0 tain dx = -c e. Eliminating the parameter c between the equation of the given family and its derived equation, we obtain = 1 - (y - x + 1) or dy = x - y. dx Step 2. We now find the D.E. of the orthogonal trajectories by replacing x - y by itf negative reciprocal 1 thereby obtainin g the D.E. d d x Y = 1 Y - x' Y - x 90 Chapter 3 Step 3. We solve this last D.E. We regard x as the dx dependent variable and write the equation in the form dy = y - x or ; + x = y. This is linear in x, with I.F. e J1dy = e Y. Multiplying ; + x = y through by this, we have e Y d dx + eYx = y e y or [x e Y] = ye Y. Integrating, we y dy obtain xe Y = eY(y - 1) + k or x = Y - 1 + ke - Y. This is the family of orthogonal trajectories of the given family. 6. Step 1. We first find the D.E. of the given family Y = cx 2/(x + 1). Differentiating we obtain, = c(x 2 + 2x)/(x + 1)2. From the given equation, c = (x + 1)yx 2; and substituting this into the derived equation and $1 \cdot f \cdot b \cdot dy - (x + 2)y$ Th \dots h D E f slmp 1 Ylng, we 0 taln dx - x(x + 1). lS lS t e. 0 the given family. Step 2. We now find the. D.E. of the orthogonal \dots b 1 (x + 2)y h D E f 1 trajectorles y rep aClng x(x + 1) ln t e \dots 0 step x (x + 1) ln t e \dots 0 step x (x + 1). lS lS t e. 0 the given family. Step 2. We now find the. D.E. of the orthogonal \dots b 1 (x + 2)y h D E f 1 trajectorles y rep aClng x(x + 1) ln t e \dots 0 step x (x + 1) ln t e the D.E. dx = (x + 2)y. Step 3. have y dy We solve this last D.E. Separating variables, we = x(x + 2 1) dx. 2 Integrating this, we have Y2 = 224 21n Ix + 21 + k 1 or x + y - 2x + In(x + 2) k. Applications of First-Order Equations 91 8. 2 Step 1. We first find the D.E. of the given family x = -2y dy 2y - 1 + ce. Differentiating, we obtain 2x = 2y dx - 2ce - 2y. We eliminate the parameter c between these two equation, c = (x 2 - 2y + 1) dy or dy = dx dx dx x 2 2y - x This lS the D.E. of the given family. Step 2. We now find the D.E. of the orthogonal trajectories by replacing x 2 by its negative $2y - x 2 \cdot 1 x - 2y h b b \cdot h D E$ recliproca, t ere y 0 talning t e . . x dy dx 2 = x - 2y x Step 3. We solve this last D.E. dy 2 Writin g it as - + -y = dx x x, we recognize that it is a linear equation in standard form with P(x) = 2 and Q(x] = x. An I.F. is eJP(x)dx = x eJ (2/x)dx = e2lnlxl = x 2 \cdot 1 x - 2y h b b \cdot h D E recliproca, t ere y 0 talning t e . . x dy dx 2 = x - 2y x Step 3. We solve this last D.E. dy 2 Writin g it as - + -y = dx x x, we recognize that it is a linear equation in standard form with P(x) = 2 and Q(xJ = x - 2y x Step 3). Multiplying through by this, we 2 dy 3 d 2 3 have x dx + 2xy = x or dx [x y] = x. Integrating, we 2 find x y 4 x = 4 + k. This IS the equation of the one-parameter family 2 3 Differentiating, obtain 2x - 2 dy x - y = cx we - . y dx - 2 From the given equation, 223 3 cx = (x - y)/x. Substituting this in the derived equation and simplifying, dy 2 2 we obtain successively 2x 2y dx = 3 (x - y) /x or 222 dy Y dy 3 y - x 2 y = -x + 3 - or - = dx x dx 2xy Step 2. We now find the D.E. of the orthogonal 2 2 3y - x trajectories by replacing by its negative 2xy reciprocal 2 2xy 2 ' thereby obtaining the D.E. x 3y dy _ dx 2 x 2xy $3y_2$ Step 3. We solve this last D.E. Yriting it in the form dy $2(y_x)$ we recognize that it is homogeneous. We dx - 1 $3(y_x)_2'$ let y = vx. Then = v + x, and the D.E. becomes v + dv 2v dv 3v 3 + v Separating variables, x = 2 or x - dx dx - 2. 1 - 3v 1 - 3v we have $\begin{bmatrix} 1 & 3V & 2 \end{bmatrix} dv = dx$ Using partial fractions, this 3v + vx becomes $\begin{bmatrix} v \\ 2v \end{bmatrix} dv = x$. Integrating, we find $\ln v = 3x + 1 - \ln(3v + 1) = \ln k = \ln k = 1$. Now replacing v by y/x and taking $3v + 1 + \ln k$, where k > 0, or $\ln -fL = \ln k = 3x + 1 + \ln k$, where k > 0, or $\ln -fL = \ln k = 3x + 1 + \ln k$. From this, $3v + 1 + \ln k = 3x + 1 + \ln k$, where k > 0, or $\ln -fL = \ln k = 3x + 1 + \ln k$. From this, $3v + 1 + \ln k = 3x + 1 + \ln k$. or + Y3 = 2 c. We first find the D.E. of this 2 2 2 given family. Differentiating x L find - + = c we + 4 3, 2y dy o. From this, have dy 3x which is the = we = -4y' 3 dx dx D.E. of the orthogonal trajectories is obtained from this by replacing -3x/4y by its negative reciprocal 4y/3x, and so it is = . Step 3. We solve this lost D.E. dy 4 dx have = 3 C. Integrating, we find In Ikl; and simplifying, we obtain y = Separating variables, we 4 In I y I = 3 In I x 1 + k x 4/3 . 13. We first find the D.E. of the given family of parabolas y = c 1 x 2 + K. Differentiating, we obtain the D.E. of the family of parabolas in the form d d Y = 2y - 2K. It will be convenient x x to rewrite this slightly, as follows: dy = 4y - 4K Then dx 2x the D.E. of the given family of ellipses x 2 + $2y^2 - Y = c^2$. Differentiating, we obtain 2x + 4y - dy = 0, from which we at once obtain the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the
two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven families 1 are orthogonal provided K = 4.14. Step 1. x 1 - c 2 x We first find the D.E. of the dx family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven family of ellipses, that is, dy = dx 2x 1 - 4y. (**) Comparing (*) amd (**) we see that the two glven family of ellipses, that is, dy = dx 2x 1 - 4y. the given D.E., 1 - c 2 x = y the derived equation, we have dy = dx Substituting this into 2 Y2. This IS the D.E. x of the given family. Step 2. The D.E. of the orthogonal trajectories is obtained from this by replacing . 1 2 / 2 d .. reciproca -x y, an so It IS 2 / 2 b . y x Y ltS 2 dy x dx - 2". y negative Step 3. We solve this last D.E. Separatin 2 2 have y dy = -x dx. Then integrating and simplifying, we 3 3 find x + y = c 1. Thus the value of n is 3. Applications of First-Order Equation, we obtain 1 + dx = 2cx. Eliminating the parameter c between the given equation, we obtain the D.E. of the given f dy x + 2y family in the orm dx = x f(x,y) + tan a = 1 - f(x, y) + 21 - 2f(x,y) + 2y in this D.E. by x x + 2y + 2x = = 1 - 2(x + 2y) x 3x +homogeneous. We let y = dx vx to obtal. n V + dv 3 + 2v (4v + 1)dv x dx = -1 + 4v or 4v 2 + 3v + 3 Integrating this (we recommend tables), we find (1/2)lnI4v 2 + 3v + 31 - (1/2) (2/{39}) arc tan 8v + 3 = {39 - In x + (1/2)c, where the final 1/2 is for conven1ence. We multiply through by 2 and replace v by y/x to obtain = x 2 2 In 4y + 3x + 3x 2 x (2) arc tan [$8Y + 3x l = {39J 39 x J 2 - In x + c.$ Thus we find the desired family of oblique trajectories in the form Inl3x 2 + 3xy + 4y21 = - () arc tan [3:8:] = c. 96 Chapter 3 Section 3.2, Page 88 1. We choose the positive x aX1S vertically downward along the path of the stone and the origin at the point from which the body fell. The forces acting on the stone are: (1) F 1, its weight, 4 Ibs., which acts downward and so IS positive, and (2) F 2, the air resistance, numerically equally to v/2, which acts upward and so is the negative quantity -v/2. Newton's Second Law gives m; = F 1 + F 2 " Using g = 32 and m = ; = 3 4 2 = , this becomes !. dv = 4 v 8 dt 2. The initial condition IS v(0) = 0. The D.E. is separable. We write it as dv 4 - v/2 = 8 dt · Integrating, we find c 1 = 4. Thus the velocity at time t 1S given by v = 8 - 4t Y .. h . dx 8 - 8e - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t Y .. h . dx 8 - 8e - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t Y .. h . dx 8 - 8e - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t Y .. h . dx 8 - 8e - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating, we find x = 8t - 4t Applying + 2e + c 2 . the I.C. x(O) = 0, by v = 8 - 4t 8e . rltlng t lS as dt = and integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 . the I.C. x(O) = 0 as the integrating + 2e + c 2 we find c 2 = -2. Thus the distance -4t fallen is x = 8t + 2e - 2. Thus the answers to part (b), we simply let t = 5 ln these f d Thus v(5) 8(1 e -20) 8 expressions or v an x. - .- ft/sec and x(5) - 2(19 + e - 20) 38 feet. 3. We choose the positive x aX1S vertically upward along the path of the ball and the orlgln at the ground level. The forces acting on the ball are: (1) F 1, its weight, (3/4) Ib, which also acts downward and so is the negative quantity - v/64. Newton's Second Law gives m = F 1 + F 2 Using g = 32, and $m = 3/4 \ 3 \ dv \ 3 \ v$ wig = 32 = 128 ' this becomes The 128 dt = -4 - 64. initial condition is v(O) = 20. The D.E. IS separable. We write it as 3dv = -dt. 2v + 96 Integrating we find (3/2) Inl2v + 961 = -t + Co which $1 \ 1 - 2t/3$ reduces to $v + 48 = c \ 1 \ e$. Applying the I.C. v(O) - 20 to this, we find $c \ 1 = 68$. Thus the velocity at time t IS given by v = 68e-2t/3 - 48. From this we obtain x - 102e-2t/3 - 48t + c2. Applying the I.C. x(O) = 6, we obtain c2 = 108. Thus we have the distance x = 102e-2t/3 - 48t + c2. Applying the I.C. x(O) = 6, we obtain c2 = 108. Thus we have the distance x = 102e-2t/3 - 48t + c2. =; and t 0.5225 (seconds). For this value of t, we find x 10.92, which IS the height above the ground that the ball will rise. 4. The forces acting on the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., which moves the ship are: (1) F 1, the constant propeller thrust of 100,000 lb., motion of the ship and so is the negative quantity -8000v. dv Newton's Second Law gives m dt = F 1 + F 2. Using g = 32 (2000 Ibs / ton) and m = wig = (32000 tons) 32 - = 2,000,000 dt = 100,000 - 8000v. The initial condition 18 v(O) = O. The D.E. IS separable. We write it as dv 1 v dv dt = - or - 500. dt 20 250 25 - 2v -Integrating we find $\ln 25 - 2vl = -2;0 + Co$ which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2;0 + Co which
reduces to 125 - 2vl = -2;0 + Co which reduces to 125 - 2vl = -2answer part (c), let $v = (0.80)(12.5) \ln (*)$ and solve for t. We obtain 0.80 = 1 - e-t/250 and hence e-t/250 = l/S. Thus - 2io = In(l/S), from which t = 402 (sec.). 6. We break the problem into two parts: (1) while the body is falling back toward earth. We consider part (1). We choose the positive X-axIS upward along the path of the object. The forces acting on the object are: (1)F 1, its weight, - 98000 dynes; and (2) F 2, air resistance, - 200 v dynes. Each is negative, since each acts in the downward direction. Newton's S d L. dv F F 100 dv 98000 econ aw gl ves m dt = 1 + 2 or dt = - 200v. The I.C. is v(O) = 150. We simplify the D.E. to read; = (2v + 980). It lS dv separable; and separating variables, we have 2v + 980 = -2t or v = -490 + ce Applying the I.C., v(0) = 150 to this, we find c = 640. Hence -2tv = -490 + 640e. (*) 100 Chapter 3 The object stops rlslng and starts falling when v = 0. -2t49Thus this happens at t such that e = 64. We find t 0.1336 (sec.). In (a) we seek the velocity 0.1 second after the object is thrown. Since 0.1 < 0.1336, we let $t = 0.1 \ln (*)$, and find $v = -490 + 640 e - 1 \cdot 2 \cdot 33.99 \text{ cm/sec}$. Now consider part (2). We choose the positive x-aX1S vertically downward from the highest point reached. Now the weight is + 98000, since it acts in this downward direct ion. The D. E. is now 100; = 98000 - 200v, and the I.C. is v(O) = O. Simplifying the D.E. and separating dv variables, we have v = 490 - c e. The I. C. gives c = 490, and so v - 490(1 - e - 2t). (**) In (b) we seek the velocity 0.1 second after the object stops rising and starts falling. So we let t - 0.1 in (**) and obtain v = 490(1 - e- 0.2) 88.82cm./sec. 8. We choose the positive x-aXIS horizontally along the given direction and so is positive, and (2) F 2, the resistance force, numerically equal to 2v, which Applications of First-Order Equations 101 acts opposite to the glven direction and so IS the negative quantity -2v. dv Newton's Second Law glves m dt = F 1 + F 2. Using g = 32 and m = : -15° 3 + 2170 = 10, this becomes 10 dv = 12 - 2v. dt The initial velocity IS 20 m.p.h. I.C. is v(O) = 8: .88 - 3 ft/sec. Thus the The D.E. 1S separable. Ye write it as 5 dv 6 - v = dt. Integrating we find 5 lnlv - 61 = -t + Co which reduces to I 61 -t/5 A l. h I C (0) 88 h.v - = c e. pp Ylng t e.. v = 3 to t lS, we find c = 73°. Thus the velocity at time t 1S given by v = 6 + (7 3 0) e -t/5. (*) To answer part (a), we let t = 15 in (*) to obtain (70) -32 to t lS = -10 to to t lS = -10 to t lS = -10 to to to t lS = v(15) = 6 + 3 e 7.16 ft/sec. To answer part (b), let $v = ()(8) = 4 \ln (*)$ and solve for (70) -t/5 -t/5 6 + 3 e = (44)(3), from which e this, -; -0.99 and t 4.95 (seconds). t. Ye have 13 From = 35. 102 Chapter 3 10. Ye choose the positive x-aX1S vertically upward along the path of the shell with the origin at the earth's surface. The forces acting on the shell are: (1) F 1, its weight, 1 lb., which acts downward and so is the negative quantity -1. and (2) F 2, the air resistance numerically equal to , 10-4 2 N ' sean so IS e negative quantity - v. ewton s Second Law g1ves m; = F 1 + F 2. Using g = 32 and m = ; = ;2 ' this becomes 1 dv - 32 dt - 42 = -1 - 10 v. The initial condition IS v(O) = 1000. The D.E. 1S separable. We write it as 1 dv 10 4 + v 2 dv 32 dt -- = or = 32 dt 10 4 2 + 10 4 10 4 v Integrating we find (10) arc tan (10) 32t = -+ c or 10 4 arc tan (10) 32t = -+ c or form arc tan (10) - arc tan 10 - (). Taking the tan of each side and multiplying by 100 glves v = 100 tan(arc tan 10 - 0.32t). Applications of First-Order Equations 103 This is the answer to part (a). To answer part (b), note that the shell will stop rising when v = 0. Setting v = 0, we at once have arctan 10 - 0.32t = 0, and thus t = ar c tan 10/(0.32) 4. 60 (s e c). 13. We choose the positive x-aX1S horizontally along the glven direction of motion and the orlgln at the point at which the man stops pushing. The forces acting on the loaded sled as it continues are: (1) F 1, the air resistance, · 11 1 3v h e h. th numerlca y equa to, w lC acts Opposite to e 3v direction of motion and so is given by - , and (2) F 2, the frictional force, having numerical value p, N = (0.04) dv Law gl ves m dt , this becomes (80), which also acts opposite to the direction of motion and so is given by -(0.04)(80) = -1: Newton's Second w 80 = F 1 + F 2 - Using g = 32 and m = g = 32 = 5 dv 3v 16 2 dt = -4 - 5. The initial velocity is 10 ft/sec., so the I.C. is v(O) = 10. The D.E is separable. Ye write it as dv 15v + 64 dt = - 50. Integrating, we find Inl15v + 641 = reduces to 115v + 641 = red Application of the condition x(0) = 0 then gives c 1 = 48. Hence the distance x is given by x = (48)(1 + 3t/10) - 61t. To answer the stated question, note that the sled will continue until the velocity v = 0. Thus we set v = 0 and $1 \text{ ft} f \cdot d \cdot -3t/10 + 32 \text{ fh} \cdot h + 402$ so ve or , $\ln \ln g = 107$ rom w lC ... We now evaluate x at t 4.02 to determine the distance which the sled will continue. We find x (4.02) (48) (1-13027) - 64 (i 502} 16.18 feet. 15. We choose the positive x direction down the slide, (2) F 2, the frictional force; and (3) F 3, the alr resistance. The case weight 24 Ibs., and the component parallel to the slide has numerical value 24 sin 450 = 24 . {2 Since this acts in the positive (downward) direction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the friction along the slide, F 1 = 24 The frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2
numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numerical value p, N, where J.L IS the coefficient of the frictional force F 2, has {2 numer 105 24 magni tude of N is 24 cos 450 = J2 Since the force F 2 acts in the negative (upward) direction along the slide, we have F 2 = -(0.4)(;:). Finally, the air resistance F3 has numerical value;, Since this also acts in the negative direction, we thus have F3 v = -3. Newton's Second Law now gives m; -F 1 + F 2 + F3' With g = 32, w 24 3 m = g = 32 = 4 and the above forces, this becomes ()(;) = (;;-) - (0.4)(;;-) -; The initial condition IS v(O) - o. Th D E o bl U e U rl " te l O t as (43) (d d V t) e. . IS separa e. w " = (-5v + 108J2) 15 or 5v - 10SJ2 I 4t (7) 1 find In 5v - 108J21 = - "9 + Co or 15v - 108J21 = - "9 + Co o the velocity is given by = 4 45 dt Integrating we dv v = 108 5 J2 (1 - e - 4t/9). (*) To answer (a), simply let t = 1 in (*). Ye obtain v(l) = (1085 (1 - e4) 10.96(ft/sec). To answer (b), more work is required. We first integrate (*) to find the distance x from the top of the slide. We have 106 Chapter 3 x - (10J:f)t + (24J:f)e-4t/9 + c. Since x = 0 at t - 0, we find c = 243J:f and hence $5' = (108 \text{ vO21 t} (243)\text{ e}^{-1}) + 5 \text{ e}^{-5}$. The slide being 30 ft. long, we let x = 30 in this to determine the time t at which the case reaches the bottom. That is, we must find t such that $30 = (1085t + (243)\text{ e}^{-1}) + 5 \text{ e}^{-5}$. The slide being 30 ft. long, we let x = 30 in this to determine the time t at which the case reaches the bottom. That is, we must find t such that $30 = (1085t + (243)\text{ e}^{-1}) + 5 \text{ e}^{-5}$. The slide being 30 ft. long, we let x = 30 in this to determine the time t at which the case reaches the bottom. error calculation with a hand calculator shows that the two sides of this are approximately equal for t = 2.49. This is the time at which the error calculator shows that time to be approximately 20.46 (ft/sec). 16. We choose the positive x direction down the hill with the origin at the starting point. The forces acting on the boy and sled are: (1) F 1, the component of their weight parallel to the hill; (2) F 2, the frictional force; and F 3, the air resistance. The boy and sled weigh 72 lbs., Applications of First-Order Equations 107 and the component parallel to the hill; (2) F 2, the frictional force; and F 3, the air resistance. The boy and sled weigh 72 lbs., Applications of First-Order Equations 107 and the component parallel to the hill; (2) F 2, the frictional force; and F 3, the air resistance. direction on the hill, F 1 = 36. The frictional force F 2 has numerical value J-L N, where J-L > 0 is the coefficient of friction and N is the normal force. The magnitude of N is 72 cos 300 = 36/3; the J-L is an unknown which will be determined in due course from the given data of the problem. Since the force F 2 acts in the negative (upward) direction on the hill, we have $F_2 = -J-L(36/3)$. Finally, the air resistance F3 has numerical value 2v. Since this also acts in the negative direction, we thus have $F_3 = -2v$. Newton's Second Law now gi yes m; = . w 72 9 F 1 + F 2 + F3. W1th g = 32, m = g = 32 = 4 ' and the above forces, this becomes (:)(;) = 36 - It 36/3 - 2v. The initial condition is v(O) = 0. Another condition is also given, that is v(5) = 10; and this extra condition will be sufficient for us to eventually determine p. The D.E. is separable. We write it as dv() (18 - J-L18/3 - v) dt = or dv 8 dt = 18 (J-L 13 - 1) 9 v + 108 Chapter 3 8t Integrating we find Inlv + 18(- 1)1 = - If + Co or Iv + IS(- 1) I = c e - 8t/9. Applying the initial condition, we at once find that c = 118(-1)1. Thus we obtain the solution in the form v + 18(-1) = 18(-1)(1 - e - 40/9) and hence 1 - p = ()(0.988)-1, from which we obtain p = 0.25. 17. This problem has two parts: (A), before the object reaches the surface of the lake; and (B), after it passes beneath the surface. We consider (A) first. We take the positive x-axis vertically downward with the origin at the point of release of the object. The forces acting on the body are: (1) F 1, its weight, 32 lbs., which acts downward and so is positive; and (2) F 2, the air resistance, numerically equal to 2v, which acts upward and so is the negative quantity -2v. Applying Newton's Second Law, with m; =; = 1, we at once obtain the D.E. lS separable. We write it dv dt. Integrating we find as 32 - 2v = In132 - 2vl = -2t + Co or 132 - 2vlYith this, we at once have Applications of First-Order Equations 109 -2t v = 16(1 - e) (*) This glves the velocity at the instant when the object reaches the surface of the lake. To solve problem (B), we will need to know the velocity at the instant when the object reaches the surface of the lake. function of time. This is found by integrating (*). Ye at once obtain 16t + -2t + k. x = 8e Since x(O) = 0, we have k = -8, and hence the distance fallen (before striking the water) is given by -2t x = 16t + 8e - 8. Since the point of release was 50 feet above the water, if we let x = 50 in this, it will determine the time at which the object hits the surface. Thus we must solve -2t 50 - 16t + Be - 8 for t. We write this in the form 58 - 16t = 8e - 2t . A little trial-and-error calculation with a hand calculator leads to the approximately 15.99 (ft/sec). We now turn to problem (B). Ye again take the positive x-axis vertically downward, but now we take the origin at the point where the object hits the surface of the lake. The forces now acting on the buoyancy) have negative signs since they act upward. Newton's Second Law leads to the D.E. = 32 - 6v - 8. By part (A), the velocity of the object at the surface of the lake is 15.99 (ft/sec). Ye take this as the I.C. here: v(O) = -6t + c - 1 or 124 - 6vl = -6t + c -11.99
e. This is the velocity after the object passes beneath the surface. We want to know what this is 2 sec. after. Hence we let t = 2 in this to obtain v 4.00 (ft/sec). Section 3.3, Page 102 1. Let x be the amount initially present, we have the I.C. x(O) = xO. Also, since 10% of the original number have undergone disintegration ln 100 years, 90% remain, and so we have the additional condition x = c e Application of the I.C. immediately gives $X_0 = c$. Hence we have Applications of First-Order Equations 111 -kt x = xOe Now apply the additional condition to 9x O -lOOk determine k. We have JR) = xOe, which reduces to e - k (190) 1/100 = Thus the solution takes the form = () t/l00 x Xo 10 (*) To answer question (a), we let t = 1000 in (*). We find x(1000) = Xo b 9 0 fO 0.3487 xo' Thus the answer to x 34.87%. To answer question (b), we let x = 40 in (*). We xo = () t/100 () () have 4 Xo 10 From this, 10 In 1 9 0 = In ' In 1 4 9 In 10 from which t = 100 1315.28. Thus t 1315 years. 4. Let x = the amount satisfies the D.E. of the first chemical present. Then x dx dt = -kx, where k > 0. Two conditions and x(4) = -. The D.E. 1S separable; are given: x(1) = and separating the first chemical present. Then x dx dt = -kx, where k > 0. Two conditions and x(4) = -. variables and integrating, we at once b. - k t A 1. h d. bt. o ta1n x = ce. pp Ylng t e two con 1t10ns, we 0 aln. - k 2 - 4k 1 respectively ce = 3 and ce = 3. These two equations will determine c and k. Dividing the first by the second gives e 3k = 2, from which e k = 21/3. Then from the second (1) 4k + 24/3 equation for c and k, c = 3 e = 3. Thus the solution of the D.E. which satisfies the two given 112 Chapter 3 (2 4 / 3) k-t conditions is x = 3 (e) = 2(4-t)/3 3 (2 4 / 3) 1/3-t 3 (2) or x = (*) To answer question (a), let t = 7 in (*). This gives 2-1 1 x(7) = 3 = 6 (kg) Now compare this with the original amount, which is x(O) = 24/3 3 (kg) Tie have x(7) x(O) 1 6 = 24/3 3 1 3 0.1984. Thus 19.8% of the first chemical remains 4 {2 at the end of seven hours. To answer question (b), we first note that one tenth (0) 24/3 of the first chemical is x 10 = 30 (kg). We thus let x 2 4 / 3 = 30 ln (*) and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10; and solve for t. The have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] 24/3 From this 3 [In 2] 24/3 From minutes. 7. Let x = the temperature of the body at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = k(x - 40). We also have the D.E. is separable. We write it in the form x x 40 = kdt. Integrating, we obtain Inlx - 401 = c e kt Since kt x 40, I x + 0. 40 I = x - 40, so we have x = 40 + c e. Application of I.C. x(O) = 100 gives 100 = 40 + c or c = 60. Thus x = 40 + 60 e kt. Then application of the Application of the Application of I.C. x(O) = 100 gives 100 = 40 + c or c = 60. Thus x = 40 + 60 e kt. Then application of the Ap (*) (a) Let $t = 30 \ln (*)$. e find x(30) - 40 + 60(6). Hence t 98 minutes, 17 seconds. 9. Let x = the temperature of the ple at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = k(x - 80). We also have the I.C. x(O) = 350 and the additional condition x(5) = dx we have $x - 80 \ 300$. Separating variables in the D.E., = kdt; and integrating, we find Inlx - 801 kt = kt + cO' and hence I $x - 80 \ I = c \ Since \ x \ 80$, this kt simplifies to $x = 80 + c \ so \ c = 270$. Thus $x = 80 + 270e \ kt$. Application of the additional condition 5k 5k 220 x(5) = 300 gives 300 = 80 + 270 27 (22)t/5 = N In(22/27) 63.54. Hence t 63 minutes, 32 seconds. 10. Let x_{-} = the temperature of the coffee at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. We also have the D.E. dt = $k(x_{-}70)$. kt + cO' and hence Ix - 701 = c e Since x 70, this kt simplifies to x = 70 + c e Applying the I.C. x(O) = 180, we have 180 = 70 + c, so c = 110. Thus x = 70 + 110 e 10k, from which e 10k = 90 k = (191) 1/10 110 or e Thus we obtain the solution (9) t/10 x = 70 + 110 IT (*). (9) 3/2 (a) Let t = 15 ln (*). We have x(15) = 70 + 110 IT 151.41°. Applications of First-Order Equations 115 (b) Let x = 140 in (*). (191) 1/10 We have 140 = 70 + 110 II ' 10 1n(7/11) N 11. Then t - 10 (7/11) N 11. Then t N II- en - In(9/11) N t 30 minutes, 12 seconds past 10 and 30 minutes, 12 seconds past 10. 11. Let x = the population at time t. Then we at once have the D.E. = kx. Letting xo denote the population at the start of the given 40 year period, we have the I.C. x(0) = xO. Since the population doubles in 40 years, we have the additional condition x(40) = 2xO. The solution of the D. E. is x = c e kt Applying the I. C _ to this we at once have c = xo' and hence x = xoe kt. Now applying the additional condition x(40) = 2xO. The solution of the D. E. is x = c e kt Applying the I. C _ to this we at once have c = xo' and hence x = xoe kt. Now applying the I. C _ to this we at once have c = xo' and hence x = xoe kt. = 2 and k = In 4 0.0173. Thus the solution of the D.E. which satisfies the two given conditions IS x 0.0173t = xOe To answer the stated question, we let x = 3x O in this, from which eO.0173t = 3 and 0.0173t = In 3. From this we find t 63.5. Thus the population triples in approximately 63.5 years. 116 Chapter 3 12. Let x = the population at time t. We at once have the dx D.E. dt = kx. We take 1970 as the zeroth year of the problem and have the I.C. x(0) = 30,000. Then 1980 is the tenth year, and we have the additional condition x(10) = 35,000. The one-parameter family of solutions of the D.E. is x = c e kt Applying the I. C. to this, we have c = 30,000. Thus x = 30,000. The one-parameter family of solutions of the D.E. is x = c e kt Applying the I. C. to this, we have c = 30,000. Thus x = 30,000. The one-parameter family of solutions of the D.E. is x = c e kt. additional 10k condition to this, obtaining 35,000 = 30,0006. From . k (7) 1/10 thlS, e = 6 (7) t/10 30,000 6 · We seek the population in 1990, which is Thus we obtain the solution, we find x = 30,000() 2 40,833. 15. The population x satisfies the D.E. dx = dt 3x - 100 3x 2 10 8. The I.C. IS x(1980) = 200,000. The D.E. IS separable. We write it ln the form 10 6 dx 3 dt = 100 · 6 x(x - 10) To integrate the left member, we use partial fractions. Thus we obtain [x 1106 - :] dX = 3 100 dt Applications of First-Order Equations 117 6 In x - 10 = x - 3t/100 T.T e ce. " 3t - 100 + cO. 6 I 3t 10! - In!x = 100 + Co or x - 10 6 From this we obtain = Integrating we find In Ix x to this. apply the I.C. x(1980) = 200,000 = (2)(105) + 1061 - 3(198)/10C - We have 5 - ce, or 2(10) 4e 59.4-3t/100. We solve this for x, obtaining x + 4x e59.4-3t/100 = 106 and hence 10 6 x = 1 + 4e59.4-3t/100 (*) This 1S the answer to part (a). To answer part (b), 10 6 we let t = 2000 in (*). We find x(2000) = $[1 + 4e \ 0.6J \ 312,966$. To answer (c), simply find limx, where x is t+CD 10 6 = -y- = 1,000,000. 16. dx 2 (a) We have the initial-value problem dt = k x - x, 2 x(t O) = xO' where we assume k x - x > 0. The D.E. is separable. We separate variables and use partial fractions to obtain $[+ k_{"} x] dx = dt$. Then integration gives $[\ln x - \ln(k - "x)] = t + \ln Co$. x kt Simplifying, we obtain $k_{-} x = c e$ Applying the 118 Chapter 3 -kt xOe 0 I.C. x(t O) = X o to this, we find $c = Before k - x \cdot o$ using this, we solve the one-parameter solutions for x. We have x = k ce kt - family of kt , \ c x e , from k kt ce kt + ce which (1 + c ekt)x = k c e kt and hence x = kc - kt. AC + e the value of c already determined from the initial condition. We find or finally x - We now substitute into this x = r - kt .. k xOe 0 k - A x 0 - kt x - VOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe 0
k - A x 0 - kt x - vOe 0 k - A x 0 - kt x - vOe) e^{t} by e^{t} in the successive forms Applications of First-Order Equations 119 dx (10)6 x - 4x 2 - 4(10)10 dt = 4(10)8 ' dx = [x - 5(10) 4 J [x - 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 J dt (10)8. We now apply partial fracting the left member and the left member and the lef simplification give In [x - 20(10)4] - x - 5(10) 15t 4 + Co (10) or x - 20(10)4 = ce - 15t /(10)4 x - 5(10)4 We apply the I.C. x(1980) = 20,000 = (20)(10)3, the left member reduces thus: (20)(10) 3 (1 - 10) = 3 6. (5) (10) (4 - 10) 4 Thus we find 6 = ce - (15) (1980)/(10)4. Thus the solution takes the form 120 Chapter 3 x - (20)(10)4 = 6 e 15 (1980-t)/(10)4 x - 5(10)4 We must solve this for x. After some algebraic manipulations, we obtain the desired result 4 x = (10)5[3 e 15 (1980-t)/(10)4 - 1 18. This falls into two problems: (A) Before the cats arrived, and (B) after the cats arrived. We let x = the number of mice on the island at time t and proceed to solve problem (A). During this time, in the absence of cats killing mice, the D.E. is simply = kx, with solution x = c e kt Choosing January 1, 1970 as time t = 0, the I.C. is x(O) = 50,000. Then, measuring t 1n years, Jan. 1, 1980 is time t = 10; and we have the additional condition x(10) = 100,000. Applying the I.C. x(O) = 50,000. 50,000, we at once find c = 50,000 and hence kt x = 50,000 e. The additional condition now gives e 10k = 2, from which $k = \ln ; 0$ ' Vith this, the solution takes the form x = 50,000 e t $\ln(2/10)$. But this solution only holds until 1980 and so will not help us to answer the stated question. It is the number k, which represents the natural rate of population increase, which we need here to solve problem (B) and answer the stated question. Turning to problem (B), Slnce the cats kill 1000 mice/ month = 12,000 from its right member. Applications of First-Order Equations 121 dx 2 Thus we have dt = kx - 12,000, where k = In 10 was found in problem (A). The D.E. is separable, and we write it In dx the form $kx_1 = 0$ for problem (B), we thus have the I.C. x(O) = 100,000. Application of this gives c = 100,000 k $\cdot 12,000$. With this value of c and k = In 1 ' we obtain the solution of problem (B): x = (102) [12,000 + (10,000 In 2 - 12,000)e t In 2/10 J The answer to the stated question is now found by letting t = 1 in this expression. We have the D.E. = kx. Since the annual rate is 6%, we let k = .06 and dx so have the D.E. dt = .06x. We also have the D.E. dt = .06x. We also have the Solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. We find x = c e. 06t The I.C. gl ves c = 1000, so we have the solution. Ye obtain 2000 = 1000 e. 06t = 2. From this .06t = 2. From this .06t = 102 and t or e N 11.55. Thus the amount will double In approximately N 11.55 years. 20. From 19, have the I.C. x(O) = xO' where X denotes the original amount. Solving the D.E., we find x kt = c e Applying the I. C. to this, we find c = Xo and so h h 1. kt ave t e so ut10n x = xOe. (a) If the original amount doubles ln two years, then x = 2x O when t = 2. Applying this to the solution, we obtain 2x O 2k From this $k = \ln 2 0.3466 = xOe 2$ Thus the annual rate is a remarkable 34.66%. (b) Here we have the additional condition x(1/2) = 3xO/2. Ye apply this to the solution. Ye have 3xO/2 = k/2 k xOe, from which e = 9/4. Then the solution becomes x = xo()t. We ask how long it will take the original amount to double, so we let x = 2x O in this solution and solve for t. We find () t = 2, from h . h In 2 0 8547 S h d bl w 1 c t = In (9/4) . . 0 team 0 un t 0 u e s 1 n approximately 0.85 years. Applications of First-Order Equations 123 21. Let x = the amount of salt at time t. We use the basic equation (3.63), = IN - OUT. We find IN = (3 lb./gal) (4 gal./min.) = 12(lb./min.); and OUT = (C lb./gal.) (4 gal./min. = lxO (lb./min.) Hence the D.E. of the problem IS = 12 - lxO ' The I.C. is x(O) = 20. The D.E. separable. We write it as dx 300 - x 15 = or dt 25 dx dt Integrating, find Inl300 - xl = 300 x. Thus we obtain the solution ln the form 300 - x = 280 e - t/25 or x = 300 - 280 e - t/25 = 140 or e - t/25Ve have IN = 0, since pure water flows into the tank; and we find OUT = (C Ib./gal.)(2gal/min), where c lb./gal. is the concentration. Since water flows in at 5 gal/min. and the mixture flows in at 5 gal/min., there is a galn of 3 gal/min of brine at time t 18 100 + 3t. Thus the ... x Th A UT 2x (Ib / concentration IS 100 + 3t. us = $100 + 3t \cdot min$.). Hence the D.E. of the problem is = 100 2 : 3t + 100 2 : 3t= 10(100) Thus we have the (100) 10 (100) 2/3 solution x = -(3t + 100)2/3. 10 = (a) Ye let t = 15 in the
solution. 10 (100) 2/3 We find x = -(145)2/3 7.81. So there is approximately 7.81 lb. The concentration is 7;4 8; 0.0539 lb./gal. (b) Since there was initially 100 gallons of brine in the tank and the amount is increasing at the rate of 3 gal./min. the number of minutes t needed to obtain a full tank of 250 gallons is given by 100 + 3t = 250; so t = 50. Ye let t = 50 in the solution and find 2/3 x = 10(100) 5.4279 250 0.0217 lb. / gal. Applications of First-Order Equations 125 25. Let x = the amount of salt at time t. Ye use the basic equation = IN -OUT. We find IN - (30 gm/liter)(4 liters/min) = 120 (gm/min); and OUT = (C gm/liter) (liters/min), where C gm/liter is the concentration. The rate of inflow is 4 liters/min of fluid in the tank. Hence at the end of t minutes the f fl ° d 0 h k 0 300 3t l O t Th th amount 0 Ul ln t e tan IS + 1 ers. us e concentration at time t is x 3t gm/liter, and so OUT - 300 + 25x 5x 3t] 3t + 600 (gm/min). 2 L 300 + 2"" = problem is = 120 - 3t x600 ' Hence the D.E. of the The I.C. IS x(O) = 50. The D.E. 1S linear. Ye write it in the standard form dx 5 120. dt + x = 3t + 600 An I.F. is e [(5/(3t+600)]dt = e(5/3)ln(3t+600) = (3t + 600)5/3. Multiplyingthrough by this we have (3t + 600)5/3 + 5(3t + 600)5/3 = 120(3t + 600)5/3. Integration gives (3t + 600)5/3 = 120(3t + 600)5/3. Thus the solution is given is given by this we have (3t + 600)5/3 + c or x = 15(3t + 600)5/3 = 120(3t + 600)5/3. Thus the solution is given is given is given in the solution in the solution is given in the solution in the solution in the solution is given in the solution in the solution is given in the solution in the solution in the solution is given in the solution in the solution in the solution is given in the solution in the solution in the solution in the solution is given in the solution in the solution in the solution is given in the solution in the solution in the solution is given in the solution is given in the solution in the so by x = 15(3t + 600) - (8950)(600)5/3(3t + 600) - 5/3. (*) The stated question asks for x at the instant when the tank overflows. Since the amount of fluid increases at the rate of liter/min and the 500 liter tank originally had 300 liters in it, this time t is given by 3; = 200 or t = 40. Ve thus let t = 40 ln (*) and obtain x (40) = () 5/3 1500(1000) - (8950) (600)5/3(1000)5/3 = 15000 - 8950 11,179.96 (grams). 26. Let x = the amount of salt at time t. Ye use the basic equation = IN - OUT. Ve find IN = (50 gm/liter)(7 liter/min), where C gm/liter IS the concentration. The rate of inflow is 5 liters/min. and that of outflow 1S 7 liters/min., so there is a net loss of 2 liters/min of fluid in the tank. Hence at the end of t minutes the amount of fluid in the tank is 200 - 2t liters. Thus the concentration at time t is 200 - 2t (gm/min). Hence the D.E. IS dt = 250 - 200 - 2t (gm/min). Hence the D.E. IS dt = 250 - 200 - 2t (gm/min). Equations 127 dx 7 dt + 200 _ 2t x = 250. An I.F. is e J [7/(200-2t)]dt = e-(7/2)ln(200-2t) - (200 - 2t)-7/2. Integration gives (200 - 2t)-7/2 x = 50(200 - 2t)-7/2 x = 50(200 - 2t)-7/2 x = 50(200 - 2t)-7/2. Integration gives (200 - 2t)-7/2 x = 50(200 - 2t)-7/2 40, we find 40 = 10,000 + c(200)7/2, from which c = -9960(200)-7/2. Thus the solution is given by (t) 7/2 = 50(200 - 2t) - 9960(1 - 100). (*) The tank will be half full when it contains 100 liters. Since it initially contained 200 liters and it loses 2 liters/min., it will contain 100 liters after 50 mln. Thus we let t = 50 in (*). Ve find $7/2 \times (50) - 5000 - 9960(1)$ 4119.65 (gm). 128 Chapter 3 30. Let x = the number of people who have the disease after t days. Then 10,000 - x people do not have it, and we have dx lJe also have the I.C. x(O) the D.E. is separable; and separating variables, we h dx k d U . . 1 f . d ave x(10,000 - x) = t. slng partla ractions an integrating, we find 1000 [lnlxl-lnll0,000 - xl] = kt + c0. Noting that x 0 and 10,000 - x 0, we x simplify and obtain In 10 000 x = ce. We apply the I.C. to , obtain c = 1/9999. 10,000 kt + c1. Then , x 10,000 kt the Applications of First-Order Equations 129 x = 10,000 [199] t/5. 1 + 9999 9999 We are asked how many people have the disease after 10 days. We let t = 10 ln this to obtain x(10) - 10,000 N N [199] 2 1 + 9999 9999 We are asked how many people have the disease after 10 days. We let t = 10 ln this to obtain x(10) - 10,000 N N [199] 2 1 + 9999 9999 478.88. Thus about 479 people have the disease after 10 days. We let t = 10 ln this to obtain x(10) - 10,000 N N [199] 2 1 + 9999 9999 We are asked how many people have the disease after 10 days. D.E. = k(IO -)(15 - 34X), the I.C. x(O) = 0, and the additional condition x(15) = 5. Separating variables and simplifying, the D.E. takes the 16 dx form 3 (x_40) (x_20) - k dt. In(x partial fractions, we have 1 [X + 40 - x + 20] dX Integrating, we obtain 1 [In(x - 40) 20] = kt + c o or In - 40 = 15 kt + C1. From - 20 4 Applying = k dt. this, x - x - 15 kt40 4 = c e 20 (*) 130 Chapter 3 We apply the I.C. x(0) = 0 to this, and at once obtain c = 2. We set c = 2 in (*), and apply the additional condition x(15) = 5. 35(1; k) 15 1; k We have 15 = 2 e and hence e = Thus we obtain the solution ()1/15 x - x - = 2()t/15 We solve this for x In terms of t, obtaining x = [(t/15 - 112) r/15 - 112] r/15 - 112part (a). To answer (b), let t = 60 in this. We have x(60) = ()4 = 11 = N 4 = 2 = 0 for all x on a x b. x 3x (a) Suppose c 1 e + c 2 e = 0 for all x on a x b. x 3x (b) Let t = 60 in this. We have x(60) = ()4 = 11 = N 4 = 2 = 0 for all x on a x b. x 3x (b) Let t = 60 in this. We have x(60) = ()4 = 11 = N 4 = 2 = 0 for all x on a x b. x 3x (c) Suppose c 1 e + c 2 e = 0 for all x on a x b. 2c 2 e 3x = 0 for all x a x < b. we on S . 2 3x f. 0 for all such x, we must have c 2 = 0. Ince e Then c 1 e x 3x = 0 reduces to x = 0 for all x on a x b - 1 - h x d 3x 1 - 1 lmp 1es t at c 1 = c 2 = 0, so e an e are ... 1near y independent on a < x < b. x 3x (b) W (e, e) = x e 3x e 4x = 2ef 0 on a < x < b, so x e 3e 3x e x and e 3x 1 - 1. d d th - - t 1 are lnearly independent, we apply Theorem 4.4. We have W(e 2x e 3x) = 2x e 3x e = 3e5x = 2e 5x - e 5x f = 0.2e 2x 3e 3x 131 = 132 Chapter 4 for all x, -CD < X < CD. Thus the two solutions are linearly independent on -CD < x < CD. (b) 2x 3x y = c + c 2 e (c) To satisfy the condition y(O) = 2, we let x = 0, y = 2 in the general solution of part (b). We have c + c 2 = 2. (*) Now we differentiate the general solution, obtaining y' = 2c 1 e2X + 3c 2 = 3x. To satisfy the condition y'(0) = 3, we let x = 0, y' = 3 in this derived equation. We have 2c 1 + 3c 2 - 3. (**) Solving the two equations (*) and c 2, - we find c = 3, c 2 = -1. Substituting these values of 1 c 1 and c 2 into the general solution of part (b), we 3 2x 3x have the desired particular solution y = e - e. This is unique by Theorem 4.1; and it is defined on -CD < x < CD. 8. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have Higher-Order Linear Differential Equations 133 x e x xe W(ex, xe x) = x e x (x + 1)e - e 2x f 0 for all x, -00 < x < 00. Thus the two solutions are linearly independent on the interval -00 < x < 00. (b) x x y = c e + c 2 xe . 1 (c) To satisfy the condition y(O) = 4, we let x = 0, y = 1 - ln the general solution, obtaining y' = c 1 e x + c 2 (x + 1) eX. To satisfy the condition y(O) = 4, we let x = 0, y = 1 - ln the general solution of part (b) . We have c 1 = 1. Now we differentiate the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (c) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x =
0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of part (b) = 4, we let x = 0, y = 1 - ln the general solution of p y' = 4 in the derived equation. We have $c \ 1 + c \ 2 = 4$. Since $c \ 1 = 1$, this yields $c \ 2 = 3$. Substituting these values of $c \ 1$ and $c \ 2$ into the general solution, we have the desired particular solution, we have the desired particular solution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have $2 1 \times 2 V(x_2, :2) \times 2 2 4$ f $0 = = - - - 2 \times x \times 2x - 3 \times 2x$ for all x, 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. the general solution of part (b). We have $c \ 2 \ 4c \ 1 + 4 = 3$. (*) Now we differentiate the general solution, obtaining $y' = c \ 1 \ e \ x + c \ 2$ (x + l)e x. To satisfy the condition y'(O) = 4, we let x = 0, y' = 4 in the derived equation. We have $c \ 1 + c \ 2 = 4$. Since $c \ 1 = 1$, this yields $c \ 2 = 3$. Substituting these values of $c \ 1$ and $c \ 2$ into the general solution, we have the desired particular solution y = eX + 3xe x. This is unique by Theorem 4.1; and it IS defined on - CD < X < (I). 10. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have $2 1 \times 2 \times (v 2 : 2) 2 2 4 f 0 x$, $= = 2 \times x \times 2x 3 x$ for all x, 0 < x < (I). Thus the two solutions are linearly independent on 0 < x < 00. Higher-Order Linear Differential Equations 135 (b) Y 2 = c 1 x c 2 + 2"- x (c) To satisfy the condition y(2) = 3, we let x = 2, Y = 3 ln the general solution, obtaining -3 y' = 2c 1 x - 2c 2 x To satisfy the condition y'(2) = -1, we let x = 2, y' = -1 in this derived equation. We have $C \ 2 \ 4c \ - = -1 \ 1 \ 4 \ (**)$. Solving the two equations (*) and $c \ 2$, 1 we find $c \ 1 = 4 \ c \ 2 = 8$. Substituting these values of C 1 and c 2, 1 we find $c \ 1 = 4 \ c \ 2 = 8$. Substituting the two equations (*) and (**) for c 1 and c 2 into the general solution of part (b) we - (4 \ 1) \ X \ 2 have the desired particular solution y $8 + 2^{"}$. This is unlque by Theorem 4.1; and it lS defined on 0 < x < (I). 136 Chapter 4 12. We use Theorem 4.4. We have -x 3x 4x e e e W(-x 3x 4x) -x 3e 3x 4e 4x e, e = -e -x qe 3x 16e 4x e 1 1 1 -x 3x 4x -1 3 4 = e e e - - 1 9 16 6x { 3 4 - 1 4 - 1 3 } = e + 9 16 1 16 19 = e 6x [12 + 20 - 12] = 20e 6x f 0 for all x, -00 < x < 00. Thus the three solutions are linearly independent on -00 < x < 00. The general solution . -x 3x 4x lS y = c 1 e + c 2 e + c 3 e 13. We have 2 4 x x 2 4 4x 3 W(x, x, x) = 1 2x 0 2 12x 2 4 4 6x 4 f 0 -= 16x - 10x = Higher-Order Linear Differential Equations 137 on 0 < x < 00. Thus by Theorem 4.4, the three glven solutions are linearly independent on 0 < x < 00. The 1 1 . . 2 4 h genera so ut10n 1S y = c 1 x + c 2 x + c 3 x, were c 1 ' c 2 and c 3 are arbitrary constants. Section 4.1.D, Page 132 L illi 1. et y = vx. Then y' = xv' + v and y = xv + 2v'. Substituting these into the glven D.E., we obtain x 2 (xv H + 2v') - 4x(xv' + v) + 4xv = 0 or x 3 v H - 2x 2 v' = 0 or xv'' - 2v' = 0. Letting w = v', we obtain x : -2w = 0 or dw = 2dx. Integrating, we find w x 2 lnlwl = 21n Ixl + Inlcl or w = cx. We choose c = 1, 3 x recall v' = w, and integrate to obtain v =. Now 4 x forming y = xv, we obtain v =. Now 4 x forming y = xv, we obtain v = 0. A true constant multiple thereof serves as the desired linearly independent solution. Choosing multiple 3, we have y = x 4. The general solution is then y = c 1 x + c 2 x 4. 4. Let y = xv. Then y' = xv' + v and yH = xv H + 2v'. Substituting these into the given D.E., we obtain (x 2 - x + 1)v H + (x 2 + 2)(1 - x)v' = 0. Letting w = v', 2 dw we obtain (x 2 - x + 1)v H + (x 2 + 2)(1 - x)v' = 0. Letting w = v', 2 dw we obtain (x 2 - x + 1)v H + (x 2 + 2)(1 - x)v' = 0. Letting w = v', 2 dw we obtain (x 2 - x + 1)v H + (x 2 + 2)(1 - x)v' = 0. Letting w = v', 2 dw we obtain (x - x + 1)v H + (x - 2 + 2)(1 - x)v' = 0. long division and partial x(x - x + 1) 138 Chapter 4 f. h " " h f dw ractlons, we express t lS ln t e orm -- = w [1 - + 22x - 1] dX. Integration now yields x - x + 1 In I wi = x - 2 In I x I + In I c I and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ We choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ We choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ We choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ We choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ We choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ we choose c = 1, recall w = y' and obtain, $y'(1 - 1 - 1) x = +x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ we choose c = 1, recall w = y' and obtain $x + 2 e + x - 2 e + x + 1 + \ln I c I$ and so Inlwl 2 = $x + \ln J c (x - x + 2x - 1) L$ we choose c = 1, recall w = y' and obtain $x + 2 e + x - 2 e + x + 1 + \ln I c I$. = e + 2" e dx, x f 1 x x x by obtain e + f : dx · Thus 2" e dx parts, we - x x y = eX - or y = eX (1 -). Now forming y = xy, we find the desired linearly independent solution is y = c 1 x + c 2 (x - 1)e · 5. 2x 2x 2x Let y = e y. Then y' = e y' + 2e y and H 2x H 4 2x . 4 2x S b ", h "Y = e y + e y, u stlutling t IS.D E () (2x u 2x glven ..., we obtain 2x + 1 e v + 4e v' + into the 4e 2x v) Higher-Order Linear Differential Equations 139 4() (2x 2 2x) 4 2x 0 - x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v' + e v + e v = or () 2x u 2x 2x + 1 e v + 4xe VI = 0 or (2x + 1) v H + 4xv' = 0 or (2x + 1) v H rewrite this as d := (-2 + 2x : 1) dx. Integrating, we have Inlw/ = -2x + Inl2x + 11 + Inlcl or -2x w = c(2x + 1)e - 2x. N f 2x bt ow orming y = ev, we 0 ain y = -(x + 1). This or any nonzero constant multiple thereof serves as the desired linearly independent solution. Choosing multiple -1, we have y = x + 1. The 2x general solution is then $y = c 1 e + c 2 (x + 1) \cdot 8 \cdot x \cdot x$ Let y = e v +
e v + e v +2v' = 0. Letting w = v', we obtain (x + x) + (x + 2x + 2)w = 0 or dw w = 2x + 2x + 2 dx. Using long division and partial x + x fractions, we express this ln the form d = [1 + z + 2] + 2 dx. Using long division and partial x + x fractions, we express this ln the form d = x + 2 dx. Using long division and partial x + x fractions, we express this ln the form d = x + 2 dx. Using long division and partial x + x fractions, we express this ln the form d = x + 2 dx. Using long division and partial x + x fractions, we express this ln the form d = x + 2 dx. 2x. (*) From the first two glven D.E.'s, F 1 (x) = 1, F 2 (x) = x, with respective given particular integrals f 1 (x) = 1/6, f 2 (x) = x/6 + 5/36. The right member of (*) is 2F 1 (x) - 12F 2 (x) = 1/3 - 2x - 5/3 = -2x - 4/3. Higher-Order Linear Differential Equations 141 We now apply the

theorem over again, this time to the third given D.E. and the D.E.(*), with F 1 (x) = 2 - 12x, F 2 (x) x x - e, f 1 (x) = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k 2 = 6. Doing so, we find the particular integral x y = -2x - 4/3, f 2 (x) = e/2, k l = 1, and k = e2x 3x Y = c l e + c 2 e Its roots are 4. The auxiliary equation is 3m 2 - 14m - 5 = 0. Its roots 5 and 1 distinct). The G.S. - and -2 l S 2 x/2 - 2x Y = c e + c e 1 2 7. The auxiliary equation lS 4m 2 - 4m + 1 = 0. Its roots are 1 (real and lS $5x - x/3 y = c l e + c 2 e \cdot 6$. The auxiliary equation lS 4m 2 - 4m + 1 = 0. Its roots are 1 (real and ls $5x - x/3 y = c l e + c 2 e \cdot 6$. The auxiliary equation lS 4m 2 - 4m + 1 = 0. Its roots are 1 (real and ls $5x - x/3 y = c l e + c 2 e \cdot 6$. The auxiliary equation lS 4m 2 - 4m + 1 = 0. Its roots are 1 (real and ls $5x - x/3 y = c l e + c 2 e \cdot 6$. $y = e(c \ 1 \sin 3x/4 + c \ 2 \cos 3x/4)$. 11. The auxiliary equation $3 \ 3m \ 2 \ 3 = 0$, which $1Sm - m + can \ be written \ m \ 2 \ (m - 3) = 0$ or (m + 1)(m - 3) = 0 or (m + 1)(m - 3) = 0 or (m + 1)(m - 3) = 0. Hence its roots are 1, -1 and 3 (real and , distinct). The G.S. $x - x \ 3x \ IS \ Y = c \ 1 \ e + c \ 2 \ e + c \ 3 \ e \ 14$. The auxiliary equation is $4m \ 3 + 4m \ 2 - 7m + 2 = 0$. Observe by inspection that -2 is a root. Then by synthetic division, -244-84-728-21 o Find the factorization (m + 2)(4m 2 - 4m + 1) = 0 and hence (m + 2)(2m - 1)2 = 0. Thus the roots are -2 (real simple 1 1 root) 2' 2 (real double root). The G.S. is Higher-Order Linear Differential Equations 143 -2x (1/2)x Y = c 1 e + (c 2 + c 3 x) e or Y - 2x + (1/2)x (1/2)x = c 1 e c 2 e + c 3 x e . 15. The auxiliary equation $3 \ 2 \ -1 \ 0$, which can $18 \ m \ m \ + m \ = be$ written $m \ 2 \ (m \ -1) \ 2 \ 1)(m \ -1) \ + \ = 0$ or $(m \ + \ = 0.$ Thus the roots are the real simple root 1 and the conjugate complex pair %i. The G.S. is $x \ y \ = \ c \ 1 \ e \ + \ c \ 2 \ sinx \ + \ c \ 3 \ cosx.$ 16 Th 1 t m $3 \ + \ 4m \ 2 \ + \ 5m \ + \ 6 \ 0 \ Db$. e auxi iary equa ion is =. serve by inspection that -3 is a root. Then by synthetic division, $-3\ 1\ 4\ 5\ 6\ -3\ -3\ -6\ 1\ 1\ 2\ 0$ find the factorization (m + 3)(m 2 + m + 2) = 0. From 2 m + m + 2 = 0, obtain = $-1\ \%\ 1\ -8\ =\ m\ 2\ 1\ \%\ 1\ -8\ +\ 1\$ o. Its roots are 4,4 (real, double root). The G.S 1S 4x Y = (c 1 + c 2 x)e or 4x y = c 1 e 4x + c 2 xe 19. The auxiliary equation 1S m 2 - 4m + 13 = o. Solving it, obtain = 4 % 16 - 52 = m 2 2 % 3i. So the roots are the conjugate complex numbers 2 % 3i. The G.S. is $y = e 2X (c 1 \sin 3x + c 2 \cos 3x)$. 22. The auxiliary equation is 4m 2 + 1 = 0. Its roots are m = *() i. The G.S. is y = clsin()x + c2COS()X. 24. The auxiliary equation is 8m 3 + 12m 2 + 6m + 1 = 0. By inspection, we find that -1/2 is a root. Then by synthetic division, $-1/2 \ 8 \ 1 \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ x + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS $Y = (c \ 1 + c \ 2 \ -x/2 \ G.S$. IS Y =3 x)e. Higher-Order Linear Differential Equations 145 27. 4 2 The auxiliary equation is m + 8m + 16 = 0 or 2 2 (m + 4) = 0. The roots are %2i, %2i. That is, each of the conjugate complex pair %2i is a double root. The G.S. is $y = (c 1 + c 2 x)\sin 2x + (c 3 + c 4 x)\cos 2x$. 28. Th "I" " 4 3 3 2 2 0 e aUXl lary equation IS m - m - m + m + = . Observe by inspection that -1 is a root. Then by synthetic division -1 1 -1 -3 -1 2 1 -2 -1 1 2 1 -2 -2 0 we find the factorization (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 which can be written (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m - 2) = 0 or (m + 1)(m 2 - 1)(m - 2) = 0 or (m + 1)(m - 2)(m - 2)(m - 2) = 0 or (m + 1)(m - 2)(m -2m + 3 = 0 gives the conjugate complex roots = -2 % 4 - 12 = m 2 - 2 % 2@ = 2 - 1:i; [2i. Thus the roots are -2, -2, -1:i; [2i. The G.S. IS -2x x Y = c 1 e + c 2 xe + e - (c 3 sin[2x + c 4 cos[2x). 31. The auxiliary equation is S 4 3 O. Factoring m - 2m + m = 3 2 3 2 gives m (m - 2m + 1) = 0 or m (m - 2m + 1)
= 0 or m (m - 2m + 1) = 0 or m (m - 2m -1 = 0. Thus the are 0, 0, 0 [triple from 3 1, roots root, factor m], 1, [double root, from factor 2 The G.S. (m - 1) J. IS (2 Ox x Y = c 1 + c 2 x + c 3 x) + (c 4 + cSx)e x Y = c 1 + c 2 x + c 3 x + c 4 e + cSx)e x Y = c 1 + c 2 x + c 3 x + c 4 e + cSx)e 5m 4 + 10m 3 + 10m 2 + Sm + 1 = 0. We recognize this as S a binomial expansion, and write it as (m + 1) = 0. Thus -1 is a five-fold root of the auxiliary equation is 6 2m 3 + 1 0 or m = (m 3 - 1)2 = 0.221)2 O. From or (m - 1)(m + m + 2)find the double real Solving the (m - 1) = 0, we root 1. quadratic m 2 + 1 = 0, we obtain + m = -1 1 - 4 = m 2 1 II 2 2 1, which is a conjugate complex palr. auxiliary equation are 1, 1, 1 - - + 2 Thus the roots of the ll. 1 ll. 2 1, -2 + 2 1, -!_lli 2 2 ' -!-lli 2 qx]. -X/2 [(). II (+ e c 3 + c 4 x Sln 2 x + c 5 148 Chapter 4 35. The auxiliary equation is 4 1 = 0. Hence seek m m + we such that m 4 = -1; that is, seek the four form z = 1 (cos 'K + i sin 'K). From complex number theory, we know that the n nth roots of z = r(cos () + i sin ()) are given by the formula 1/n n r:: [(() + 2k 'K)] = 1 r cos n ... (() + 2k 'K)] + 1 sln , n k = 0,1,2,...,n - 1. (*) Since z = 1 (cos 'K + i sin 'K), we have r = 1, () = 'K here; and since we want the four fourth roots, we take n = 4. Thus the formula (*) becomes 1/4 4 f 1 1 [(K + 2k 'K)] z = 1.1. COS 4 + 1 S ln 4 'k = 0,1,2,3. Letting k = 0, 1, 2, and 3 successively ln this, we find 'K 'K $\cos - + 1 \sin = + 14422$ ' 3'K 3'K $\cos - + 1 \sin = + 14422$ ' 5'K 5'K $\cos - + 1 \sin = + 14422$ ' 7'K 7'K $\cos - + 1 \sin = + 14422$ ' 5'K 5'K $\cos - + 1 \sin = + 14422$ ' 5'K 7'K $\sin - 14422$ ' 5'K 7'K $\sin - 14422$ they are in fact the two palrs of conjugate complex numbers * i and * i. 4 Alternatively, the auxiliary equation is m + 1 = 0, which can be written as (m + 2 + 1) - 2m = 0 or (m + 2 + 1) - 2m =1 gives the conjugate complex roots $m = f_2 + f_2 +$ auxiliary equation is $m \ 6 + 64 = o$. So we seek m such that $m \ 6 = -64$; that is, we seek the SIX sixth roots of z = -64. In the so-called polar form, this is $z = 64(\cos + i \sin)$. We use the formula (*) given in the solution of Exercise 35, with r = 64, 0 = r, and n = 6. The formula becomes zl/6 = 6.64 [COS(r + 6.2kr) + 1 Sin(r + 6.2kr)], k = 0, 1, 2, ..., 5Letting k = 0, 1, 2, 3, 4, and 5 successively ln this, we find 2[COS] 2[q + i()] .[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1, 2 [cos 7 sin 7 6 r] 2[-E + i(-)] -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1 sln = 2 - [5 2 cos If + 1 sln 5 6 r] = 2[-q + iq)] = -.[3 + 1 sln 5 6 r] = -.[3 + 1 slnrespectively. These are the six sixth roots of -64. Thus they are the six roots of the auxiliary equation m 6 + 64 = 0. Note that they are in fact the three pairs of Higher-Order Linear Differential Equations 151 conjugate complex numbers 2i, [3 1, and -[3 1. Thus the G.S. of the D.E. is $y = c 1 \sin 2x + c 2 \cos 2x + e[3x(c 3 Sinx + c 4 \cos x) - [3x + e (c 5 Sinx + c 4 \cos$ $\sin x + c \ 6 \ \cos x$). 37. The auxiliary equation is m 2 - m - 12 = 0. Its roots are 4, -3 (real
and distinct). The G.S. is $4x \ Y = c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ From this, $4x \ y' = 4c \ e \ 1 \ -3x \ 3c \ 2 \ e \ (A)$ Thus the solution of the stated I.V.P. is 4x - 3x y = 2e + e 40. The auxiliary equation is 3m 2 + 4m - 4 = 0, which can be written (3m - 2)(m + 2) = o. Its roots are 2/3, -2 (real and distinct). The G.S. IS 2 - x - 2x 3 (A) y = c + c 2 = 152 Chapter 4 From this, 2 2 3 x y' = c + c 2 = 152 Chapter 4 From this, 2 2 3 x y' = c + c 2 = 152 Chapter 4 From this, 2 2 3 x y' = c + c 2 = 2. Apply condition y(0) = 2 to (A) to obtain c + c 2 = 2. Apply condition y(0) = 2 to (A) to obtain c + c 2 = 2. Apply condition y(0) = 2 to (A) to obtain c + c 2 = 2. condition y'(O) = -4 to (B) to obtain c 1 - 2c 2 = -4. From these two equations 1n c 1 and c 2, find c 1 = 0, c 2 = 2. Thus the solution of the stated . -2x I.V.P. 1S Y = 2e . 41. The auxiliary equation is m 2 + 6m + 9 = O. Its roots are -3, -3 (real double root). The G.S. is $-3x y = (c 1 + c 2 x)e \cdot (A)$ From this, $-3x y' = (-3c 1 + c 2 - 3c 2 x)e \cdot (B)$ Apply condition y(O) = 2 to (A) to obtain $c_1 = 2$. Apply condition y'(O) = -3 to (B) to obtain $-3c_1 + c_2 = -3$. From this, find $c_2 = 3$. From this, find $c_2 = 3$. Thus the solution of the stated . -3x I.V.P. IS $Y = (2 + 3x)e_1 - 3x I.V.P.$ IS Y = (2 + 3Differential Equations 153 1 3 x y = (c 1 + c 2 x)e (A) From this, 1 y' = (c 1 + c 2 + c 2 x)e 3 X (B) Apply condition y(O) = -1 to (B) to obtain c 1 + c 2 = -1. From this, find c 2 = -2. Thus the solution of the stated 1 - x I.V.P 1S Y - (3 - 2x)e 3 45. 2 The auxiliary equation is m - 4m + 29 = 0. obtain the conjugate complex roots Solving, = 4 16 - 116 = m 2 4 10i 2 = 2 5i. The G.S. 1S y = e 2x (c 1 sin 5x + c 2 cos 5x) (A) From this, 2x y' = e [(2c 1 - 5c 2)sin 5x + (5c 1 + 2c 2)cos 5x]. (B) Apply condition y(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 0) = ° or c 2 = o. 154 Chapter 4 Apply condition y'(O) = 5 to (B) to obtain eO(c 1 sin 0 + c 2 cos 0) = ° or c 2 = o. 154 Chapter 4 Apply condition y'(O) = 5 to (B) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). (B) Apply condition y'(O) = 0 to (A) to obtain eO(c 1 sin 0 + c 2 cos 5x). 1 + 2c 2) cos OJ = 5 or 5c 1 + 2c 2 = 5. From this, c 1 = 1. Thus the solution of the stated I.V.P. . 2x . 5 IS Y = e Sln x. 49. The auxiliary equation is 9m 2 + 6m + 5 = 0. Solving, obtain the conjugate complex roots m = -6 36 - (36)(5) = 18 - 6 12i 18 = 1 2 3 3 1. The G.S. IS 1 Y = e - 3 X(c 1 sin x + c 2 cos x). (A) From this, 1 y' - 3 X[(1 2) 2 = e c 1 - 2 3 3 1]. $3 c 2 Sln 3 x 3 + (1 c 2) cos x] \cdot c - 31 (B)$ Apply condition y(O) = 6 to (A) to obtin c 2 = 6. Apply condition y'(O) = 0 to (B) to obtain c 1 - ; c 2 = 0. Higher-Order Linear Differential Equations 155 From this, c 1 = 3. Thus the solution of the stated I.V.P. is 1 - 3 X(3sin 2 + 6 CDS x) Y = e -x 3 or 1 3e - 3 X(. 2 + 2 CDS x). Y = sln - x 3 50. The auxiliary equation is 4m 2 + 4m + 37 - 0. Solving, obtain the conjugate complex roots m = -4 16 - 16(37) = 8 - 4 24 8 = 1 2 3i. -x/2 The G.S. IS Y = e (c 1 sin 3x + c 2 cos 3x). (A). From this, y' = e - X / 2 [(-; c 1 - 3C 2) sin 3x + (3C 1 - C 22) CDS 3X]' (B). Apply condition y(O) = 2 to (A) to obtain 3c 1 - = -4 to (B) to From this, c1 = -1. Thus the solution of the stated I. V. P. is y = e - x/2 (-sin $3x + 2 \cos 3x$). 51. Th. 1... 3 6 2 11 6 0 e aUXl lary equation IS m - m + m - =. by inspection that 1 is a root. Then by synthetic division, Observe 1 1 - 6 1 - 5 6 1 - 5 6 0 156 Chapter 4 find the factorization 2 = 0, which can (m - 1)(m - 5m + 6) be written (m - 2)(m - 2) Equations 157 find the factorization (m + 1)(m - 2) = 0. Thus the roots are -1 (real simple root) and 2,2 (real double root). The G.S. IS -x 2x Y = c 1 e + (c 2 + c 3 x)e / I -x () 2x Y = c 1 e + 4c 2 + 4c 3 x e (B) (C) Apply condition y(0) = 1 to $e^{-} e^{+} - xe^{9} 9 3 54.32$ The auxiliary equation lS m - 5m + 9m - 5 = 0. Observe by inspection that 1 is a root. Then by synthetic division, 1 1 - 5 1 1 - 4 9 - 4 5 - 5 5 0 158 Chapter 4 Find the factorization (m - 1)(m 2 - 4m + 5) - 0. From 2 m - 4m + 5 = 0, obtain = 4 :!: 16 - 20 m 2 2 :!: 1. Thus the roots are the real simple root 1 and the conjugate complex roots 2 :!: 1. The G.S. is x 2x(.) y = c 1 e + e c 2 s 1n x + c 3 cos x (A) From this, y' = c 1 e x + e 2X [(2c 2 - c 3) sin x + (4c 2 + 3c 3) cos x] (C) Apply condition y(O) = 1 to (B) to obtain c 1 + c 2 + 2c 3 = 1. Apply condition y(O) = 0 to (A) to obtain c 1 + c 3 = 0. Apply condition y'(O) = 1 to (B) to obtain c 1 + c 2 + 2c 3 = 1. Apply condition y'(O) = 1 to (B) to obtain c 1 + c 2 + 2c 3 = 1. Apply condition y'(O) = 0 to (A) to obtain c 1 + c 3 = 0. yH(O) = 6 to (C) to obtain c 1 + 4c 2 + 3c 3 = 6. The solution of these three equations ln c 1 ' c 2 ' c 3 lS c 1 = 1, c 2 = 2, c 3 = -1. Thus the solution of the stated I.V.P. is y = eX + e 2x (2sinx - cos x). Higher-Order Linear Differential Equations 159 The auxiliary equation is 4 3 2 o or 55. m - 3m + 2m = 2 and so its m (m - 1)(m - 2), roots are m = 0, 0, 0. The solution of the stated I.V.P. is $y = eX + e^{2x} (2sinx - cos x)$. , 2.
Thus the general solution of the D.E. IS x 2x Y = c 1 + c 2x + c 3e + c 4e(1) Its first three derivatives are y' = c 2 + x 2x c 3e + 2c 4e 2x + 4c 4e, (2) (3) (4) "x y = c 3e and ", x Y = c 3e respectively. Apply I.C. y(O) = 2 to (1), I.C. y'(O) = 2 to (2), I.C. yH(O) = 2 to (3), and I.C. y''(O) = 2 to (4). We obtain respectively c 1 + c 3 + c 3e4 = 2, c = 3 + 4c = 2, c = -2 + 0 = -2. Finally, the first equation gives c = -2 + 0 = -2. part of the general solution corresponding to it is 2 3 4x (c 1 + c 2 x + c 3 x + c 4 x)e. 160 Chapter 4 Since the conjugate complex numbers 2 + 3i and 2 - 3i are each 3-fold roots of the auxiliary equation, the corresponding part of the general solution 1S $2x^2 \cdot 2 e [(c 5 + c 6 x + c 7 x) sln 3x + (c 5 + c 7$ 3 4x Y - (c 1 + c 2 x + c 3 x + c 4 x)e + e 2x [(C s + c 6 x + c 7 x 2) sin 3x 2 + (c S + cgx + c 10 x) cos 3x]. 61. Since Sln x 18 a solution of the D. E., m = :!: i must be roots of the auxiliary equation m 4 + 2m 3 + 6m 2 + 2m + 5 = 0, and hence (m - i)(m + i) = m 2 + 1 must be a factor of m 4 + 2m 3 + 6m 2 + 2m + 5. By long division, we find the other factor is m 2 + 2m + 5. Hence in factored form the auxiliary equation are the two pairs of conjugate complex numbers :!: and -1 :!: 2i. The G.S. of the D.E. is -x) y = c 1 sin x + c 2 cos x + e (c 3 sin 2x + c 4 cos 2x + c Higher-Order Linear Differential Equations 161 62. Since eXsin 2x is a solution of the D. E., m = 1 : !: 2i must be a factor of m 4 + 3m 3 + m 2 + 13m + 30 = 0, and hence [m - (1 + 2i)][m - (1 - 2i)] = m 2 - 2m + 5 must be a factor of m 4 + 3m 3 + m 2 + 13m + 30 = 0, and hence [m - (1 + 2i)][m - (1 - 2i)] = m 2 - 2m + 5 must be a factor of m 4 + 3m 3 + m 2 + 13m + 30 = 0. m + . lnce m + m + = (m + 2)(m + 3), the auxiliary equation in factored form is (m + 2)(m + 2 and its roots 3)(m - 2m + 5) - o. are m = -2, -, -31: I: 2i. Thus the general solution is , -2x - 3x + e X (c 3 sin $2x + c 4 \cos 2x$). y = c 1 e + c 2 e Section 4.3, Page 159 Exercises 1 through 24 follow the pattern of Examples 4.36, 4.37 and 4.38 on Pages 154-159 of the text. The five steps of the method outlined on pg. 152 are indicated in each solution. 1. The corresponding homogeneous D.E. is " - 3y' + 2y = 0, with roots 1, - 2. The complementary function x 2x The lS y = c e + c 2 e c 1 NH term is a constant multiple of the DC function given by 2 x. Step 1 Form the DC set of 2 x . 2 It is Sl = {x,x,l}. Step 2 : This step does not apply, since there IS only one UC set present. Step 3 : An examination of the complementary function shows that none of the functions in Sl is a solution of 162 Chapter 4 the corresponding homogeneous D.E. Hence Sl does not need revision. Step 4 : Thus the original set Sl remains. Form a linear combination of its three members. Step 5 : Thus we take 2 Y = Ax + Bx + C p as a particular solution. Then y , = 2Ax + B, p y " = 2Ax + C p as a particular solution. Then y , = 2Ax + B, p y " = 2Ax + C p as a particular solution. Then y , = 2Ax + B, p y " = 2Ax + C p as a particular solution. Then y , = 2Ax + B, p y " = 2Ax + C p as a particular solution. Then y , = 2Ax + B, p y " = 2Ax + C p as a particular solution. Then y , = 2Ax + B + C p as a particular solution. obtain 2A = 4, -6A + 2B - 0, 2A - 3B + 2C - 0. From these, we find A = 2, B = 6, C - 7. Thus we obtain the particular integral Y p = 2x + 6x + 7. Higher-Order Linear Differential Equations 163 The G.S. of the D.E. is x 2x 2 Y = c 1 e + c 2 e + 2x + 6x + 7. Higher-Order Linear Differential Equations 2 m + 20. with roots IS m + = -1::: 1. The complementary function IS - x c 2 cos x). The NH Yc - e (c 1 sin x + term IS a constant - multiple of the UC function shows that none of the functions in 4x. It $IS SI = \{sin 4x, cos 4x\}$. Ste p 2: This step does not apply. Steo 3 : An examination of the complementary function shows that none of the functions in 4x. It $IS SI = \{sin 4x, cos 4x\}$. Ste p 2: This step does not apply. Steo 3 : An examination of the complementary function shows that none of the functions in 4x. It $IS SI = \{sin 4x, cos 4x\}$. Sl is a solution of the corresponding homogeneous D.E. Hence Sl does not need reVlslon. Step 5 : Thus the original set Sl remains. Form a linear combination of its two members. Step 5 : Thus we take $y = A \sin 4x + B \cos 4x$. 164 Chapter 4 We substitute in the D.E., obtaining -16A sin 4x - 16B cos 4x + 8A cos 4x - 8B sin 4x + 2B cos 4x = 10 sin <math>4x - 14B = 0.7 From these, we find A = -13 ' B - the particular integral 4 - 13. Thus we obtain Y p = -(13) sin 4x - (14A - 8B) sin 4x - (14A - 8B) sin <math>4x - 10 sin 4x - 10 si) cos 4x. The G.S. of the D.E. is $Y = e \cdot x$ (c 1 sin $x + c 2 \cos 3x$) - (1 7 3) sin $4x - (1) \cos 4x$. 8. The corresponding homogeneous D.E. is y'' + 2y' + 10y = 0. The auxiliary equation is $x \cdot y \cdot c = e(c 1 \sin 3x + c 2 \cos 3x)$. The NH term IS a constant -2x multiple of the UC function given by xe -2x SteD 1 : Form the UC set of xe -2x St = {xe,e}. It IS SteD 2 : This step does not apply. Higher-Order Linear Differential Equations 165 Step 3 : An examination of the corresponding homogeneous D.E. Hence St does not need . . reV1Slon. Step 4 : Thus the -2x 10Axe + (-2A + 10B)e = 5xe. We equate coefficients of like terms on both sides of this to obtain 10A = 5, -2A + 10B = 0. From these, we find A = 1 1 2 ' B = 10. Thus we obtain the particular integral -2x xe Y p = 2 + -2x e 10. The G.S. of the D.E. is -2x + 2x e 10. The corresponding the particular integral -2x xe Y p = 2 + -2x e 10. The definition of the definition of the particular integral -2x xe Y p = 2 + -2x e 10. The definition of homogeneous D.E. is y'' + 4y = 0. The auxiliary equation is m 2 + 4 = 0, with conjugate complex roots m = 2i. The complementary function IS $y C = c 1 \sin 2x + c 2 \cos 2x$. The NH member is a linear combination of the UC functions sin 2x and cos 2x. Steo 1 : Form the UC set of each of these two DC functions: SI = {sin 2x, cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x, sin 2x + c 2 cos 2x}, S2 = {cos 2x}, 2x}. Steo 2 : Each set is identical with the other, so we only keep one of them, say 81. Steo 3 : Observe that each member of Sl is included ln the corresponding homogeneous equation. Thus we multiply each member of Sl by x to
obtain the revised set 81' = {xsin2x,xcos2x}, whose members are not solutions of the homogeneous D.E. SteD 4 : We now have only the revised set Sl' = { $x \sin 2x + Bx \cos 2x + Asin 2x + Bx \cos 2x + Bx$ $\sin 2x$. p Higher-Order Linear Differential Equations 167 We substitute into the D.E., obtaining [-4Ax sin 2x + 4A cos 2x - 4B sin 2x] + 4 [Ax sin 2x + 8 cos 2x] + 4 [Ax sin Thus we obtain the particular integral $y = 2x \sin 2x + c 2 \cos 2x$. The G. S. is $y = c 1 \sin 2x + c 2 \cos 2x + 2x \sin 2x - x \cos 2x$. The corresponding homogeneous D.E. is y'' - 4y - 0. The auxiliary equation is m 2 4 = 0 or (m + 2)(m 2) = 0, with roots m = 2, -2. The complementary function IS 2x - 2x The NH member is a constant multiple Yc = c 1 e + c2 e of the UC functions xe 2x. SteD 1 : Form the UC set of xe 2x 2x SI = {xe ,e }. SteD 2 : This step does not apply. Step 3 : Observe that the member of e 2x of SI is included in the revised set , 2 2x 2x SI = {xe ,e }. $= \{x e, xe\}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = \{x e, xe\}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x e, xe}, whose members are not solutions of the homogeneous D.E. 168 Chapter 4 Step 4: We now have only the revised set, 2 2x 2x Sl = {x$ 2x, = 4Ax 2 e 2x + (8A + 4B)xe 2x + (2A + 4B)e 2x - (2A + 4B)e = 16xe or 2x 2x 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A + 4B)e = 16xe or 2x 8Axe + (2A obtain the particular integral y = p 2x - 2x 2 2x The G.S. is y = c 1 e + c 2 e + 2x e 2 2 2x 2x x e - xe 2x xe 15. The corresponding homogeneous D.E. m 4y " + y' - 6y IS Y + - - o. The auxiliary equation 3 4m 2 - 6 O. By IS m + + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = inspection note that m = 1 IS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - 1)(m + m = 2(m + 3). Thus the roots of the auxiliary equation are m = 1, -2, -3. The complementary ft. x - 2x -3x Th NH t. unc 10n lS Yc = c 1 e + c 2 e + c 3 e e erm lS a inear combination of the UC functions: $2 Sl = \{x, x, l\}, S2 = \{1\}$. SteD 1 : Form the UC functions: $2 Sl = \{x, x, l\}, S2 = \{1\}$. SteD 1 : Form the UC functions: $2 Sl = \{x, x, l\}, S2 = \{1\}$. SteD 1 : Form the UC functions: $2 Sl = \{x, x, l\}, S2 = \{1\}$. 2: Set S2 is completely included ln 51' so S2 IS omitted, leaving just S1. SteD 3: An examination of the corresponding homogeneous D.E. Hence S1 does not need revision. SteD 4: Thus the original set S1 remains. We form a linear combination of its three members. SteD 5: Thus we take y p 2Ax + B, y H = 2A, Y m p p obtaining = $Ax^2 + Bx + C$. Then y, = p = 0. We substitute into the D.E., o + 4(2A) + (2Ax + B) - 6(Ax 2 + Bx + C) = -18x 2 + 1 or 2 2 - 6Ax + (2A - 6B)x + (8A + B - 6C) = -18x + 1. We equate coefficients of like terms on both sides of this to obtain -6A = -18, 2A
6B = 0, 8A + B - 6C = 1. From these, we synthetic division or otherwise) we obtain the factored form (m - 2 5) From this, the roots of l)(m + 2m + = o. the auxiliary equation are 1 and -1 % 2i. The x -x complementary function is $Yc = c 1 e + e (c 2 \sin 2x + c 3 \cos 2x)$. The NH term lS a linear combination of the UC functions given by sin 2x, x 2, x, and 1. Step 1 : Form the UC set of each of these four UC 2 functions: $SI = \{sin 2x, cos 2x\}, S2 = \{x, x, 1\}, S3 = \{x, 1\}, S4 = \{1\}, S4 =$ Hence neither Sl nor S2 needs revision. Step 4 : Thus the original sets Sl and 52 remain. Ye form a linear combination of their five members. Higher-Order Linear Differential Equations 171 SteD 5 : Thus we take $y = A \sin 2x + B \cos 2x + Cx^2 + Dx + E p$ as a particular solution. Then Yp, = 2A cos 2x + 2Cx + D, "-4A sin 2x 4B cos 2x + Cx 2 + Dx + E p as a particular solution. Then Yp, = 2A cos 2x + Cx 2 + Dx + E p as a particular solution. 2C, Yp = III -SA cos 2x + SB sin 2x. Yp = We substitute ln the D. E , obtaining (-SA cos 2x + C) + 3(2A coequate coefficients of like terms on both sides of this to obtain -9A + 2B = 5, -2A - 9B = 0, -5C = 10, 6C - 5D = 3, 2C + 3D - 5E = 7. From these, we find A - -77 ' B = 2.17 ' C = -2, D = -3, E = -4. Thus we obtain the particular integral $y = p.9 \sin 2x.17.2 \cos 2x.2.2.9x.82 + 17 - x - 1r - 25 - 172$ Chapter 4 The G.S. of the D.E. IS x - xy = c.1 e + e.(c.2 sin 2) $2x + c 3 \cos 2x$) 9 sin 2 x 17 + 2 cos 2x 2 - 2x - 3x - 4. 17 19. The corresponding homogeneous D.E. The auxiliary equation is m 2 + m - -3. The complementary function is lS y" + y' - 6y = O. 6 = 0, with roots 2, 2x - 3x Y c = c 1 e + c 2 e The NH term is a linear combination of the UC functions 2x 3x e , e , x and 1. SteD 1 : Form the UC set of each of these four functions: $2x 3x Sl = \{e\}, S2 = \{e\}, S3 = \{x,l\}, S4 = \{1\}$. SteD 2 : Set S4 is completely included in S3; so S4 lS omitted, leaving the three sets Sl' S2' and S3. Step 3 : Observe that the only member e 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x of 8 1 = $\{e 2x\}$ 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus a solution of multiply the member e of Sl by x to obtain the revised set S ' 1 $2x = \{xe\}$, whose only member IS not a solution of the homogeneous D.E. 2x SteD 4 : We now have the three sets Sl' = $\{xe\}$, 8 $2 = \{e 3x\}$, 8 $3 = \{x,l\}$. Ve form a linear combination of the ir four elements. Higher-Order Linear Differential Equations 173 Steo 5 : Thus we take 2x 3x $Y p = \{xe\}$, 8 $2 = \{e 3x\}$, 8 $3 = \{x,l\}$. Ve form a linear combination of the ir four elements. equate coefficients of like terms on both sides of this to obtain 5A = 10, 6B = -18, -6C = -6, C - 6D = -11. From this, we find A = 2, B = -3, C = 1, D = 2. We thus obtain the particular integral $2x \ 3x \ Y = 2xe - 3e + x + 2$. The corresponding homogeneous D.E. is y'' - 4y' + 5y = 0. The auxiliary equation is m 2 - 4m + 5 = 0, with roots 2 % 1. The complementary function is 2x (.) Y c = e c 1 Sln x + c 2 cos x. The NH term lS a constant 2x multiple of the UC function glven by e cosx. 2x Steo 1 : Form the UC set of e cos x. 2x Steo 1 : Form th : ot mem ers e cos x an e SIn x 0 1 are included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of set SI by x to obtain the revised S ' { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } h . h h b set 1 = xe cos x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { now a vet e set l' = xe cos x, xe s 1 n x, and we form a linear combination of its two members. Steo 5 Thus we take A 2x. B 2x Y = xe Sln x + xe cos x p as a particular solution. Then y, p 2x. $2x = (2A - B)xe \cos x 2x$. $2x + Ae slnx + Be \cos x = (3A - 4B)xe 2x \sin x + (4A - 2B)e 2x \sin x + (2A + 4B)e 2x \cos x$. 4A = 3, B = 0 and A = 3, B = 0. The set of this is the particular solution of the set of th $1 \sin x + c 2 \cos x + xe Slnx$. 176 Chapter 4 24. The corresponding homogeneous D.E. is ym 2yH y' + 2y = 0. The auxiliary equation lS 322 m - 2m - m + 2 = 0, that is (m - 1)(m - 2) = 0. From this, the roots are 1, -1, 2. The complementary function x - x 2x lS Yc = c 1 e + c 2 e + c 3 e The NH term is a linear combination of the UC functions given by e 2x and e 3x. SteD 1: Form the UC set of each of these two functions: $2x 3x Sl = \{e\}$, $S2 = \{e\}$ we multiply the member e 2x of Sl by x to obtain the revised 2x set Sl' = $\{xe\}$ whose only member lS not a solution of the homogeneous D.E. S u S $\{2x\}$ S $\{e 3x\}$. teo 4 : we now have the two sets l' = xe , 2 = We form a linear combination of their two elements. 2x 3x Steo 5 : Thus we take y = Axe + Be as a particular p 2x 2x 3x solution. Then y , = $2Axe + Ae + 3Be \cdot p + 4Ae 2x + 9Be 3x = + m SAxe 2x + 27e 3x = + 27e 3x = - 2(4Axe 2x + 4Ae 2x + 9Be 3x) = 2(4Axe 2x + 4Ae 2x + 9Be
3x) = 2(4Axe 2x + 4Ae 2x + 9Be 3x) = 2(4Axe 2x + 4Ae$ Ve equate coefficients of like terms on both sides of this to obtain 3A = 9, B = -8, and hence A = 3, B = -1. Thus $2x \ 3x \ y = c \ 1 \ e + c \ 2 \ e + c \ 3 \ e + 3xe - e \ 25$. The corresponding homogeneous D.E. is $x - x \ 2x \ 3x \ y = c \ 1 \ e + c \ 2 \ e + c \ 3 \ a \ 3 \$ % i. The complementary function is y c = c 1 + c 2 sin x + c 3 cos x. The NH term is a linear combination of the UC functions: $2 Sl = \{x, x, l\}$, $S2 = \{sin x, cosx\}$. SteD 2 : Neither set is identical with nor included ln the other, so both are retained. SteD 3 : Observe that the member 1 of Sl is included in the complementary function (in the c 1 = c $1 \cdot 1$ term) and so is a solution of the corresponding homogeneous D.E. Thus we 178 Chapter 4 multiply each members of S2 are included in the complementary function. Thus we multiply each member of S2 by x to obtain the revised set S2' = {x sin x, xcosx}, which has no members which are solutions of the homogeneous D.E. 3 2 Step 4 : Ve now have the two sets S1' = {x sin x, xcosx}. We form a linear combination of their five elements. Step 5 : Thus we take yp cos x. Then = Ax 3 + Bx 2 + Cx + Dx $\sin x + Ex 2 - Ex \sin x yp = -3E \cos x + Ex \sin x - 3D \sin x + E \cos x + Ex \sin x - 3D \sin x + E \cos x + Ex \sin x - 3D \sin x + E \cos x + Ex \sin x - 3D \sin x yp = -3E \cos x + Ex \sin x - 3D \sin x - 3E \cos x + Ex \sin x - 3D \sin x + E \cos x + Ex \sin x + D \sin x + E \cos x + E \sin x + D \sin x + E \cos x + E \sin x + D \sin x + E \cos x + E \sin x + D \sin x + E \sin$ 2 + 2Bx + (6A + C) - 2D sin x - 2E cos x = 2x 2 + 4 sin x. or Higher-Order Linear Differential Equations 179 We equate coefficients of like terms on both sides of this to obtain 3A = 2, 2B = 0, 6A + C = 0, -2D = 4, -2E = 0, and hence 2 B 0, C - 4 D E o. Thus A = -2, = -2 = we 3, -(23)X 3 obtain the particular integral yp The G.S. of the D.E. is 4x $-2x \sin x$. (23) X3 - 4x - 2x S1 " n X. Y = C1 + c2 s1n x + c3 cos x + 26. The corresponding homogeneous D.E. is ylV ym + 2yH = o. Th . 1 . . . 433220 e aUXl lary equation is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c1 + c2 x + c3 e + c4 e The NH term is a linear - 1 (m - 2) = 0, with roots 0,0 (real double root), 1, 2. The x 2x combination of the UC functions given by e, e, and x. Step 1 : Form the UC set of each of these three UC -x 2x functions: $51 = \{e\}, S2 = \{e$ corresponding homogeneous D.E. Thus we multiply each 2x member of 52 by x to obtain the revised set $S2' = \{xe\}$, whose member of 53 by x 2 to obtain the revised set 180 Chapter 4 $83' = \{x3, x2\}$, whose members are not solutions of the homogeneous D.E. (note that multiplication by x, instead of x 2, is not sufficient here). -x SteD 4: Ve now have the three sets $51' = \{e\}$, $83' = \{x3, x2\}$. Ve form a linear combination of their four elements. x 2x 3 2 Step 5: Thus we take Y p = Ae- + Bxe + Cx + Dx as a particular solution. Then -x + 2Bxe 2x 2x 2 Yp , = -Ae + Be + 3Cx + 2Dx, " -x 4Bxe 2x 4Be 2x + 6C + 2D, Yp = Ae + HII -x + SBxe 2x 12Be 2x + 6C, Yp = -Ae + . 2x 32Be 2x . 1V -x yp = Ae + 16Bxe + Ve substitut into the D.E., obtaining -x 2x 2x -x 2x (Ae + 16Bxe + 32Be) - 3(-Ae + SBxe + 12Be 2x + 6C) + 2(Ae - x + 4Be2x + 4Be 2x - x 2x + 6Cx + 2D) + 2(Ae - x + 4Be2x + 4Be 2x - x 2x + 6Cx +
2D) + 2(Ae - x + 4Be2x + 4Be 2x - x 2x + 6Cx + 2D) + 2(Ae - x + 4Be 2x + 2D = 3e + 6e - 6x or -x 2x 6Ae + 4Be + 12Cx + (-ISC + 4D) - x 2x = 3e + 6e - 6x. Ve equate coefficients of like terms on both sides of this to obtain 6A = 3, 4B = 6, 12C = -6, -18C + 4D = 0, and Higher-Order Linear Differential Equations 181 1 B 3 C 1 D 9 Thus obtain the hence A = 2' = 2' = -2' = -4. we particular integral 1 - x 3 2x 1 3 9 2 y = -e + 4. -xe - x - x - x - p 2 2 2 4 x The G.S. of the D.E. is $y = c 1 + c 2 x + c 3 e 1 - x 3 2x 1 3 9 2 + 2 e + 2 x - 4 x \cdot 2x + c 4 e 27$. The corresponding homogeneous D.E. IS III - 6y = 0. The auxiliary equation y + 1S 3 2 11m - 6 = 0. By inspection we find that m 1 m 6m + = 1S a root. Then by synthetic division, we have (m 1)(m 2 - 5m + 6) = 0 or (m - 2x + c 4 e 27) and (m - 2x + c 4 l(m - 2)(m - 3) = 0. Thus the roots are m = 1, 2, 3. The complementary x 2x 3x function IS Yc = c l e + c 2 e + c 3 e The NH member IS a x 2x 4x functions: Sl = {xe, e}, Sl = {e}, Sl = {e In another, so each IS retained. x SteD 3 : Observe that member of SI by x to obtain the revised set S ' { 2 x X } h b t 1 t o f th 1 = x e , xe , w ose mem ers are no so u 10ns 0 e homogeneous D.E. Similarly, member of S2 is included in Yc' so we multiply each member of 52 by x to obtain the revised set S2' = {x 2 x 3. 182 Chapter 4 Step 4 : Ve now have the two revised sets S1' = {x 2 e x, xe x} and 52' = Step 5 : Thus we take y = Ax e + Bxe + Cxe + De as a p particular solution. Then 2 x + B)xe x x 2 x y = Ax e + (4A + B)xe + (2A + 2B)e + 4Cxe 2x 4Ce 2x + 16De 4x + ", 2 x x x Y p = Ax e + (6A + 3B)e + SCxe 2x 2x 64De 4x + 12Ce + We substitute ln the D.E., obtaining 2 x x x 2x [Ax e + (6A + B)xe + (6A + B)xe + (6A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe x + Bex + 2Cxe 2x + Ce 2x + 4De 4x] = 6[Ax 2 e x + (2A + B)xe + (2A + B)xeHigher-Order Linear Differential Equations 183 "We equate coefficients of like terms on both sides of this to obtain 4A = 1, -6A + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1. Thus = 4 + 2B = 0, -c = -4 6D = 6. From , the equation A = 1, -c = -4 6D = 6. From A = 1 A = 1, -c = -4 6D = 6. From A = 1 A = 1, -c = -4 6D = 6. From A = 1 A = 1. 3 e 1 2 x + - x e 4 3 x 2x 4x + 4 xe + 4xe + e = 0. By inspection we find that m = 1 1S a root. Then by synthetic division, we have (m - 1)(m 2 - 3m + 2) = 0 or (m - 1)2(m - 2) = 0. Thus the roots are m = 1, 1 (real double root), and 2. The 1 $f..() x 2x comp ementary unction IS Yc = c 1 + c 2 x e + c 3 e The 2 x NH member is a linear combination of the UC functions: 2x x x x 51 = {x e, x e, e}, 52 = {e}. SteD 2 : "We have S2 (SI' so we discard 52' leaving only SI. 184 Chapter 4 x x Step 3 : Observe that the members$ e and xe of S1 are both included in the complementary function and so are solutions of the corresponding homogeneous equation. Thus we multiply each member of 51 by x 2 to obtain the revised 4 x 3 x 2 x set S1' = {x e, x e}, x e }, whose members are not solutions of the homogeneous D.E. Step 4 : Ve now have
the set S1' 4 x 3 x 2 x . = {x e, x e}, x e } }, WhICh was revised in Step 3. Ve form the linear combination 4 x 3 x 2 x Ax e + Bx e + Cx e of the three elements of 51'. 4 x 3 x 2 x Step 5: Thus we take y = Ax e + Bx e + Cx e as a p particular integral. Then y, p 4 x 3 x 2 x x = Ax e + (4A + B)x e + (3B + C)x e + 2Cxe, "Yp = Ax 4 e x + (SA + B)x 3 e x + (12A + 6B + C)x 2 e x x x + (6B + 4C)xe + 2Cxe2Ce, III Yp 4 x 3 x 2 x = Ax e + (12A + B)x e + (36A + 9B + C)x e x (A + 1SB + 6C)x e + (6B + 4C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (6B + 4C)x e + (24A + 1SB + 6C)x e + (24(4A + B)x e + (3B + C)x e x 4 x 3 x 2 x + 2Cxe] - 2[Ax e + Bx e + Cx e] - 2[C = 1/2. Thus we obtain the particular integral 1 4 x yP = -4 x e 3 x - x e 1 2 x + 2 x e · The G.S. of the D.E. IS x 2x Y = (c 1 + c 2 x) e + c 3 e 1 4 x - x e 4 3 x x e 1 2 x + 2 x e · 31. The corresponding homogeneous D.E. is yH + Y = O. The auxiliary equation is m 2 + 1 = 0, with roots *i. The complementary function is y c = c 1 Sln x + c 2 cos x. The NH term is the UC function x sin x. SteD 1 : Form the UC set of this UC function. It IS SI = {x sin x, xcosx, sin x, cosx}. SteD 2 : This step does not apply. SteD 3 : Observe that the members sin x and cos x of SI are included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of SI by x to obtain the revised set S ' { 2. 2 } h b 1 = x slnx, x cosx, xSlnx, xcosx W ose mem ers are not solutions of the homogeneous D.E. 186 Chapter 4 Step 4 : Ve now have the set Sl'. We form a linear b . . f . f 1 2 . 2 . com lnatlon 0 ltS our e ements x Sln x, x cos x, x Sln x, and x cos x. Step 5 : Thus we take A 2 . B 2 C . D y = XSlnx+ xcosx+ XSlnx+ xcosx p as a particular solution. Then y, - Ax 2 cos x - Bx 2 sin x + (2A - D)x sin x p + (2B + C)x cos x + C sin x + D cos x and = -Ax 2 sin x + (2A - D)x in x + (2A - D)x cos x + (2A - 2D)sin x + terms on both sides of this to obtain -4B = 1, 4A = 0, 2A - 2D = 0, 2B + C = 0. From this, A = 0, B = -, C = -, D = 0. Thus we obtain the particular integral $yp = x2\cos x + 4x \sin x$. The G.S. of the D.E. is .121. y = c1 Sln $x + c2\cos x + 4x \sin x$. The G.S. of the D.E. is .121. y = c1 Sln $x + c2\cos x + 4x \sin x$. The G.S. of the D.E. is .121. y = c1 Sln $x + c2\cos x + 4x \sin x$. The G.S. of the D.E. is .121. y = c1 Sln $x + c2\cos x + 4x \sin x$. lng omogeneous .. IS Y + Y - Y = Th "I"" 4 2 3 3 2 0 e aUXI lary equation IS m + m - m = or 2 m (m - 1)(m + 3) = 0, with roots 0, 0 (real double root), 1, -3. The complementary function is x -3x Yc = c 1 + c 2 x + c 3 e + c 4 e The NH term is a linear "2 x 3x combination of the UC functions given by x, xe, e, and 1. Step 1: Form the UC set of each of these four UC functions: Sl 2 S - {xe x x = {x,x,}}, e }, 2 - 3x S4 {1}. S3 = {e }, = Step 2 : Set S4 is completely included in Sl' so S4 lS omitted, leaving the three sets Sl' S2' 53. Step 3 : Observe that members x and 1 of 51 are included III the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of Sl by x 2 to obtain the revised set Sl' = 432 {x, x}, whose members are not solutions of the homogeneous D.E. (Note that multiplication by x, instead 2 of x, IS not sufficient here.) Next observe that the member eX of 52 is included in the complementary function. Thus we multiply each member of 52 by x to obtain the 2 x x revised set S2' = {x e, xe}, whose members are not solutions of the homogeneous D.E. Step 4: e now have the three sets SI' = {x 4, x 3, x 2}, 2 x x { $3x S2 = {x e, xe}, S3 = e$ }. We form the linear 188 Chapter 4 combination x 4 + Bx 3 + Cx 2 + Dx 2 e x + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x of their six elements. SteD 5: Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe x + Fe 3x o Exe + Fe. p Then y, 4Ax 3 2 2x = + 3Bx + 2Cx + Dx e p + (2D + E)xe x 3Fe 3x, + Ee + "2D 2 x 9F = 12Ax + 6Bx + 2C + x e + (4D + E)xe x 9F = 3x, + (6D + 3E)e + III 24Ax + 6B + D 2 x (6D + E)xe x 9F = 3x, + (2D + x 9Fe 3x) + (2D + x 9Fe 3x)obtaining $2 \times 24A + Dx = (8D + E)xe + 3x + 81Fe + 2[24Ax + 6B + x 3x + (6D + 3E)e + 27Fe] 2 \times x + Dx e + (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 28Fe = 18x 2 + 16xe x + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + Dx = (4D + E)xe + 2 \times 3x = 18x + 16xe + 4e - x (12D + 4E)e + 27Fe] 2 \times x + 2x + 4e - x (12D + 4E)e + 27Fe] 2 \times x + 2x + 4e - x (12D + 4E)e + 27Fe] 2 \times x + 2x + 4e - x (12D + 4E)e + 27Fe] 2 \times x + 2x + 4e - x (12D + 4E)e + 27Fe$ 3x - 9. Higher-Order Linear Differential Equations 189 We equate coefficients of like terms on both sides of this to obtain -36A = 18, 48A - 18B = 0, 24A + 12B - 6C = -9, B = -4/3, C = -19/6, D = 2, E = -9, F = 1/27. Thus we obtain the particular integral 1 4 4 3 19 2 2 x yp = -x x - x + 2x e 2 3 6 - 9xe x 1 3x + 27 e The G.S. of the D.E. is x - 3x y = c 1 + c 2 x + c 3 e + c 4 e
1 4 - x 2 4 3 - x 3 19 2 2 x x 1 3x - If <math>x + 2x e - 9xe + 27 e 34. The corresponding homogeneous D.E. IS 1V 5 III 7" 5y' + 6y = O. The auxiliary equation y - y + y - IS 4 3 2 - 5m + 6 = O. By inspection find that m m - 5m + 7m we = 2 is a root. Then by synthetic division, we have (m - 2)(m - 3) = 0 or (m - 2)(m - 3) = 0 or (m - 2)(m - 3)(m - 3)(m - 2)(m - 3)(m - 3)(m - 2)(m - 3)(m - 3){sin2x, cos2x}. SteD 2 : Neither set IS identical with nor included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S1 by x to obtain the revised set Sl' = {x sin x, x cos x}, whose members are not solutions of the homogeneous D.E. Step 4 : Ye now have the two sets S1' = {x sin x, x cos x}, S2 = {sin 2x, cos 2x}. Ye form a linear combination of their four elements. Step 5 : Thus we take y = Ax sin x + B cos x + C sin 2x p + D cos 2x as a particular solution. Then y, = p Ax cos x + B cos x + C sin 2x p + D cos 2x as a particular solution. Then y = p Ax cos x + B cos x + B cos x + C sin 2x p + D cos 2x as a particular solution. - 2Dsin2x, y" = -Ax sin x - Bxcosx p + 2A cos x - 2B sin x - 4C sin 2x - 4D cos 2x, Y III = -Ax cos x + Bx sin x - 3B cos x p - 8Ccos2x + 8Dsin2x, y IV = Axsinx + Bxcosx p - 4A cos x + 4B sin x + 16C sin 2x + 16D cos 2x, Y III = -Ax cos x + 4B sin x + 16C sin 2x + 16D cos 2x + 8Dsin2x, y IV = Axsinx + Bxcosx p - 4A cos x + 4B sin x + 16C sin 2x + 16D cos 2x + 8D sin x - 3B cos x + 4B sin x + 16C sin 2x + 16D cos 2x + 8D sin x - 3B cos x + 4B sin x + 16C sin 2x + 4B sin x + 16C sin 2x + 4B sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 4B sin x + 16C sin 2x + 4B sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x + 8D sin x + 16C sin 2x + 16D cos 2x - 3Asinx - 3Bcosx - 8Ccos2x + 8Dsin2x) + 7(-Axsinx - Bxcosx + 2Acosx - 2Bsinx - 4Csin2x - 4Dcos2x) - 5(Axcosx - Bxsinx + Asinx + Bcosx + 2Ccos2x - 2Dsin2x) + 6(Axsinx + Bx cos x + C sin 2x + D cos 2x) = 5 sin x - 12 sin 2x or (IOA - IOB) sin x + (IOA + IOB) cos x + (-6C - 30D) sin 2x + (-6C - 30D) sin like terms Higher-Order Linear Differential Equations 191 on both sides of this to obtain IOA - IOB = 5, IOA + IOB = 0, -6C - 30D = -12, 30C - 6D = 0. The equations in A and Bare equivalent to 2A - 2B = 1, A + B 0, from which A = 1 B 1 The = 4' = 4. equations IOA - IOB = 2, 5C - D = 0, from which C = 1 5 Thus we obtain the 13' D = 13. particular integral 1 1 1. 2 yP = 4 x Sln x - 4 x cos x + 13 Sln x + 5 13 cos 2x. The G. S. of the D. E. is 2x Y = c 1 e 3x + c 2 e + c 3 s 1nx + c 4 cos x 1. 1 1. 2 + 4 x Sln x. 5 + 13 cos 2x. 37. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 8m + 15 = 0, with roots 3, 5. The 3x 5x complementary function IS $y_c = c \ 1 \ e + c \ 2 \ e \ The UC$ set $2x \ 2x \ 2x \ of UC$ function $xe \ IS \ SI = \{xe, e\}$. This does not $2x \ 2x \ need$ revision, so we take y = Axe + Be as a $p \ 1 \ 1 \ TA \ 2x \ (A + 4B)e \ 2x \ x \ (A + 4B)e \ 2x \ x \ (A + 2B)e \ 2x \ (A + 2B)e$ + 15(Axe + Be = 9xe or 2x 2x 3Axe + (-4A + 3B)e = 9xe. 192 Chapter 4 We equate coefficients of this to obtain 3A = 9, -4A + 3B = 0. From these, we find A = 3, B = 4. Thus we obtain the particular integral 2x 2x y = 3xe + 4e. The G.S. of the D.E. is p 2x 5x 2x 2x 2x Axe + (-4A + 3B)e = 9xe. 192 Chapter 4 We equate coefficients of this to obtain 3A = 9, -4A + 3B = 0. From these, we find A = 3, B = 4. Thus we obtain the particular integral 2x 2x y = 3xe + 4e. The G.S. of the D.E. is p 2x 5x 2x 2x 2x Axe + (-4A + 3B)e = 9xe. to this. We find c 1 + c 2 + 4 = 5 or c 1 + c 2 = 1. (*) We next differentiate the G.S. 3x 5x 2x 2x to obtain y' = 3c 1 e + 5c 2 - 1. (*) From (*) and (**) we find c 1 - 3, c 2 = -2. Thus we obtain the particular solution 3 3x 2 5x 3 2x 4 2x Y = e - e + xe + e - 38. -2, B = -1. Thus we find the particular -3x - 3x integral y = -2xe - e and the general solution p - 2x - 5x - 3x - 3x y = c 1 e + c 2 e - 2xe - e We now apply the initial condition y(O) - 0 to this. Ye have c 1 + c 2 = 1. Differentiating the general solution, we find $-2x y' = -2c e 1 - 5x - 3x - 3x 5c 2 e + 6xe + e \cdot Applying$ the initial condition y(O) = -1 to this, we find set of UC function 8e IS c 1 {e- 2x}. -2x Ye assume the particular integral y = Ae. p - 2x H - 2x Then yp' = -2Ae, yp = 4Ae. Substituting into the given D.E., we find 4Ae - 2x - 2ke - 2x + 16Ae - 2x - 2ke - 2x + 16Ae - 2x - 2ke -+ 2e. Thus Ye apply the initial condition y(0) = 2 to this, obtaining c 1 = 0. Ye differentiate the general solution to obtain y' - 2x - 4x - 2x = (-4c 1 + c 2) - 4 = 0. Since c 1 = 0, this gives c 2 = 4. Thus we obtain -4x - 2x = (-4c 1 + c 2) - 4 = 0. Since c 1 = 0, this gives c 2 = 4. Thus we obtain -4x - 2x = (-4c 1 + c 2) - 4 = 0. Since c 1 = 0, this gives c 2 = 4. Thus we obtain -4x - 2x = (-4c 1 + c 2) - 4 = 0. 9 = 0, with double real root -3. The -3x complementary function is y = (c 1 + c 2 x)e. The UC set -6x - 6x of the UC function 27e is $\{e\}$. We assume the Higher-Order Linear Differential Equations 195 -6x - 6x - 36Ae- 6x = 27e-6x - 6x or 9A = 27e. From this, 9A = 27, and so A = 3. Thus we obtain the particular integral y = 3e - 6x and the p -3x - 6x general solution. Ye obtain c 1 + 3 = -2 so c 1 = -5. Differentiate the general , solution to obtain y' (-3c 1 - 3c 2 x)e - 3x 18e - 6x = +c 2 - Apply the I.C.y'(0) = 0 to this. Ye obtain -3c 1 + c 2 - 18 = 0. Since c 1 = -5, we find c 2 = 3. Thus -3x - 6x we obtain the solution y = (3x - 5)e + 3e. 42. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 10m + 29 = 0, with roots 5 2i. The complementary function is Yc = e 5X ($c 1 \sin 2x + c 2 \cos 2x$). 5x . 5x The UC set of the UC function e IS SI $= \{e\}$. This d d k - Ae 5x oes not nee revision, so we ta e y as a p 5x 5x 8 5x Ye at once see that A 2 and hence or 4Ae = e = 196 Chapter 4 obtain the particular integral y = 2e 5x . The G.S. of the p D. E. is 5x . 5x ye at once see that A 2 and hence or 4Ae = e = 196 Chapter 4 obtain the particular integral y = 2e 5x . The G.S. of the p D. E. is 5x . 5x ye at once see that A 2 and hence or 4Ae = e = 196 Chapter 4 obtain the particular integral y = 2e 5x . The G.S. of the p D. E. is 5x . 5x ye at once see that A 2 and hence or 4Ae = e = 0. = e (c 1 s1n 2x + c 2 cos 2x) + 2e. Ye apply the I.C. y(O) = 0 to this. We find c 2 + 2 = 0, so c 2 = -2. Ye next differentiate the G.S. to obtain 5x. $5x y' = e [(5c 1 - 2c 2) Sln 2x + (2c 1 + 5c 2) cos 2x] + 10e \cdot We$ apply the I.C. y'(O) = 8 to this. Ye find 2c 1 + 5c 2 + 10 = 8 or 2c 1 + 5c 2 = -2. Since c 2 = -2, this gives c 1 = 4. Thus we obtain the particular solution $y = 2e 5x (2 \sin 2x - \cos 2x + 1)$. 44. The auxiliary equation of the corresponding homogeneous D.E. 2 - m - 6 = 0 (m - 3)(m + 2) 0, with roots IS m or -3, -2x IS Y = c 1 e + c 2 e c The UC sets of the UC functions in the right member of the 2x 3x D.E. are $SI = \{e\}$, $S2 = \{e\}$. Neither is completely contained ln the other so both are retained. The member 3x e of S2 lS contained ln the complementary function and so lS a solution of the homogeneous D.E. Higher-Order Linear Differential Equations 197 2x, 3x Now we have the two sets $SI = \{e\}$, $S2 = \{xe\}$. We take a linear combination of their two members as a 2x 3x particular integral. That is, we take y = Ae + Bxe. p 2x 3x 3x " 2x 3x Then y, = 2Ae + Be + 3Bxe and y = 4Ae + 6Be p p 3x + 9Bxe. We substitute into the D.E., obtaining 4Ae 2x + 6Be 3x + 9Bxe 3x (2Ae 2x + Be 3x + Be 3x) + 3Bxe 3x) _ 6(Ae 2x + Bxe 3x) = Se 2x _ 5e 3x . or We equate coefficients of like terms on both sides of this to obtain the particular integral y = -2e - p 3x xe The G.S. of the D.E. is 3x - 2x 2x 3x Y = c 1 e + c 2 e - 2e - xe Ve apply the I.C. y(O) = 3to this. We find c 1 + c - 2 = 3 or c 1 + c - 2 = 3 or c 1 + c - 2 = 5 (*). We next 2 differentiate the G.S. to obtain y' 3c 1 = 4, c 2 = 10 (**). 198 Chapter 4 From the two equations (*) and (**) in the unknowns c 1 and c 2, we find that c 1 = 4, c 2 = 1. Thus we obtain the 4 3x -2x 2 2x 3x particular solution y = e + e - e - xe. 45. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 2m + 1 = 0 or (m - 1)2 = 0, with roots 1, 1 (double root). The complementary function is x x Yc = c 1 e + c 2 xe. The UC sets of the UC function ln the right member of the D.E. are Sl 2x 2x S2 x = {xe, e}, e }, = {e } Neither is completely contained in the other, so both are retained. The member of 52 by x2 to obtain the revised set $52' = \{x 2 \in x\}$, whose member lS not a solution of the homogeneous D.E. (note that 2Cxe an y = 4 xe + p + 4Cxe x + 2Ce x. We substitute into Higher-Order Linear Differential Equations 199 2x 2x 2 x x 4Axe + (4A + 4B)e + Cx e + 4Cxe + 2Ce 2x 2x 2 x x - 2[2Axe + (A + 2B)e + Cx e + 2Cxe] + Axe 2x + Be 2x + Cx 2 e x - A 2x (A) 2x C x xe + 2 + Be + 2 e = 2x X 2xe + 6e 2x x 2xe + 6 both sides of this to obtain A = 2, 2A + B = 0, 2C = 6. From these, we find A = 2, B = -4C = 3. Thus we obtain the particular, integral 2xe 2x 2x 2x 1S y = c 1 e + c 2 xe + -4e + 3x e. We apply the I.C. y(O) = 1 to this. We find c 1 - 4 = 1 or c 1 = 5. We next differentiate the G.S. to obtain y'x x x+ 4xe 2x 2x 8e 2x = c e + c 2 xe + c 2 e + 2
e + 2 e corresponding homogeneous D.E. is m 2 + 1 - 0, with roots *i. The complementary function is y c = c 1 sin x + c 2 cos x. The UC sets of the D.E. are Sl = {x 2, x, 1}, S2 = {sin x, cosx}. Neither is completely contained in the other, so both are retained. The members sin x and cos x of 52 are included in the complementary 200 Chapter 4 function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each members are not solutions of the homogeneous D.E. 2 Now we have the two sets $SI = \{x, x, l\}, S2' = \{x \sin x, x \cos x\}$. We take a linear contract of the homogeneous D.E. 2 Now we have the two sets $SI = \{x, x, l\}, S2' = \{x \sin x, x \cos x\}$. We take a linear contract of the homogeneous D.E. 2 Now we have the two sets $SI = \{x, x, l\}, S2' = \{x \sin x, x \cos x\}$. their five members as a particular integral. That 1S, we take $y = Ax 2 + Bx + C + Dx \sin x + Excosx$. then $pyP_{,} = 2Ax + B + Dx \cos x + D \sin x + Excosx$. then $pyP_{,} = 2Ax + B + Dx \cos x + D \sin x + Excosx$. then $pyP_{,} = 2Ax + B + Dx \cos x + D \sin x + Excosx$. $Ax 2 + Bx + (2A + C) + 2D \cos x - 2E \sin x = 3x 2 - 4 \sin x$. or We equate coefficients of this to obtain A = 3, B From these we find Thus we obtain the cosx. The G.S. of the D.E. is $y = c l \sin x + c 2 \cos x + 3x 2 - 6 + 2x \cos x$. = 0, 2A + C = 0, 2E = -4. A = 3, B = 0, C = -6 D = 0, E = 2., particular integral 3x 2 6 2x yp = -4. - + Higher-Order Linear Differential Equations 201 We apply the I.C. y(O) = 0 to this. We find c 2 - 6 = 0 or c = 6. We next differentiate the G.S. to obtain 2 y' = c 1 cos x - c 2 sin x + 6x - 2x sin x + 3x 2 - 6 + 2x cos x. 48. The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 4 = 0, with pure imaginary conjugate complex roots :2i. Thus the complementary function 8 sin 2x is $S = \{sin 2x, cos 2x\}$. Since each member of this set is a solution of the corresponding homogeneous equation, we multiply each by x and replace S by S' = { $x \sin 2x + A \cos 2x + A \sin 2x + 4A \cos 2x$. We then form the particular integral $y = -4Ax \sin 2x + 4A \cos 2x$. Then p y, = $-2Bx \sin 2x + 4A \cos 2x + A \sin 2x + 4A \cos 2x$. Then p y, = $-2Bx \sin 2x + 4A \cos 2x + A \sin 2x + 4A \cos 2x$. $4Bx \cos 2x = 8 \sin 2x \text{ or } -4B \sin 2x + 4A \cos 2x = 8 \sin 2x$. From this, 4A = 0, -4B = 8, and hence A = 0, B = -2. Thus we obtain the particular integral $y = -2x \cos 2x$. 202 Chapter 4 We apply the I.C. y(O) = 6 to this, obtaining c 2 = 6. Differentiating the general solution, we obtain $y' = -2x \cos 2x$. $2c_1 \cos 2x - 2c_2 \sin 2x + 4x \sin 2x - 2 \cos 2x$. Applying the I.C. y'(0) = 8 to this, we find $2c_1 - 2 = 8$, from which $c_1 = 5$. Thus we obtain the solution $y = 5\sin 2x + 6\cos 2x - 2x\cos 2x$. 49. The auxiliary equation of the corresponding homogeneous D.E. 3426 = 0 or (m + 1)(m - 3)(m - 2) 0, 1Sm - m + m + = with roots -12, 3. The complementary function is , -x 2x 3x The UC sets of the UC y = c 1 e + c 2 e + c 3 e c functions in the right member of the D.E. are SI {xe x x and S3 = {sin x, cos x}. Since = ,e }, S2 = {e }, S1 > S2' we omit S2' retaining SI and S3. Neither of these need revision, so we take a linear combination of their four members as a particular integral. That IS, we take y = Axe $x x C \sin x + D \cos x$. Then + Be + p, Axe x (A + B)e x + C cos x + D sin x, yp = + III Axe x (2A + B)e x C cos x + D sin x, yp = + III Axe x (2A + B)e x C cos x + D sin x, yp = + III Axe x (2A + B)e x + C cos x + D sin x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + Be x + C cos x] + 6[Axe x + $\sin x + D \cos x J = 3xe x + 2e x - \sin x$ or $4Axe x + (-4A + 4B)e x + 10D \cos x = 3xe x + 2e x - Slnx$. We equate coefficients of like terms on both sides of this to obtain 4A = 3, -4A + 4B = 2, 10C = -1, 10D = 0. Thus we obtain the particular integral, $3 \times y = 4 \times 5 \times + -e 41$. 10 Slnx. The G.S. of the D.E. lS -x 2x Y = c1 e + c2 e 3x + c3 e 3x + -xe 45 x + -e 41. - 10 Sln x. We apply the I.C. y(O) = 33/40 to this. We find c1 + c2 + c3 = -17/40. (*) We next differentiate the G.S. to obtain -x 2x 3x y' = -c1 e + 2c2 e + 3c3 e x 1 + 2e - 10 cosx. 3x + -xe 4 We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 2c 2 + 3c 3 + 2 - 10 = 0 or - c 1 + 2c 2 + 3c 3 19 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 e + 9c 3 e 3 x + -xe 4 11 x + -e 4 1. + 10 Slnx. We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 11 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 e + 9c 3 e 3 x + -xe 4 11 x + -e 4 1. + 10 Slnx. We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 11 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 e + 9c 3 e 3 x + -xe 4 11 x + -e 4 1. + 10 Slnx. We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 11 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 e + 9c 3 e 3 x + -xe 4 11 x + -e 4 1. + 10 Slnx. We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 11 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 e + 9c 3 e 3 x + -xe 4 11 x + -e 4 1. + 10 Slnx. We apply the I.C. y'(O) = 0 to this. We find 1 - c 1 + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 11 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 We differentiate once more, obtaining H - x 2x 3x y = c 1 e + 4c 2 + 9c 3 + 4 = 0 or c 1 + 4c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -10. (**) 204 Chapter 4 + 2c 2 + 9c 3 + 10 = -1 - 4. (***) The three equations (*), (**), (***) determine c 1, c 2, c 3. Adding (*) and (**), we have 3c 2 + 4c 3 = -93/40. Adding (**) and (***), we have 6c 2 + 12c 3 = -93/20. Solving these two resulting equations in c 2 and c 3, we find c 2 = -31/40, c 3 = 0. Then (*) gives c 1 = -c 2 - c 3 - 17/40 = 7/20. Thus we obtain the desired particular solution 7 -x
0 or 2 (m - 1) (m - 4) = O. Thus the roots are 1, 1 (double real root) and the simple real root 4. The complementary x 4x function is Yc = (c 1 + c 2 x)e + c 3 e. The UC sets of the UC functions in the right member of the S2' we omit S2' retaining S1 and S3. Neither of these need reVIslon, so we can be called use to the UC function is Yc = (c 1 + c 2 x)e + c 3 e. The UC sets of the UC s take a linear combination of their four members as a particular integral. That is, we take y = Ax + B + 2De 2x, p p y H = 2A + 4De 2x, p p y4Bx + (-12A + 9B - 4C) - 2De = Sx 2 + 3 - 6e 2x. We equate coefficients of like terms on both sides of this to obtain -4A = S, ISA - 4B = 0, -12A + 9B - 4C = 3, -2D = -6. From these, we find A = -2, B = -9, C = -15, D = 3. Thus we obtain the particular integral 2 2x yp = -2x - 9x - 15 + 3e. The G.S. of the D.E. IS x 4x 2 2x y = (c 1 + c 2 x)e + c 3 e - 2x + 2 e $9x - 15 + 3e \cdot 206$ Chapter 4 We apply the I.C. y(O) = 1 to this. We find C 1 + c 2 + 4c 3 - 9 + 6 = 7 or c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. We find c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. We find c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. We find c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. We find c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. We find c 1 + c 2 + 4c 3 = 10. (*) We differentiate once more, obtain y H is C. y'(O) = 7 to this. () x 4x 2x Y = c 1 + 2c 2 + c 2 x e + 16c 3 e - 4 + 12 = 10 or c 1 + 2c 2 + 16c 3 e - 4 + 12 = 10 or c 1 + 2c 2 + 16c 3 - 4 + 12 = 10 or c 1 + 2c 2 + 16c 3 - 4 + 12 = 10 or c 1 + 2c 2 + 16c 3 = -2. (***) Solving the three equations (*), (**), (***) for c 1 , c 2 , c 3 , we find c 1 = 122/9, c 2 = -4/3, c 3 = -5/9. particular solution y = (12 x)e X e 4x 2x 2 9x 15 + 3e 2x . 53. The auxiliary equation of the corresponding homogeneous 2 D.E. IS m + 4m + 5 = 0 with conjugate complex roots -2x - 2% 1. The complementary function is $Yc = e(c 1 \sin x + c 2 \cos x)$. The NH member can be written as the -2x - 2% 1. The complementary function is $Yc = e(c 1 \sin x + c 2 \cos x)$. The NH member can be written as the -2x - 2% 1. The complementary function is $Yc = e(c 1 \sin x + c 2 \cos x)$. The NH member can be written as the -2x - 2% 1. -2x -2x functions e and e cos x. The UC sets of these -2x functions are, respectively, SI = {e } and S2 = -2x . -2x {e Slnx, e cosx}. Neither is contained ln the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S2 by the lowest integral power of x so that the resulting revised set will contain no members that are solutions of the corresponding homogeneous D.E. It turns out that the first power of x, x itself, will accomplish this. So we multiply each members are not solutions of the homogeneous D.E. -2x - 2x. We now have sets SI = {e} and 8 2 ' = {xe Slnx, $-2x xe cosx}$. We form a linear combination of their members 20S Chapter 4 -2x - 2x. -2x yp = Ae + Bxe Slnx + Cxe cosx. 54. The auxiliary equation of their members 20S Chapter 4 -2x - 2x. -2x yp = Ae + Bxe Slnx + Cxe cosx. 54. The auxiliary equation of the corresponding homogeneous D.E. is m 2 -6m + 9 = 0, with roots 3, 3 (double root). 3x The complementary function is y = 3x and e 3x of S3 are included In the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S3 by x 2 to obtain the revised set , 4 3x 3 3x 2 3x S3 = {x e , x e }, whose members are not solutions of the homogeneous D.E. (Note that multiplication by x, 2 instead of x, is not sufficient here.) We now have the three sets Sl' S2' and S3'. We form a linear combination of their twelve members. Thus: $4 \times 3 \times 2 \times + Dxe \times Ee \times yp = Ax + x + 4 \times 2x = + x + 2 \times$ obtain the revised set. S ' 1 2 -3x . 2 -3x . - $x = x \in Sln 2x$, $x \in cos 2x$, $x \in Sln 2x$, $3x \cdot 2 -3x \cdot 2$ Dxe cos 2x + Ex e Sln 3x + Fx e cos 3x - 2x - 2x + Gxe sin 3x + Hxe cos 3x + Ie sin 3x - 2x + Je cos 3x + Ie sin 3x - 2x + Je cos 3x - 2x + Gxe sin 3x - 2x + Je cos 3x + Ie sin 3x +combination of the two UC functions are of these functions are, respectively, $2x 2^3 x^3 x^3 S = \{x e, e\}$ and $S2 = \{x e, e\}$. Neither lS contained in the other, so each is retained. Both members of Sl are contained in the other, so each is retained. Both members of Sl are contained in the other, so each is retained. each member of Sl by the lowest integral power of x so that the resulting revised set will contain no members that are solutions of the corresponding homogeneous D.E. By trial, we see that multiplying by x or x 2 will not accomplish this. Rather, we must multiply each member of Sl by x 3, obtaining S ' { 4 2x 3 2x } h b 1 . f 1 = x e, x e, w ose mem ers are not so utlons 0 the homogeneous D.E. u h S { $4 \ 2x \ 3 \ 2x \ } d$ we now ave sets l' = x e , x e , e }. We form a linear combination of Higher-Order Linear Differential Equations 211 h " f " b A 4 2x B 3 2x C 2 3x D 3x t elr lve mem ers y = x e + x
e + x e + homogeneous D.E. is m 6 + 2m 5 + Sm 4 = 0, with roots 0, 0, 0, 0 (four-fold root) and -1 % 2i. The complementary functions are Sl = {x,x} + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 2 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x + c 6 cos 2x). The NH member is a linear combination of the three UC 3 - x - x + c 6 cos 2x + c 6 cos ,x,1}, S2 = {x e, xe, e}, and -x -x S3 = {e sin2x, e cos2x}. None is completely contained ln any other, so all three are retained. All four members of S1 are contained in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S1 by the lowest positive integral power of x so that the resulting revised set will contain no members that are solutions of the corresponding homogeneous D.E. By actual trial, we see that multiply each 4 member of Sl by x, x 2, or x 3 will not accomplish this. Rather, we must multiply each 4 members of Sl by x, obtaining SI 1 7 6 S 4 - {x, x, x} } whose members are not solutions of the homogeneous D.E. 212 Chapter 4 Also, both members of S3 are included in the corresponding homogeneous D.E. Thus we multiply each members are not solutions of the homogeneous D.E. We now have the three sets Sl', S2' and S3'. We form a linear combination of their nine members. Thus: 7 6 5 4 2 -x -x yp = Ax + Bx + Cx + Dx + Ex e + Fxe + Ge- x + Hxe- x sin2x + Ixe-xcos2x. 63. The auxiliary equation of the corresponding homogeneous .4222 D.E. IS m + 3m - 4 = 0 or (m + 4)(m - 1) = 0, with roots 1, -1, %2i. The complementary function is x -x y c = c 1 e + c 2 e + c 3 sin 2x + c 4 cos respectively. Higher-Order Linear Differential Equations 213 Both members cos 2x and sin 2x of S2 are contained in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S2 by x to obtain the revised set S2' = {x cos 2x, x sin 2x}, whose members are not solutions of the homogeneous D.E. Thus we replace S2 by the revised set S2'. The situation is exactly the same for each of S3 and S4' so we replace S3 by the revised set S3' = {xe x}. We form a linear combination of their five members Thus $Y = A + Bx \sin 2x + Cx \cos 2x + Dxe x + Exe - x$. p Alternatively, we regard the NH member of the D.E. as a linear combination cos $hx = e - x + e^2$. 1 S { . 2 2 respective $y = 1 - x + e^2$. 1 substitution that the member sin x cos x of Sl lS a solution of the corresponding homogeneous D.E. Thus we multiply each member of Sl by x to obtain the revised set 8 1 '. In like manner, one finds coshx is 214 Chapter 4 a solution of the homogeneous D.E.; and so we replace S2 by the revised set S2' = {x sinh x, xcoshx}. So we now have the two revised sets S1' and S2'. We form a linear combination of their five members. Thus: Y p = Ax sin 2x + Bx cos 2x + Cx sin x cos x + Dxsinhx + Excoshx. 64. The auxiliary equation of the corresponding homogeneous D.E. is m 4 + 10m 2 + 9 = 0 or (m 2 + 9)(m 2 + 1) = 0 with roots %i, %3i. The complementary function IS y C = c 1 sin x + c 2 cos x + c 3 sin 3x + c 4 cos 3x. The NH member IS the UC function sin x sin 2x, sin x cos 2x, cos x sin 2x and constraints and constraint sin 2x of Sl is a solution of the corresponding homogeneous D.E. Thus we multiply each members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x sin 2x, x sin x cos x sin 2x, x sin x cos 2x}, whose members are not solutions of the homogeneous D.E. Thus we multiply each members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. Thus we multiply each members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x
cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised set S' + {x sin x cos x cos 2x}, whose members are not solutions of the homogeneous D.E. So we replace Sl by the revised s Thus: y = Axsinxsin2x + Bxsinxcos2x + Cxcosxsin2x + Dxcosxcos2x. (*) Alternatively, using the trigonometric identify sin u sin v =; [cos (u - v) - cos(u + v)] with u = 2x, Higher-Order Linear Differential Equations 215 v = x, we see that the NH member is equal to ; [cos x - cos 3x] and thus is a linear combinat ion of the simple UC functions cos x and cos 3x. The UC sets of these two functions are S2 = {cos x, sin x} and S3 = {cos 3x, sin 3x}, respectively. Both members of S2 are contained ln the corresponding homogeneous D.E. Thus we multiply each members of S2 by x to obtain the revised set S2' = {xcosx, xsinx} whose members are not solutions of the homogeneous D.E. Thus we replace S2 by the revised set S2'. The situation lS exactly the same for the set S3' so we replace S2 by the revised set S2'. The situation of their four members. Thus: $y = Ex \cos x + Fx \sin x + Gx \cos 3x p + Hx \sin 3x$ (**) The student should conVlnce himself that the y given by p(*) can be expressed in the form given by p(*), and vice versa. Section 4.4, Page 169. 3. The complementary function IS defined by y c (x) = c 1 sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) sin x + v 2 (x) cos x. (1) Then y p'(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) sin x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) cos x - v 2 (x) cos x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x - v 2 (x) cos x + c 2 cos x. We assume a particular integral of the form 216 Chapter 4 Yp(x) = v 1 (x) cos x + c 2 cos $x + v l'(x) \sin x + v 2'(x) \cos x$. We impose the condition $v 1'(x) \sin x + v 2'(x) \cos x + v l'(x) \cos x + v 2'(x) \sin x$. (3) Substituting (1) and (3) into the given D.E., we obtain $v 1'(x) \cos x + v 2'(x) \sin x + v 2'(x) \sin x$. (4) We now have conditions (2) and (4) from this y p''(x) = -v 1 (x) \sin x + v 2'(x) \sin x + v 2'(x) \sin x + v 2'(x) \sin x. (5) Substituting (1) and (3) into the given D.E., we obtain $v 1'(x) \cos x + v 2'(x) \sin x + v 2'(x) \sin x$.

which to determine v 1 '(x) and v 2 '(x): sln x v 1 '(x) + cos x v 2' (x) = 0, cos x v 1 '(x) - sln x v 2' (x) = sec x. Solving, we find Higher-Order Linear Differential Equations 217 0 cos x sec x - sln x v 1 '(x) - sec x cos x - sec x. Solving, we find v 1 (x) = x, the sec x - sec x - sln x v 1 '(x) - sec x cos x - sec x - sln x - sec x - sec x - sln x - sec x - sln x - sec x - sec x - sln x - sec x - sec x - sln x - sec x - sec x - sln x - sec x - s $v 2 (x) = Inl \cos xl$. Substituting into $yp(x) = v 1 (x)\sin x + v 2 (x) \cos x$, we find $y(x) = x \sin x + \cos x [Inl \cos x I]$. 6. The complementary function IS defined by $y c (x) = c 1 \sin x + c 2 \cos x + x Sln x + c$ (1) Then yp'(x) = v 1 (x)cosx - v 2 (x)sinx + v 1 '(x)cosx - v 2 (x)sinx + v 1 '(x)cosx - v 2 '(x)cosx + v 1 '(x)cosx - v 2 (x)sinx + v 1 '(x)cosx - v 2 '(x)sinx + v 2 '(x)cosx + v 1 '(x)cosx - v 2 (x)sinx + v 2 '(x)cosx + v 1 ' t an x s e C x. (4) We now have conditions (2) and (4) from which to determine v 1 '(x) and v 2 '(x): { sin x cos x v 1 '(x) = sln x eos x cos x + sln x see x = -1 v 1 '(x) = sln x eos x cos x + sln x see x sin x tan x see x v 2 '(x) = -1 Sln x eos x eos x eos x eos x eos x eos x + sln x see x = -1 v 1 '(x) = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + sln x = -1 Sln x eos x eos x + -1 sln x eos x eos x + -1 sln x = -1 Sln x eos x eos x + -1 sln x eos x - Sln x - tan 2 + 2 = x = -see x. Higher-Order Linear Differential Equations 219 Integrating, we find vi (x) = -In I cos x I, v 2 (x) = x - tan x . Substituting into y p (x) = vi (x) sin x + v 2 (x)cosx, we find yp(x) = -sinxlnlcosx I + xcosx - Slnx, which may be more simply written as y = cOsln x + c 2 cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. We assume a particular integral of the form y(x) - 2x + c 2 e cos x. $\sin x + v 2$ (x)e $\cos x \cdot (1) - 2x - 2x$ Then yp' (x) = vi (x)e $\cos x - 2v 1$ (x)e $\sin x - 2x - 2x - 2x + v 2$ (x)e $\cos x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\cos x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\cos x - 2v 1$ (x)e $\sin x - 2v 2$ (x)e $\sin x$ differentiating this and collecting like terms, we find y "(x) p -2x -2x = 3v 1 (x) e cos x - 2x + vi' (x) [e cos x - 2e sin x] - v 2' (x) [e sin x + 2e cos x]. (4) Then substituting (1), (3), and (4) into the given D.E. and collecting like terms, we obtain -2x - 2x + vi' (x) [e cos x - 2e sin x] - v 2' (x) [e cos x - 2e sin x] - v $\sin x - 2x - 2x + 2e \cos x = e \sec x$. (5) We now have conditions (2) and (5) from which to determine vi'(x) and v 2 '(x): -2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) +
e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x - 2 \cos x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x - 2 \cos x$) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x - 2 \cos x$) vi'(x) + e (-sin x - 2x e \cos x - 2 \sin x) vi'(x) + e (-sin x - 2x e $\cos x - 2 \sin x - 2 \cos x$) vi'(x) + e (-sin x - 2x e (\cos x - 2 \sin x) + e (-sin x - 2 \cos x) $(x) = -2x \cdot e Sln x - 2x e \cos x - 2x e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2 \cos x = -4x \cdot e Sln x - 2 \cos x - 2x e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2 \cos x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot e (\cos x - 2 \sin x) - 2x \cdot e (-sln x - 2 \cos x) = -4x \cdot e Sln x - 2x \cdot$ Into y p x = v i xe slnx+v 2 xe cosx, we find y (x) = xe- 2x sinx + [lnlcosx] e- 2 xeosx. The p G.S. of the D.E. 1S - 2x . Y = e (c i s1nx + c 2 cosx) + xe Slnx I I - 2x + [In cos x] e eos x. 10. The auxiliary equation m 2 - 2m + 1 = 0 has roots -1, -1, x so the complementary function is $y_c(x) = (c 1 + e 2 x)e$, 222 Chapter 4 x x which we rewrite slightly as [(x + l)e x J. (4) Then substituting (1), (3), and (4) into the glven D.E., we obtain x x x v 1 (x)e + v 2 (x) [(x + 2)e J + v 1 (x)e x + v 2 (x) [$(x + 1)e J - 2\{v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 1 (x)e x x + v 2 (x) [(x + 1)e J + v 2 (x) [(x + 1)e J$ (5) from which to determine v 1 '(x) and v 2 '(x): x x v 1 '(x) e + v 2 '(x) = 0, v 1 '(x) + xv tables or integration by parts), we find v 1 (x) 1 3 1 3 = -x 3 x In x 9 v 2 (x) 1 2 1 2 = 2 x ln x - x 4 224 Chapter 4 Substituting into Yp(x) = v 1 (x)e x + v 2 (x)xe x, we find Yp = - 36 x e + 6 x e In x. The G.S. of the D.E. IS x x 5 3x 13x y = c 1 e + c 2 xe - 36 x e + 6 x e In x. 12. The complementary function 1S defined by y c (x) = c 1 sin x + c 2 cos x. We assume a particular integral of the form y p' (x) = vi (x) cos x + v 2' (x) cos x + v $-vi(x) \sin x - v2(x) \cos x + vi'(x) \cos x - v2'(x) \sin x$. (3) Substituting (1) and (3) into the given D.E., we obtain 3 vi'(x) - Sinx v 2'(x) = 0, cosx v 1'(x) - Sinx v 2'(x) = 0, cosx v 1'(x) - Sinx v 2'(x) = 1, cosx v 1'(x) - Sinx v 2'(x) = 0, cosx v 1'(x) - Sinx v 2'(x) = 1, cosx v 1'(x) - Sinx v 2 Solving, we find $0 \cos x 3 3 \tan x - S \ln x v 1'(x) - \tan x \cos x = -1$ Sln x $\cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x 3 3 \cos x \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) - S \ln x (1 2 2 - \cos x) 1 2 = - = + - \cos x - S \ln x (1 2 2 - \cos x) - S \ln x (1 2 2 -
\cos x) - S \ln x (1 2 2 - \cos x)$ tables), we find v l (x) = (cosx)-l + cosx, v 2 (x) = 1 2 tan x sec x + Inlsecx + tan x 1 tan x 3 I I 2 + 2 eos x In see x + tan x 1 tan x 3 I I 2 + 2 eos x In see x + tan x 1 tan x 3 I I 2 + 2 eos x In see x + tan x 1 tan x 3 I I 2 + 2 eos x In see x + tan x 1 tan x 3 I I y - c 1 sin x + c 2 cos x + 2 c + 2 cos x In sec x + tan x . 13. 2 The auxiliary equation m + 3m + 2 = 0 has roots -1, -2, -x - 2x so the complementary function is y (x) = c e + C 2 e c 1 We assume a particular integral of the form y (x) p - x - 2x - v 1 (x)e + v 2 (x)e + 0 (2) leaving y (x) p -x -2x = -v 1 (x)e - 2v 2 (x)e · (3) From this, Higher-Order Linear Differential Equations 227 y "(x) p -x -2x - x = v 1 (x)e + 4v 2 (x)e - v 1 '(x)e - 2v 2 '(x)e + 4v 2 (x)e + = 11x + e or -x - 2x + v 1'(x) = -2v 2'(x) = 1 (x) = x + e V 2 + (x) = -2x + e x v 1'(x) = -2x + e x ++ e e e -x -2e -2x -e Integrating, we find v 1 (x) x = In(1 + e) v 2 (x) = f e 2x x dx = f [e X - ex] dx 1 + e 1 + e x x = In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we find x x x x - 2x Yp(x) = [In(1 + e) e - x - 2x Substituting into Yp(x) = v 1 (x)e + v 2 (Differential Equations 229 or -x - 2x + 2x + 2x + 2x = c 3 = c 1 - 1. 15. The complementary function 1S defined by y c (x) = c 1 sin x + c 2 cos x. We assume a particular integral of the form y p (x) = v 1 (x) sin x + v 2 (x) cos x - v 2 (x) sin x + v 2 (x) cos x. We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) sin x + v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x - v 2 (x) cos x . We impose the aparticular integral of the form y p (x) = v 1 (x) cos x condition v l' (x) sin x + v 2' (x) cos x - v 2 (x) sin x. Then from this yp'' (x) = v 1 (x)cos x - v 2 (x) sin x. (3) Substituting (1) and (3) into the glven D.E., we obtain v 1 '(x)cos x - v 2 '(x) sin x. (3) Substituting (1) and v 2 '(x): 230 Chapter 4 sin x v 1 '(x) + cosx v 2 '(x) = 0, cosx v 1 '(x) = $1 + \sin x \cdot 2 + \sin x$ COS x = -x - 1 + Sln x Substituting into Yp(x) = v 1 (x)sinx + v 2 (x)cosx, we find y (x) p 2 cos x - 1 + Sln x [In(1 + sin x)] - x cos x - 1 + Sln x (In(1 + sin x)] - x cos x - 1 + Sln x (In (1 + sin x)] - x cos x - 1 + Sln x (In (1 + sin x)) - x $(x) = x + v^2(x) [(x + 1)e x].$ (3) Then Y "(x) () x () [(2) X] () x p = v 1 x e + v 2 (x) [(x + 1)e] + v 1 (x)e + v 2 (x) [(x + 1)e] + v 2 (x 1'(x) = x + v 2'(x) [(x + 1)e x] x - 1 = e Sln x. (5) We now have conditions (2) and (5) from which to determine v 1'(x) + x v 2'(x) = 0, x x - 1 e v 1'(x) + x v 2'(x) = 0, x x - 1 e v 1'(x) + (x + 1)v 2'(x) - 1 = Sln x. Subtracting the first from the second, we find v 2'(x) = -1 = Sln x; Higher-Order Linear Differential Equations 233 and then the first equation glves v 1 '(x) = -x 2 '(x) - 1 = - Sln x + h - v 2 (x) 2 x . Substituting into yp(x) = v 1
(x) x - 1 = - Sln x + x , -1 = - Sln x + 4 xe Sln x ()xex 1 - x 2 2 x - 1 + x e Sln x + xex l x 2 x - 1 e Sln x 4 The G.S. of the D.E. IS = + 2 x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x 2 3xe x J l + 4 2 - x x - 1 x e Sln x - 1 x - 1 x e Sln x - 1 x e Sln x - 1 x - 1 x e Sln x - 1 x - 1 x e Sln x - 1 xintegral of the form + c 2 e 234 Chapter 4 y (x) p - x - 2x = v 1 (x)e + v 2 (x)e . (1) Then y '(x) p - x - 2x - x - 2x = -v 1 (x)e + v 2 '(x)e = 0 (2) leaving y '(x) p - x - 2x - x = v 1 (x)e + v 2 '(x)e + v Substituting (1), (3), and (4) into the glven D.E., we obtain -x -2x -x + 2(x)e - v 1 '(x)e - 2v 2 '(x)e - v 1 '(x)e - 2 elementary functions, we simply leave it -x yp(x) = v i (x)e + y (x) = e-xlnlxl p as indicated. -2x v 2 (x)e , we e-2X()dX' Substituting into find The G.S. of the D.E. lS -x y = c i e -2x -x I I -2x r (e) + c 2 e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y (x) = e -xlnlxl p as indicated. -2x v 2 (x)e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y (x) = e -xlnlxl p as indicated. -2x v 2 (x)e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y (x) = e -xlnlxl p as indicated. -2x v 2 (x)e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y (x) = e -xlnlxl p as indicated. -2x v 2 (x)e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y (x) = e -xlnlxl p as indicated. -2x v 2 (x)e + e In x - e J xX dx. 236 Chapter 4 18. The auxiliary equation m 2 - 2m + 1 = 0 has the roots 1, 1 (double root), so the complementary function is Yc(x) = v i (x)e + y i (x) = (c 1 + c 2 x)e x, which we rewrite slightly as x x Yc(x) = c 1 e + c 2 xe. We assume a particular integral of the form y(x) p x x = v 1 (x)e + v 2'(x)xe = 0, (2) leaving Yp'(x) = v 1 (x)e + v 2(x)[(x + 1)e] + v 1'(x)e + v 2(x)[(x + 1)e] + v 1'(x)e + v 2'(x)xe = 0, (2) leaving Yp'(x) = v 1 (x)e + v 2(x)[(x + 1)e] + v 1'(x)e + v 2'(x)xe = 0, (2) leaving Yp'(x) = v 1(x)e + v 2(x)[(x + 1)e] + v 1'(x)e + v 2'(x)xe = 0, (2) leaving Yp'(x) = v 1(x)e + v 2'(x)xe = 0, (2) leaving Yp'(x) = v 1(x)e + v 2'(x)xe + v 2'(x x - v 1 (x)e + v 2 (x) [(x + 2)e] + v 1 '(x)e x + v 2 '(x)[(x + 1)e] + v 1 '(x)e x $= x \ln x$. (5) We now have conditions (2) and (5) from which to determine v 1 '(x) + xe v 2 '(x) = 0, x x e v 1 '(x) + xe v 2 '(x) = 0, v 1 ' equation gives -x 2v 1'(x) = -e x Inx. 238 Chapter 4 2 Th h () f X In x d d us we ave v 1 x = -x x an e f x In x v 2 (x) = x dx; e and since neither of these integrals can be expressed ln closed form in terms of a finite number of elementary functions, we simply leave them as indicated. Substituting into y (x) p x x = v 1 (x)e + v 2 (x)xe, we find y (x) p X f $x 2 \ln x X f x \ln x = -e dx + xe dx$. x x e e The G.S. of the D.E. $lS 2 x e X f x l x n x X f x \ln x y = (c 1 + c 2 x) e - dx + xe x dx$. e e 21. Since x + 1 and x 2 are linearly independent solutions of the corresponding homogeneous equation, the complementary 2 function is defined by Yc(x) = c 1 (x + 1) + c 2 x. We assume a particular integral of the form 2 yp(x) = v 1 (x)(x + 1) + v 2 (x)x. (1) Then y'(x) = v 1 (x) + 2v 2 (x)x + v 1'(x)(x + 1) + v 2'(x)x. (2) leaving Yp'(x) = v 1 (x) + 2v 2 (x)x. (3) Then from this, YpH(x) = v 1 (x) + 2v 2 (x) + 2v 2 (x)x. (4) Substituting (1), (3), and (4) into the given D.E., we obtain 2 (x + 2x)[v 1 '(x) + 2v 2 (x) + 2v 2 '(x)] - 2(x + 1) [v 1 (x) + 2v 2 (x)] + 2[v 1 (x)(x + 1) 2 2 + v 2 (x)] + 2[v 1 (x)(x +
1) 2 2 + v 2 (x)] + 2[v 1 (x)(x + 1) 2 2 + v 2 (x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1 (x)(x + 1) 2 + v 2 (x)(x)] + 2[v 1'(x) + 2x v 2'(x) = x + 2x Solving, we find $0 2x x + 22x x \cdot x(x + 2) v 1'(x) = -1 \cdot x(x + 2) = -1 \cdot x(x$ $x \ge 21 + y \ge (x)x$, we have = $x \ge -2x + x \ge 1$ lnlxl. $y = c + x \ge 1$ (x + 1) + $c \ge x \ge x \ge -2x + x \ge 1$ nlxl. $22 \cdot x \ge x + x \ge 1$ nlxl. $22 \cdot x \ge -2x + x \ge 1$ nlxl. $22 - 2x + x \ge 1$ nlxl. 22 - 2x = 1 nlx nlxl. 22 - 2x = 1 nlx nlx = 1 nlxl. 22 - 2x241 x Yp(x) = v 1 (x)x + v 2 (x) (x + 1)e J + v 2 (x) [(x + 1)e J + v 2 ($x = \{v \mid (x) + v \mid (x) \mid (x)$ + v 2 '(x)[(x + 1)e J = x. Solving, we find 0 x xe x x (x + 1)e v 1 ' (x) = -1, x x xe (x x 1 + 1)e x 0 1 x v 2' (x) = -e x Substituting into yp(x) = v1 (x)x + v 2 (x)xe, we have 2 y (x) = -x · x. The G.S. of the D.E. is p x y = c 1 x + c 2 xe 2 · x · x. Higher-Order Linear Differential Equations 243 24. Since x and (x + 1)-1 are linearly independent solutions of the corresponding homogeneous equation, the complementary function is defined by -1 YC(x) = v 1 (x) + v 2 (x) (x + 1) + v 1 '(x) + v 2 (x) (x + 1) + v 2 (the condition -1 v 1 '(x) x + v 2 ' (x) (x + 1) = 0, (2) leaving -2 Yp'(x) = v 1 (x) - v 2 (x) (x + 1) - 2 - v 2 '(x)(x + 1) - 2 - (x) + v = (x) + 1, or -2 = (2x + 1) + (x + 1) = (2x + 1) + (x + 1) + (x-1 - (x + 1) - 2 l)(x + 1) + (x + 1) - 2 - 1 - x(x + 1) - 2 -(x)x + v 2 (x) (x + 1), we have y (x) p = 2 (2x 3 + 3x 2) (x + 1) - 1 x 6 The G.S. of the D.E. lS -1 2 (2x 3 + 3x 2) (x + 1) + x 6 25. Since Sln x and x sin x are linearly independent solutions of the corresponding homogeneous equation, the complementary function is defined by Yc(x) = c 1 s + c 2 (x + 1) + x 6 25. Since Sln x and x sin x are linearly independent solutions of the corresponding homogeneous equation, the complementary function is defined by Yc(x) = c 1 s + c 2 (x + 1) + x 6 25. Since Sln x and x sin x are linearly independent solutions of the corresponding homogeneous equation. particular integral of the form yp(x) = v 1 (x)sin x + v 2 (x) sin x. (1) 246 Chapter 4 Then Yp'(x) = v 1 (x)cosx + v 2 (x) [xcosx + sinx] + v 1 '(x)sinx + v 2 '(x)si $2\cos x + v^2(x)[x\cos x + \sin x]$. (4) Substituting (1), (3), and (4) into the glven D.E., we obtain $\sin 2x \{ -v \ 1 \ (x) \sin x + v^2(x) [x\cos x + \sin x] \} + (\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + v^2(x) [x\cos x + \sin x]] + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] \} + (\cos x + v^2(x) [x\cos x + \sin x] + (\cos x + \sin x]$ $(x)\cos x + v 2'(x) [x \cos x + \sin x] + .3 = Sln x, or finally v 1'(x) \cos x + v 2'(x) [x \cos x + \sin x] = Sln x.$ (5) We now have conditions (2) and (5) from which to obtain v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x x Sln x cos x + Sln x v 1'(x) = -x, Sln x v 1' Sln x 0 cos x Sln x v 2' (x) = = 1. Sln x x Sln x cos x + Sln x 248 Chapter 4.2 x Sln x 2 7. + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 + x Sln x or y (x) = p 2. x Sln x 0 In x; and (as on page 172 of the Let x text) dy dy x dx = dt ' X 2 d2y = d 2 y d or dt 2 + 3 y = 0 dt d 2 y d or dt 2 - 4 at + 3y = 0. The
auxiliary equation of this is m 2 - 4 m + 3 = 0, and it has the roots m = 1, 3. Thus the general solution of the D.E. ln y and t is t 3t y = c 1 e + c 2 e We return to the original independent Higher-Order Linear Differential Equations 249. bl d 1 e t by X and e 3t by 3 var1a e x an rep ace x. we find that the general solution of the given 3 y = c 1 x + c 2 x · Doing this, D. E. is 3. Let x t = e; then t d d 2 d d 2 y d = In X X Y - Y X 2y = , dx - dt ' dx 2 dt 2 dt . into 4 [d 2y d + 3y = 0 or dt 2 dt dt The D.E. transforms 2 4 dy -s dt2 dt 2 4m - Sm + 3 + 3y = 0. The auxiliary equation of this 1S 0, and it has the roots m = 1 3 Thus the = 2 ' 2 . general solution of the D.E. in y and t 1S t/2 3t/2 Y = c 1 e + c 2 e · We return to the original independent variable x and replace e t / 2 by x 1 / 2 and e 3t / 2 by x 3 / 2. Doing this, we find that the general solution of h $D E \cdot \frac{1}{2} \frac{3}{2} t e glven \cdot 1S Y = c 1 x + c 2 x \cdot 6$. Let x t = e; then t = In dy x, x dx = X 2 d2y = d 2 y dy dt' dx 2 dt 2 dt . d 2 d 2 - 4 d + 13y = 0. The auxiliary equation of this is dt t 2 m - 4m + 13 = 0, and it has the conjugate complex roots 2 % 3i. The general solution of the D.E. in y and t is y = e 2t (c l sin 3t + c 2 cos 3t). 'We return to the original. d d. bl d 1 2t by 2 d b 1n epen ent var1a e x an rep ace e x an t y In x. Doing this, we find that the general solution of the given D.E. is y = x 2 [c 1 sin(3nx) + c 2 cos(3nx)], or y = x 2 [c 1 sin(1n x 3) + c 2 cos(3nx)]. x = dt + x... = dt - x... = dt - x... = dt - x... = dt + d2 The D.E. transforms into d2y - dy + 9y = 0. The auxiliary equation of the D.E. in y and t 1S Y = c1 sin 3t + c2 cos 3t + V we return to the original independent variable x and replace t by In x. Doing this, we find that the general solution of the given D.E. IS Y = c l sin(3 ln x) + c 2 cos(3 ln x) or y = c l sin((ln x 3) 3 + c 2 cos((ln x)) 0 + c 2 cos(3 ln x)) or y = c l sin(3 ln x) + c 2 cos(3 ln x) or The general solution of the D.E. in y and t is t/3 y = (c 1 + c 2 t)e · We return to the original independent variable x and replace e t / 3 by x 1 / 3 and t by In x. Doing this, we find that the general solution of the given D.E. is y = (c $1 + c 2 \ln x$) x 1 / 3. Higher-Order Linear Differential Equations 251 11. Let x t = e · then t = , In x, dy x - dx = dy x 2 d 2 y = dy x dt ' dx2 d 2 y _ dt2 dt ' 3 d 3 and (as on page 174 of the text) x dx 3 = d 3 y _ 3 d 2 y dt 3 dt 2 dy d 3 y _ d 2 + 2 dt . The D.E. transforms into 3 + dt 3 dt 2 dy dt 11 dy dt - 6y = 0 or d 3 y _ 6 d 2 y + dt 2 dt dt dt 3 dt 2 dy dt 3 d solution of the D.E. in y and t 1S t 2t 3t Y = c l e + c 2 e + c 3 e We return to the original independent variable x and replace e t by x, e 2t by x 2, and 3t 3 e by x. Doing this, we find that the general solution 2 3 of the given D.E. is $y = c l x + c 2 x + c 3 x \cdot 14$. Let x t = e; then t = In x, dy x - dx - dy and dt' X 2 d2y = d 2 y dy dx 2 dt 2 dt . 4 d 4 y To transform x, we apply the dx four-step procedure outlined in the Remarks on page 175 of the text. With n = 4, we determine r(r - 1)(r - 2)(r - 3). We expand this to obtain 4 3 2 6r. Replacing r k by dky for k = 1, 2, r - 6r + 11 6 dt. This lS dt 3 dt 2 4 d 4 y 4 d 4 4 d 3 y d 2 y _ dy d Y _ 6 x 4. That 1S, x J = + 116 dt - dx 4 dt 4 dt 3 dt 2 dx 252 Chapter 4 d 4 d 3 d 2 y d The D.E. transforms into 4 - 6 Y 3 + 112 - 6 dt dt dt 4 dt 3 dt 2 dy + 6 dt - Sy = 0. The auxiliary equation of this IS m 4 _ 6m 3 + 7m 2 + 6m - S = 0, and its roots are 1, 2, 4, and -1. The G.S. of the D.E. in y and t is t 2t 4t - t Y = c 1 e + c 2 e + c 3 e + c 4 e t original variable x and replace e by x, 4 - t - 1 x, and e by x Doing this, we find 2 4 the given D.E. IS Y = c 1 x + c 2 x + c 3 x 'We return to the 2t 2 e by x that the -1 + c 4 x 4t b e y G.S. of 15. t dy dy 2 d 2 y d 2= dt2 dt t 4e - 6 (1) (note that x 1n the right member has transformed into e t; see Remark 1, page 174 of text). The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 5m + 6 = 0, with roots m = 2, 3. Thus the complementary function of . 2t 3t (1) 1S yc = c 1 e + c 2 e \cdot We find a particular integral by the method of undetermined coefficients. 'We assume t B. Then y, t "t and substituting yP = Ae + e Ae, p = Ae + e Ae + e1. Returning to the original independent variable x by replacing t by et c., find the general solution of e x, we the D.E. : 2 3 + 2x - 1. given y = c 1 x + c 2 x 16. t Let x = e; then t = In x, dy x - dx = $x 2 d 2 y dt' dx^2 = d 2 y$ dt dt. The D.E. transforms into d 2 y dt ' dx^2 = d 2 y dt' dx^2 = d 2 y D.E. is m 2 - 6m + 8 = 0, with roots m = 2, 4. Thus the 2t 4t complementary function of (1) 1S yc = c 1 e + c 2 e We find a particular integral by the method of undetermined coefficients. Ve assume yp = Ae 3t. Then yp' = 3Ae 3t, II 9A 3t d b . . . D E (1). kl Y = e, an su stlutting 1nto .. , we qU1C y P have _Ae 3t = 2e 3t. From this, -A = 2 and so A = -2. Thus 3t 1S Y = -2e, and its p 4t 3t + c 2 e - 2e. Returning the particular integral of (1) 2t general solution is y = c 1 e to the original independent variable x by replacing e t by x, etc., we find the general solution is <math>y = c 1 e to the original independent variable x by replacing e t by x, etc., we find the general solution
of the given D.E.: 243 Y = c 1 x + c 2 x - 2x + 18. Let x = t then t = In x, dy dy and e x = , dx dt ' 2 d 2 d 2 y x - L = The D.E.transforms into dx 2 dt 2 dt. 254 Chapter 4 d 2 t Y 2 + 4y = 2t e \cdot dt (1) The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 4 = 0 with roots m = :i: 2i. Thus the complementary function of (1) IS Y c = c 1 sin 2t + c 2 cos 2t. We find a particular integral by the method of undetermined coefficients. We assume y = Ate t + Bet. p Then y = Ate t + (A + B)e t , y H = Ate t + (2A + B)e t , y H = Ate t + (2A + B)e t , and p p substituting into the D.E. (1), we have 5Ate t + (2A + 5B)e t = 2te t . From this 5A = 2, 2A + 5B = 0, hence 2 B - 4 Thus the particular A = -, = 25 . 5 integral of (1) 2 t 4 t and its general IS yP - te - e, -5 25 2 t 4 t solution is y = c 1 sin 2t + c 2 cos 2t + 5 te - 25 e · Returning to the original independent variable by t replacing e by x, t by In x, we find the general solution of the given D.E.: y = c 1 sin(lnx) + c 2 cos(lnx) + 5 4x - 25. 20. Let x = t then t = In x, dy dy and, e. x = , dx dt ' 2 d 2 y 2 dy The D.E. transforms into x = dx 2 dt 2 dt. Higher-Order Linear Differential Equations 255 2 dy 5e 2t !.L dy 5y 3 dt + = or dt 2 dt d 2 y 5y - 5e 2t . (1) dt 2 4 dt + - The homogeneous equation of (1) is Yc = e 2t (c l sint + c 2 cost). We find a particular integral of (1) by the method of undetermined coefficients. We assume y = Ae 2t p and readily find A = 5. Thus the G.S. of (1) 1S 2t. 2t Y = e(c 1 Sln t + c 2 cos t) + 5e. We return to the original independent t by In x. Thus the y=x 2[c l sin(lnx) + variable x by replacing e 2t G.S. of the given D.E. 1S 2 c 2 cos(lnx)] + 5x . 2 by x and 21. Let $x t = e \cdot then t$, dy dy X 2 dy - ln x, $x dx - dt' dx^2 = d 2$. correspon 1ng to IS 1S m - m + m - =. W1t roots 2, 4, 5. The complementary function of (1) 1S 2t 4t 5t 0 Yc = c 1 e + c 2 e + cae We find a part1cular integral of (1) by the method of undetermined coefficients. We -t assume vp = Ae. Differentiating and substituting into (1), we find -A - 11A - 38A - 40A = -9 or -90A = -9: 1 and so A = 10. Thus the G.S. of (1) 1S 2t Y = c 1 e 4t + c 2 e 5t + c 3 e 1 -t + 10 e Returning to the original independent variable x by replacing e 2t by etc., we have the G.S. of the glven D.E., that 1S, 2 x, 2 Y = c 1 x 4 + c 2 x 5 + c 3 x 1 -1 + 10 x 23. Let x t then t In x, dy _ 2 d 2 y d 2 dt auxiliary equation of this is m 2 - 3m - 10 = 0, with roots m = 5, -2. The general solution of the D.E. 1n y and t is 5t - 2t Y = c 1 e + c 2 e Returning to the original independent variable x, t et c., and obtain the we replace e by x, general solution of the given D.E. 1n the form 5 - 2 (1) Y = c 1 x + c 2 x Differentiating (1), we find $\frac{1}{4}$ y' = 5c 1 x - 3 2c 2 x (2) Higher-Order Linear Differential Equations 257 Now apply the I.C.'s. Applying y(l) = 5 to (1), we have $c \ 1 + c \ 2 = 5$; and applying y'(l) = 4 to (2), we have $5c \ 1 - 2c \ 2 = 4$. Solving these two equations $ln \ c \ 1 = 2$, $c \ 2 = 3$. Substituting these values back into (1), we find the particular solution of the stated 5 - 2 I.V.P.: y = 2x + 3x. 26. Let x = t then t = In x, $dy_d and e x$, dx - dt' 2 d 2 d 2 y dy = The D.E. transforms into x dx 2 dt 2 - dt. $d 2 y_d y = dt 2 dt t 4e - 8$. (1) The auxiliary equation of the corresponding homogeneous D.E. is m 2 - m - 2 = 0 with roots m = -1, 2. Thus the -t 2t complementary function of (1) 1S Yc = c 1 e + c 2 e We find a particular integral by the method of undetermined coefficients. We assume y = Ae t + B. Then $y_{,} = Ae t_{,} p_{,} p_{,} = Ae t_{,} p_{,} p_{,} = Ae t_{,} and its p_{,} 2t + c_{,} 2e t_{,} 2e + 4$. Thus the $t_{,} p_{,} p_{,} = Ae t_{,} p_{,} p_{,} = Ae t_{,} p_{,} p_{,} p_{,} = Ae t_{,} p_{,} p_{,} p_{,} = Ae t_{,} p_{,} p_{,$ etc., we find the general solution of the given D.E. $-1\ 2\ Y = c\ 1\ x + c\ 2\ x - 2x + 4$. (2) 258 Chapter 4 Differentiating (2), we find: $-2\ y' = -c\ 1\ x + 2c\ 2\ x - 2$. (3) Now apply the I.C.'s. Applying y(l) = 4 to (2), we have $c\ 1 + c\ 2 = 2$; and applying y'(l) = -1 to (3), we have $-c\ 1 + 2c\ 2 = 1$. Solving these two equations in c 1 and c 2, we find c 1 = 1, c 2 = 1. Substituting these values back into (2), we find the particular solution of the stated -1 2 I.V.P.: y = x + x - 2x + 4. 27. Lt t th t 1 dy dy and e x = e; en = n x, x dx = dt 2 d 2 x --L = dx 2 d 2 y _ dy dt 2 dt - 4 dt + 4y The D.E. transforms into 4 2t 6 3t e - e or d 2 y _ 5 dy + 4y = 4e 2t _ 6e 3t. (1) dt2 dt The auxiliary equation of the homogeneous D.E. corresponding to (1) is m 2 - 5m + 4 = 0 with roots 1, 4. t 4t The complementary function of (1) lS Yc = c 1 e + c 2 e Ye find a particular integral of (1) by the method of 2t 3t undetermined coefficients. Ye assume y = Ae + Be. p Differentiating, substituting into (1), and simplifying, we obtain 2Ae 2t - 2Be 3t = 4e 2t - 6e 3t . Thus A = -2, t 4t 2t 3t B = 3. The G.S. of (1) 1S Y = c 1 e + c 2 e - 2e + 3e . Returning to the original independent variable x, we Higher-Order Linear Differential Equations 259 t replace e by x, etc., and obtain the G.S. of the given D.E., y = c 1 x + c 2 x 4 - 2x 2 + 3x 3. (2) Applying the I.C. y(2) = 4 to this, we obtain 2c 1 + 16c 2 - 12. Differentiating (2), we have y' = c l + 4c 2 x 3 - 4x + 9x 2. Applying the I.C. y'(2) = -1 to this, we obtain c 1 + 32c 2 = -29. The system of { C 1 + 32c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 32c 2 - 6 equations c 1 + 32c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 32c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3, c 2 = -23/24. Thus we obtain the solution c 1 - 29 - 5/3. dy = dt 2 dt - 6y t. (1) The auxiliary equation of the corresponding homogeneous equation is m 2 - m - 6 = 0, with roots 3, -2. The complementary function of (1) by the method of undetermined coefficients. We assume yp = At + B. Differentiating and substituting into (1), we find -A - 6At 6B = t. From this, -6A = 1, -A - 6B = 0. 260 Chapter 4 1 1 Thus A = -6 ' B = 36. 3t The G.S. of (1) IS Y = c 1 e - 2t + c 2 e 1 1 6 t + 36. Returning to the original t independent variable x, we replace e by x, etc., and obtain the G.S. of the given D.E. $3y = c 1 x - 2 + c 2 x 1 1 6 \ln x + 36$. (2) Ve apply the I.C. y(l) = 1/6 to (2), obtaining 5 c 1 + c 2 = 36Differentiating (2), we have y' 3c 1 x 2 - 2c x - 31 Applying the I.C. y'(l) -1/6 = - - = 26x to this, we obtain 3c 1 - 2c 2 = 0, we find c 1 = 1/18, c 2 = 1/12. Thus we obtain the solution 1 3 1 - 21 In x + 1 y = 18 x + 12 x or $6 36 y = (x 3 3 + -2x - 2 \ln x +) = 31$. Although this is not a Cauchy-Euler equation, it is t similar to one. Let x + 2 = e. Then t = In(x + 2), $d d 2 d 2 = d 2 y_d (x + 2) = and(x + 2)$ Th D E d dt' 2 dt 2 dt - dt or 2 U - 2 - 2 dt 3y = 0. dt Intercorresponding auxiliary equation is m 2 - 2m - 3 = 0, with roots 3, -1. $D E \cdot dt \cdot 3t + 1$. 1n y an IS y = c 1 e + c 2 e The G.S. of the corresponding auxiliary equation is m 2 - 2m - 3 = 0, with roots 3, -1. $D E \cdot dt \cdot 3t + 1$. 1n y an IS y = c 1 e + c 2 e The G.S. of the Returning to the Higher-Order Linear Differential Equations 261 original independent variable x, we replace e 3t by 3 -t -1 (x + 2) and e by (x + 2). Thus the G.S. of the given . 3 -1 D.E. IS Y = c 1 (x + 2) + c 2 (x + 2). Chapter 5 Section 5.2, Page 197 1. This is an example of free, undamped motion; and equation (5.8) applies. Since the 12 lb. weight stretches the spr1ng 1.5 in. = ft., Hooke's Law F = ks gives 12 = (), w 12 3 so k = 96 Ib./ft. Also, m = g = 32 = 8 (slug). Thus by (5.8) we have the D.E. x'' + 96x = 0 or x'' + 256x = 0. (1) Since the weight was released from rest from a
position 2 1n. = ft. below its equilibrium position, we also have 1 the I.C. x(O) = 6' x'(O) = 0. The auxiliary equation of the D.E. (1) is r 2 + 256 = 0 with roots r = 16i. The G.S. of the D.E. is $x = c 1 \sin 16t + c 2 \cos 16t \cdot (2)$ Differentiating this, we obtain $x' = 16c 1 \cos 16t \cdot 16c 2 \sin 16t \cdot (2)$ Applying the first I.C. to (2), we find $c 2 = c \sin 16t \cdot (2)$ and applying the second to (3) gives c 1 = 0. Thus the solution of the D.E. is $x = c 1 \sin 16t + c 2 \cos 16t \cdot (2)$. Linear Equations 263 x = () cos 16t. The amplitude is (ft.); the period =; (sec); and the frequency is (8) = oscillations/sec. 2. This is an example of free, undamped motion; and equation (5.8) applies. Since the 16 lb. weight stretches the spring 6 in. =; foot, Hooke's Law F = ks gives 16 = k(), w 16 1 so k = 32 lb./ft. Also, m - g = 32 = 2 (slug). Thus by (5.8), we have the D.E.; x'' + 32x = 0 or x'' + 64x = 0. (1) The auxiliary equation corresponding to (1) is r = 8i. The G.S. of the D.E. (2) Differentiating this, we obtain x' = 8c l cos 8t - 8c 2 sin 8t. (3) The three cases (a), (b), (c) lead to different I.C.'s. In (a), the ; ft. below its 1 I.C. x(O) = 3. weight is released from a position, so we have the first Since it is released with initial velocity of 2 ft./sec., directed downward, we have the second to (3), we have 8c 1 = 2, so 264 Chapter 5 1 c 1 = 4. Thus we obtain the particular solution $x = () \sin 8t + () \cos 8t$. In (b), we again have the first I.C. x(O) = . But here the weight is released with initial velocity of 2 ft./sec., directed upward, so the second I.C. 1 to (3) gives c 1 = -4. Thus we obtain the particular solution $x = -() \sin 8t + () \cos 8t$. In (c), the weight is released from a position 4 In. - 1 3 ft. above its equilibrium position, so we have the first 1 I.C. x(O) = -3. Since it is released with initial velocity of 2 ft./sec., directed downward, we have the second I.C. x'(O) = -3. Since it is released with initial velocity of 2 ft./sec., directed downward, we have the second I.C. x'(O) = -3. particular solution $x = () \sin 8t - () \cos 8t$. 5. Equation (5.8) applies. Since the 4 lb. weight stretches the spring 6 in. = ft., Hooke's law F = ks gives 4 = (1) w 4 1) k 2' so k = 8 [b]/ft. Also, m = g = 32 = 8 (slug. By (5.8) we have the D.E. Applications of Linear Equations 265 x" + 8x = 0 or x" + 64x = o. (1) Since the weight was released from its 2 = 0; and applying the second x'(0) = 2 to (3), we get $8c \ 1 = 2$, 1 so $c \ 1 = 4$. Thus the solution of the D.E. satisfying the glven I.C.'s is x = () sin 8t, (4) and its derivative IS x' = 2cosSt. (5) These are the displacement and velocity, respectively, and hence provide the answer to part (a). From the solution 266 Chapter 5 (4), we see that the amplitude is (ft.), the period 1S 2 1 4 8 = 4 (sec.), and the frequency IS (/4) = oscillations/sec. These are the answers to (b). To answer (c), we seek times t at which x = 1.5 In. = 1 and x' o. Thus first 1 1. solution (4) 8 ft. > we et $x = -\ln 8$ and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and hence t n or t 5 n where = +-= 48 + 4' n = 48 4 0, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and find sin 8t 1 From this, 8t 2nif 8t 5 = 2.--+ or = -+-6 6 2n, and find sin 8t 1 From 1,2,... Ye must choose these t for which x' > 0. From (5), we see that x' (4 + 7") = 2 cos (5 6 1" + 2n1") > 0, but x' (; + 7") = 2 cos (5 6 1" + 2n1") > 0, 1, 2, ... 8. Equation (5.8) applies, with m = ; = 3 = (slug) and the spring constant k > 0 to be determined. Thus (5.8) becomes 1 " k - x + x = 0 4 or x" + 4kx - 0. (1) Since the weight is released from a position A ft. below its equilibrium position A ft. below i the I.C.'s x(O) = A > 20, x'(O) = 3. The auxiliary equation of D.E. (1) is r + 4k = 0 with ro 0 t s r = :: 2 Jk i. The G. S. of the D. E. is x - c1 sin 2 Jkt + c 2 cos 2 Jkt. (2) Differentiating we obtain x' = 2 Jkc 1 cos 2 Jkt + c 2 cos 2 Jkt. (3) Applying the first I.C. x(O) = A to (2), we find c 2 = A; and applying the second x'(O) = 3 to (3), we have 2 Jkc 1 - 3 3 Jkt + c 2 cos 2 Jkt. (3) Applying the first I.C. x(O) = A to (2), we find c 2 = A; and applying the second x'(O) = 3 to (3), we have 2 Jkc 1 - 3 3 Jkt + c 2 cos 2 Jkt. so c 1 = ---. Thus the solution of the D.E. satisfying 2Jk the given LC.'s lS x = (2) sin2Jk t + A cos2Jk t. Ve express this ln the form (5.18) of the text. Multiplying and dividing by c = J(2 t + A 2 j g + 4kA 2 = 2Jk we have $x = c[(2; c) \sin 2Jk t + () \cos 2Jk t]$. Then letting A cos "3" we have $x = c = -.Sln - 2Jk c c \cos(2Jk t + .)$. From this, h. d. 21" 1" but t e perlO lS--- - . - , 2Jk Jk the period 1" Thus Jk = 2, and hence k lS g1, ven to be 2 . = 4. The amplitude lS c = J(2) 2 + A 2 = 5, from which A = {ff 4. 9. There are two different D.E.'s of form (5.8) here, one involving the 8 lb. weight and the other involving the other weight. Concerning the 8 lb. weight, m = r = 3 = (slug), and the corresponding D.E. of form (5.8) IS x'' + kx = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. The auxiliary equation is 2 + 4k = 0 or x'' + 4kx = 0, where k > 0 IS the spring constant. as 4. Thus 21" = 2Jk 4, from which k 2r = 16. Now let w be the other weight. w w For this, m = g = 32 (slugs), and the corresponding D.E. of form (5.8) lS w " 32 x 2 1" + 16 x = 0 or " x 21"2 + -x = 0. w Th $0 1 \circ \circ 2 21"2 0 \circ th t e aUXI lary equation <math>lS r + -- = WI$ roos w r = % w 1" 1. The G.S. of the D.E. is x = c l Sln r t + c 2 cos r t. Applications of Linear Equations 269 From this the period of the motion 21" V 2w. But the IS 1" = period of this motion is glven to be 6. Thus V2w = 6, from which we find w = 18 (lb.). Section 5.3, Page 208 1. (a) This is a free damped motion, and Equation (5.27) applies. Since the 8 lb. weight stretches the spring 0.4 ft., Hooke's Law F = ks gives 8 = k(0.4), so k = w 8 1 20 lb.jft. Also, m = g = 32 = 4 (slug), and a = 2. Thus equation (5.27) becomes x'' + 2x' + 20x = 0. (1) 1 Since the weight is then pulled down 6 in. = 2 ft. below its
equilibrium position and released from rest at t = 0, we have the I.C.'s x(0) 1 = 2 'x'(0) = 0. (2) (b) The D.E. (1) may be written as " + 8 x' + 80x = 0. x The auxiliary equation 2 + 8r + 800, with roots 1Sr = r = -4: t: 8i. The G.S. of the D.E. $1S - 4t \sin 8t + c 2 \cos 8t$. (4) 270 Chapter 5 1 Applying the first I.C. (2) to (3), we find c 2 = 2' and applying the second to (4), we have 8c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0, c 2 - 1 from which c 1 - 4c 2 = 0. = 2 = 4. Thus the solution is $x = e - 4t [() \sin 8t + (;) \cos 8t] (5) - J(41) 2 + (21) 2 - (c). We first multiply and divide (5) by c 4, obtaining <math>x = (1) e - 4t \cos(8t -), 21$ where , is such that $\cos x = (1) e - 4t \cos(8t -), 21$ where , is such that $\cos x = (1) e - 4t \cos(8t -), 21$ where , is such that $\cos x = (1) e - 4t [() \sin 8t + (;) \cos 8t] (5) - J(41) 2 + (21) 2 - (c). We first multiply and divide (5) by c 4, obtaining <math>x = (1) e - 4t \cos(8t -), 21$ where , is such that $\cos x = (1) e - 4t \cos(8t -), 21$ where , is such that $\cos x = (1) e - 4t \cos(8t -), 21$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where $x = (1) e - 4t \cos(8t -), 31$ where x =damped motion, and Equation (5.27) applies. 1 Since the 8 lb. weight stretches the spring 6 in. = 2 ft., Hooke's Law F - ks gives 8 = k(;), so k = 16 lb./ft. w 8 Also, m = = g 32 (5.27) becomes 1 - 4 (slug), and a = 4. Thus equation XU + 4x' + 16x = 0. Applications of Linear Equations 271 The I.C.'s are x(O) 3 - 4 'x/CO) = o. The D.E. may be written in the form X II + 16 / 64 0 x + x = . 2 The auxiliary equation lS r + 16r + 64 = 0 with roots -8, -8 (double root). The C.S. of the D.E. is x = (t) -8t O ff 0 0 h o f o d dx c 1 + c 2 e . 1 erentlating t lS, we ln dt = -8t (-8C 1 - 8c 2 t + c 2) e . Applying the first I.C. to the G.S., we find c l = ; and applying the second to its derivative, we have -8c 1 + c 2 = 0 from which c 2 = 6. Thus we find the solution $x = (+6t)e^{-8t}$, 6. This is a free damped motion, and Equation (5.27) applieso The mks system of units is used. Since a force of 4 newtons stretches the spring 5 cm = 0.05 meters, Hooke's Law F = ks gives 4 = k(0.05), so k = 80 newtons/meter. Also, we are given that m = 2 and a = 16. Thus equation (5.27) becomes 2x'' + 16x' + 80x = 0 The I.C.'s are x(0) = 0.02 (in meters), x/(0) = 0.04 (in meters), x $2y_{6}t + (2,; 6 c 1 - 4 c 2) \cos 2,; 6 t J$. (2) Applying the first I.C. x(0) = 0.02 to (1), we find 2,; 6 c 1 - 4 c 2 = 4/100, from which c 1 = ,; 6/100. Thus the solution IS -4t $x = e 100 [,; 6 sin 2,; 6 t + 2 cos 2,; 6 t J \cdot 7.$ This is a free damped motion, and Equation (5.27) applies. Since a force of 20 lb. would stretch the spring 6 in. = ; ft., Hooke's Law F = ks glves 20 = k(;), so k = 40 lb./ft. w 4 1 The weight is 4 lb., so m = g = 32 = 8 (slug); and a = 2. Thus equation (5.27) becomes XU + 2 x' + 40x = 0. Since the weight is 4 lb., so m = g = 32 = 8 (slug); and a = 2. Thus equation (5.27) becomes XU + 2 x' + 40x = 0. Since the weight is released from rest from a position, we have the 2 I.C.'s x(O) = 3 ' x'(O) = 0. The D.E. may be written x H + 16 x' + 320x = 0. 2 The auxiliary equation lS r + 16r + 320 = 0, with roots -8 * 16i. The G.S. of the D.E. is -8t) x = e (c 1 sin 16t + c 2 co s 16t · Applying the first I.C. to the G.S., we find c 2 2 = 3; and applying the second to the derived equation, we have 16c 1 c 2 1 - 8c 2 = 0, from which c 1 = 2 = 3. Thus we have the displacement (-8t) x = e 3 (sin 16t + 2 cos 16t). To put this ln the form (5.32), we multiply and divide by c = I(1)2 + (2)2 = J5, obtaining x = (J5 e; 8t) [[I sin <math>16t + [] cos 16t]. We can now write this as x = (J5 e; 8t) cos (16t - ?). (1) where $\cos_{1} = 1$, and hence, 0.46 (rad.). J5 J5 We have thus answered part (a). To answer part (b), we see from (1) that the period is 21" 1" 16 = 8 (sec.). In the notation of page 202 of the text, 274 Chapter 5 the logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = 8 and A = - = m m logarithmic decrement is 2rb j $\sqrt{2}$ b2 Here b = 2: = 2 2 k 40 = 320. Thus we find the (2) () = decrement IS (211") 8 - %'. - ";256 To answer part (c), we let $x = 0 \ln (1)$ and solve for We have $\cos(16t - ;) 0$, 16t %' from which t = so - , = 2 + k', + - t 2 With; 0.46, this t o . 127 (sec.). - glves = 16. - 8. This is a free damped motion, and Equation (5.27) applies. Since the 24 lb. weight stretches the spring 1 ft., Hooke's Law F = ks gives 24 = k(l). so k = 24 lb.jft. w 24 3. Also, m = g = 32 - (slug), and a = 6. Thus equation (5.27) becomes x'' + 6x' + 24x = 0. or x'' + 8x' + 32x - 0. The I.C.'s are x(0) - 1, x'(0) = 0. The auxiliary equation is m 2 + 8m + 32 = 0, with roots m = -4 * 4i. The G.S. of the D.E. IS -4t () $x = e(c \ 1 \ sin \ 4t + c \ 2 \ cos \ 4t)$. 1 Applications of Linear Equations 275 Differentiating this, we find -4t x' = e [(-4c 1 - 4c 2) sin 4t + (4c 1 - 4c 2) cos 4t]. (2) Applying the first I.C. x(O) = 1, to (1), we find c 2 = 1; and applying the second, x'(O) = 0, to (2), we have 4c 1 - 4c 2 = 0, from which c 1 = 1. Thus the solution IS -4t x = e (sin 4t + cos 4t). (3) This is the first answer to part (a). Top determine the a lternate form (5.32), we multiply and divide (3) by c = j(1)2 + (1)2 = 12, obtaining x = 12 e -4t (1 - sin 4t + 1 - cos 4t). Ye can express this as 12 12 f;) -4t x = v2 e cos(4t - 1:/2), and the time-varying 1 0 d o f;) 2 -4t amp ltu e lS v e (c) The weight first attains a relative maXlmum displacement. above its equilibrium position at the first time t > 0 at which the derivative x' of (4) equals zero. Differentiating (4) and equating it to zero, we obtain $\{i\} - 4t - 4 = -14 + k$, that lS, t = k/4 where k IS an integer. The first t > 0 for which this is valid IS for k = 1, and this gives t = r/4. This IS the time desired. The displacement at this time is found by letting t = /4 in (4). Doing so, we find x = J2 e -r cos 31f' / 4 -0.0432. The minus sign simply indicates that the mass is above the equilibrium position. Thus the first maximum
displacement above it is approximately 0.04 feet. 11. w 10 5 (Equation (5.27) applies, with m = g = 32 = 16 slug), a > 0, and k = 20 Ib./ft. Thus we have the D.E. 5 " 16 x + a x' + 20x = 0. Writing this as 5 x" + 16ax' + 320x = 0, the auxiliary . . 5 2 16 3 0 0 Th b equation IS r + ar + 2 = . e roots are given y = -16a * j 256a 2 - 6400 r 10 (1) In part (a), we seek the smallest value of a for which damping is nonoscillatory. This is the value of a for which damping is critical. It occurs when the two roots given by (1) are real and equal. Thus we set 2 . 2 6400 80 256a - 6400 = 0 and flnd a = 256 and hence a = 16 = 5. In part (b), we let a = 5 in (1) and obtain the roots r = -8, -8 (double root). Then the displacement is given Applications of Linear Equations 277 -8t by $x = (c 1 + c 2 t)e \cdot -8t$ (-8c 1 - 8c 2t + c 2)e. Differentiating this, we have x' = 1 The I.C.'s are x(O) = 2'x'(O) = 1. Applying them to the preceding expressions for x and x', we find c l = ; and -8c l + c 2 = 1, from which c 2 = 5. Thus we obtain the displacement x = (; + 5t)e-8t. For part (c), we note that the extrema of the displacement are found by setting x' = (1 - 40t)e- 8t equal to zero and solving for t. We at once have t 1 Then x() - (5) - 1/5 N 0.51(sec.). For $t > 1 = 40 \cdot 40 + 8 e N 40 + 1 - 40t < 0$ and x' < 0. Also using L'Hospital's Rule, lim x' = 0. Thus the weight approaches its position t - +00 monotonically. 12. This is a free damped motion, and Equation (5.27) applies. Since the 64 lb. weight stretches the spring 4/3 foot, Hooke's Law F = ks gives 64 = k(4/3), so k = 48 lb./ft. w 64 Also, m = = 2 (slugs). The damping constant is a > g 32 O. Thus equation (5.27) becomes 2x'' + ax' + 48x = O. (1) The I.C.'s are x(O) = 2, x/CO) = O. The auxiliary . . 2 2 48 0 . h equation IS m + am + =, Wit roots -a :t: j a 2 - 384 4 (2) m = (a) We must determine a so that the resulting motion is critically damped. This occurs if and only if a 2 - 384 = 0 and hence if a = V384 = 8J6 (recall a > 0). In this case the auxiliary equation has the double root m = -2J6, and the G.S. of the D.E. $(1) IS - 2J6 t x = (c 1 + c 2 t) e \cdot (3)$ Differentiating this, we obtain Iii -2J6 t x' = [(C 2 - 2 y 6 c 1) - 2y6 c 2 t]e \cdot (4) Applying the I.C. x(O) = 2 to (3), we find c 1 - app derivative of this is x' = -48 t e - 2J6t. Since x' < 0 for all t > 0, x decreases monotonically for all t > 0. (b) The motion is underdamped if 0 < a < 8J6. Then a 2 384 < 0, the roots (2) are the conjugate complex Applications of Linear Equations 279 numbers [-a * j 384 - a 2 i) /4, and the G. S. of D. E. (1) IS of the form -at/4 [. j 384 - a 2 t x = e c 1 Sln 4 384_2 + c 2 cos 4 a t. (5) From the discussion on pages see that the quasi period is 200-201 o f the text, w e gi ven by I 21f]. j 384 - a 2 4 Equating this to the glven value /2 in the present case, we obtain j 384 - a 2 = 16, from which a = 8J2. This is the desired value of a in part (b). Vith this value of a, the solution (5) becomes -2J2t x = e (c 1 sin 4t + c 2 cos 4t), (6) with derivative x' = e- 2J2t [(2J2 c 1 4 c 2) sin 4t + (4 c 1 - 2J2 c 2) cos 4t]. Applying the I.C.'s x(O) = 2, x' (0) = o to these gives c 2 = 2, 4 c 1 - 2J2 c 2 = 0, from which c 1 = J2. Thus the solution (6) becomes 280 Chapter 5 - 2/it Ii) x=e (y2sin 4t + 2cos 4t) = .../6 e -2/it [12 sin 4t + cos 4t] .../6 .../6 -2/it = y6 e cos(4t -), where cos = 2.0 cos 4t] = .../6 e -2/it [12 sin 4t + cos 4t] .../6 e -2/it [12 sin 4t 2/../6, sin = 12/../6. From this we see that the time-varying amplitude is ../6e- 2 /it. 13. Here m = w = g 32 32 = 1 (slug). If there were no resistance, the D.E. would be m x" + kx = 0, where k > 0. The auxiliary equation of this lS r 2 + k this 27r undamped motion is , and hence the natural frequency IS Jk Jk 2r. But this IS given to be 4. 7r Hence we have t = ;, from which Jk = 8 and k = 64. This answers (a). To answer (b), we take the resistance into account and have the D.E. mx" + ax' + 64x = 0. The auxiliary equation of this is r 2 + ar + 64 - o with roots -a) a 2 256 where 2 < 256. r = 2 a G.S. of this D.E. IS The x = e- at / 2 (c l sin) 256 a 2; + c 2 cos) 256 - a 2;). Applications of Linear Equations of Linear Equations of Linear Equations 281 The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281 The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281 The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281 The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281
The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281 The period of the trigonometric factor of this is 2r 47r Hence the frequency of this applications of Linear Equations 281 The period of the trigonometric factor of this applications of Linear Equations 281 The period of the trigonometric factor of this applications of Linear Equations 281 The period of the trigonometric factor of this applications of Linear Equations 281 The period of the trigonometric factor of this applications a frequency 4 and so is IS glven as 7r 2j 256 - 222 Thus a Thus 256 - 64, from which 4r = a = r 7r a = 8J3. Section 5.4, Page 2171. The D.E. is of the form (5.58) of the text, with w 6 3 a = 0 (since damping is m = g = 32 = 16 (slugs), negligible), k = 27 lb.jft., and F(t) = 12 cos 20t. Thus we have 3" 7 16 x + 2 x = 12 cos 20t or " 4 x + 144. x = 6 cos 20t. (1) The I.C.'s are x(O) = 0, x/(O) = 0. The auxiliary equation of the homogeneous D.E. corresponding to (1) IS x = c 1 sin 12t + c 2 cos 12t. Ve use undetermined coefficients to find a particular integral of (1). Ve let x = A sin 20t + B $\cos 20t$. Then x' = 20A $\cos 20t - p p 20B \sin 20t + 256B \cos 20t = 64 \cos 20t$. Substituting p into (1), we obtain -256A $\sin 20t - 256B \cos 20t = 64 \cos 20t$. (2) Differentiating, we find x' = 12 c 1 cos 12t - 12 c 2 sin 12t + 5 sin 20t. (3) Applying the I.C. x(0) = 0 to (2), we have c 2 - (1/4) = 0, so c = 0. Thus we have 1 x = cos 12t - cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the form (5.58) of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of the text, with m = = g 10 5 32 = 16 (slug), a = 5, k = 20, and F (t) = 10 cos 20t 4 3. w The D.E. is of text, wit s 8 t. Th us we have 5 " 16 x + 5 x' + 20x = 10 cos 8t or x" + 16 x' + 64x = 32 cos 8t. (1) The I.C.'s are x(O) = 0, x/(O) = 0. The auxiliary equation of the homogeneous D.E. corresponding to (1) IS r 2 + 16r + 64 = 0 or $(r + 8)^2 = 0$, with roots -8, -8 (double root). Thus the complementary function of (1) 18 Applications of Linear Equations 283 -8t Xc = 0 or (r + 8)^2 = 0. (c 1 + c 2 t)e. Ve use undetermined coefficients to find a particular integral. Ve let X = A cos 8t + p B sin 8t. Then Xl = -8A sin 8t + 8B cos 8t - 128A sin 8t = 32 cos 8t. 1 Hence -128A = 0, 128B = 32, so A = 0, B = 4. Thus we have the particular integral x p = () sin 8t, and the G.S. of D.E. (1) is x = (c + c + c + 2)e + 8t + () sin 8t. Differentia -8t ting this, we obtain Xl = (-Sc + c + c + 2)e + 2 cos 8t. Applying the stated I. C. to these, we have c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + c + 2 = 0, and hence c + 2 = -2. Thus we obtain the solution x = -2te - 8t + (-) = -8t + (-) = -832 = 8 (slug), a = 2, and F(t) = 13sin4t. Also, by Hooke's Law F = ks, we have $1 = 128x 104 \sin 4t$. (1) x + t = 1 The I.C.'s are x(O) = 2 and x/(O) = 0. The auxiliary equation of the homogeneous D.E. corresponding to (1) lS r 2 + 16r + 128 = 0 with roots r = -8 8i. Thus ementary function of (1) IS 284 Chapter 5 -8t x = e (c 1 sin 8t + c 2 cos 8t) · Ve use undetermined coefficients to find a particular integral. We let $x = A \sin 4t + B \cos 4t$. P Substituting into (1) and simplifying, we find (112A - 64B) sin 4t + (64A + 112B) cos 4t = 104 sin 4t. Thus we have the equations 112A - 64B = 104 and 64A + 112B = 0. These reduce to 14A - 8B = 13 and 8A + 14B = 0, from which we find A - ;0'B = -;. Thus we have the particular integral $x p = (170) \sin 4t - (;) \cos 4t$ and the G.S. of the D.E. (1) lS x = e - 8t (c l sin $8t + c 2 \cos 8t$) + (170) sin $4t - (;) \cos 4t$. Differentiating this, we obtain -8t x' = -8t (c l sin $8t + c 2 \cos 8t$) + (170) sin $4t - (;) \cos 4t$ and the G.S. of the D.E. (1) lS x = e - 8t (c l sin $8t + c 2 \cos 8t$) + (170) sin $4t - (;) \cos 4t$. $e [(-8c \ 1 \ 8c \ 2) \sin 8t + (8c \ 1 - 8c \ 2) \cos 8t] + (15 \ 4) \cos 4t + (3c \ 1 - 8c \ 2) \cos 8t] + (17 \ 0) \sin 4t - (2c \ 2) \cos 8t] + (17 \ 0) \sin 4t - (2c \ 1 - 8c \ 2) \cos 8t] + (17 \ 0) \sin 4t - (2c \ 1 - 8c \ 2) \cos 8t] + (17 \ 0) \sin 4t - (2c \ 1 - 8c \ 2) \sin 8t + (12 \ 0) \sin 8t + (12$ part (a). Concerning part (b), we note that the steady-state term is (170) sin 4t - () cos 4t. The amplitude of this IS given by $c = (2, 0)^2 + (2, 0)^2 + (2, 0)^2 + (3, 0)^2$ are x(O) = 0, x'(O) = 0. The auxiliary equation of the homogeneous D.E. corresponding to (1) IS m 2 + 2m + 10 = 0, with conjugate complex roots -1 % 3i. Thus the complementary function of (1) is -t Xc = e (c 1 sin 3t + c 2 cos 3t) . (2) The problem tells us to find x uSlng Theorem 4.10 of p Chapter 4. Ve first find the particular integral x p,k of " + 2x' + lox = sin kt, (k constant), (3) x by the method of undetermined coefficients. Ve let x k - p, kA cos kt - kB sin kt + B cos kt. Then x I k - p, x, k = k 2 A sin kt + B cos kt. Then x I k - p, x, k = k 2 A sin kt + B cos kt. Then x I k - p, x, k = k 2 A sin kt + B cos kt. Substituting into (3) and simplifying, we obtain
[(10 - k 2) A - 2kB] sin kt + [2kA + (10 - k 2) B] cos kt = sin kt. From this, 286 Chapter 5 A - 10 - k 2 (10 - k 2) 2 + 4k 2 ' B -2k(10 - k2)2 + 4k2. Thus we find 10 - k2xk = 222 sin kt p, (10 - k) + 4k2k222 cos kt (4) (10 - k) + 4k as the particular integral $9 \cdot 2(5)x = 85$ Sln t $85 \cos t p$, l of the D.E. " + 2X' lOx = sin t. Letting k 2 (4), x + = ln we have the particular integral $3 \cdot 21(6)x = 26$ Sln t $13 \cos 2t$ p_{2} of the D.E. " x + 2x I + lOx = sin 2t. Letting k = 3 ln (4), we have the particular integral $x p_{3} = 1.6 sin 3t - 37 cos 3t 37 (7)$ of the D.E. x'' + 2X' + lOx = sin 3t. We are now ready to apply Theorem 4.10 of Chapter 4, and we essentially do so twice. We first use the theorem with (5) and (6) to obtain the particular integral Applications of Linear = $85 \sin t - 85 \cos t + [2 3 6 \sin 2t - 113 CDS 2t]$ (8) fh D E /I 2 '10. 1. 2 ot e... x + x + x = S ln t + 4 s ln t. We then use the theorem with (7) and (8) to obtain the particular integral [9.21(3.212)] x p = $85 \operatorname{Sln t} - 85 \cos t + 426 \operatorname{Sln t} - 13 \cos t + [3 \sin 3t - 367 CDS 3t]$ of the D.E. x" + 2x' + lOx = sin t + sin2t + sin3t (1) Thus the G.S. of (1) IS -t x = e (c 1 sin 3t + c 2 cos 3t) 9 2 3 1 + 85 sin t - 85 , cos t + 104 sin 3t - 333 cos 3t. (9) Differentiating this, we obtain -t x' = e [(-c 1 - 3c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 52 cos 2t + 26 sin 2t - 3c 2) sin 3t + (3c 1 - c 2) cos 3t] 923 1 + 85 cos t + 85 sin t + 85 cos t to (9), we obtain $2 \ c \ 2 \ -85 \ -52 \ 6 \ 333 = 0$, so $29,819 \ c \ 2 = 490,620$. Applying the I.C. x'(O) = 0 to (10), we obtain $3c \ 1 \ -c \ 2 \ +; 5 \ +; 2 \ +11 = 0$, from which $c \ 1 = 54,854 \ 1,471,860$. Substituting these values of c 1 and c 2 into (9) gives the desired solution. 7. There are two problems here, one for 0 t < If, and the other for t > '/r. We first consider that for which o < t < If. Here the D.E. is of the form (5.58), with - w 32 1 (slug), a = 4, k = 20, and F(t) 40 cos 2t. The auxiliary equation of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0, with roots -2 % 4i. Thus the complementary function of the homogeneous D.E. corresponding to (1) is r 2 + 4r + 20 = 0. (1) is $x = e^2 t$ (c l sin4t + (1) c 2 cos 4t). We use undetermined coefficients to find a particular integral. We let $x = A \sin 2t + B \cos 2t$. P Substituting into (1) and simplifying, we find (16A - 8B) sin 2t + (8A + 16B) cos 2t = 40 cos 2t. P Substituting into (1) and simplifying, we find (16A - 8B) sin 2t + (8A + 16B) cos 2t - 2B sin 2t, $x^{"} = -4A \sin 2t - 4B \cos 2t$. P Substituting into (1) and simplifying, we find (16A - 8B) sin 2t + (8A + 16B) cos 2t = 40 cos 2t. P Substituting into (1) and simplifying, we find (16A - 8B) sin 2t + (8A + 16B) cos 2t = 40 cos 2t. which we find A = 1, B = 2. Thus we have the particular integral of (1), $x = \sin 2t + 2 \cos 2t$; and the G.S. of D.E. (1) is p Applications of Linear Equations 289 x = e - 2t (c l sin4t + c 2 cos4t) + sin2t + 2 cos2t. Differentiating this, we obtain $-2t x' = e [(-2c 1 - 4c 2) \sin 4t + (4c 1 - 2c 2) \cos 4t] + 2 \cos 2t - 4 \sin 2t$. Applying the stated I.C. to these, we have $c^2 + 2 = 0$ and $4c_1 - 2c_2 + 2 = 0$, from which we find $c_1 = -$, $c^2 = -2$. Thus we obtain the solution $x = e^{-2t}[(-) \sin 4t - 2\cos 4t] + \sin 2t + 2\cos 2t(2)$ valid for 0 < t < . Now we consider the problem for which t > . The D.E. 1S again of the form (5.58), where m = 1, a = 4, k = 20, but here F(t) = 0. Thus we have "x + 4x' + 20x - 0. (3) Assuming the displacement lS continuous at t =, the solution of (3) mt take the value given by (2) at t =. That lS, we must impose the I.C. -2 x() = -2e + 2 (4) on the solution of (3). Similarly, assuming the velocity lS continuous at t =, the derivative of the solution of (3) mt take the value given by the derivative of (2) at t =. The derivative of (2) is x' = -2t e [11 sin 4t - 2 cas 4t] + 2 cos 2t - 4 sin 2t. From this, we see that we must therefore Impose the I.C. X/() -2 = -2e + 2 (5) on the solution of (3). The auxiliary equation of (3) -2t 1 s x = e (k 1 sin 4t + k 2 cos 4t), and its der i vat i ve is -2t x' = e [(-2k 1 - 4k 2) sin 4t + k 2 cos 4t] + 2 cos 2t - 4 sin 2t. From this, we see that we must therefore Impose the I.C. X/() -2 = -2e + 2 (5) on the solution of (3) -2t 1 s x = e (k 1 sin 4t + k 2 cos 4t), and its der i vat i ve is -2t x' = e [(-2k 1 - 4k 2) sin 4t + k 2 cos 4t). t + (4k 1 - 2k 2) cos 4 t] . -2, - Applying the I.C. (4) to this G.S., we have k 2 e = -2 2 - 2e + 2, from which k 2 = 2(e - 1). Applying the I.C. (5) to the derivative of this G.S., we have (4k 1 - 2k2)e-2 = -2e-2 + 2, from which k 1 = ()(e2 - 1). Thus we obtain the solution x = (e 21r - 1)e -2t [() sin 4t + 2 cas 4t], valid for t > 'Ir. Section 5.5, Page 224 1 . 1 (a) Since the 12 lb. weight stretches the spring 6 in. = 2 ft., Hooke's Law, F = ks, gives 12 = k(), so k = 24 w 12 3) lb./ft. Then since m - g = 32 = 8 (slugs, a = 3, and F(t) = 2 cas wt, The D. E. is Applications of Linear Equations 291 3 " 8 x + 3x / + 24x = 2 cos wt . (1) The I.C.'s are x(O) = 0, x/CO = 0. The resonance frequency is given by formula (5.69) of the text. It lS 21 24 9 2() $2 = 2\{2 \text{ tr w 1 This is } 21r' \text{ where w l is the value of w for which the forcing function is in resonance with the system. w 1 = 4 <math>\{2 \text{ ln (1)}, \text{ obtaining XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ The auxiliary equation of the corresponding XU} + 3x' + 24x = 2 \text{ CDS } 4 \{2t \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2) \text{ or } "x + 8x/ + 64x 16 = 3 \cos 4 \{2t \cdot (2)
\text{ or } "x + 8x/ + 64x$ homogeneous D.E. is r 2 + 8r + 64 = 0, with roots r = -4% 4J3i. Thus the complementary function of (2) IS -4t n Xc = e (c l sin 4 y 3t + c 2 cos 4 y 3t). We use undetermined coefficients to find a particular integral. e let 292 Chapter 5 x - A sin 4/it + B cos 4/it. p Differentiating twice and substituting into (1), we find { 32A - 32/iB 32/iA + 32B = 0, 16 = -3, /i 1 and from these A = 18 ' B = 18. Thus we obtain the general solution of (2) in the form $-4t x = e(c 1 \sin 4 y 3t + c 2 \cos 4 y 3t) + \sin 4/it + 118 \cos 4/it + 2f \sin 4/it$. Applying the I.C. 's to these expressions for x and x', we obtain 10, 4J3c 1 - 4c 4 o. From c 2 + = + - - 18 2 9 - these, find -J3 - 1 Thus we obtain the we c 1 = 18 ' c 2 = IS. solution of (2) in the form Applications of Linear Equations of Linear Equations 293 -4t -/3 sin 4/3t - cos 4/3t x - e 18 / 2 s corresponding homogeneous equation is () $r^2 + 24 = 0$ with roots r = % 8i. Thus the complementary function is $Xc = c \ 1 \sin 8t + c \ 2 \cos 8t$. (4) w Undamped resonance occurs when the frequency - of the 21f impressed force equals the natural frequency 4 and If hence when w = 8. Letting w = 8 in (3), we have the D.E. $x'' + 24x = 2 \cos 8t$. (5) The complementary function is given by (4). We find a particular integral using undetermined coefficients. We modify the UC set {s in 8t, cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of cos 8t by multiplying each member by t, obtaining {t sin8t, t cos 8t} of co general solution of (5) in the form 294 Chapter 5 x - cl sin 8t + c 2 cos 8t + t sin 8 3 t. Differentiating this, we find, 8 8 8t. St x = c 1 cos t - c 2 Sln t + t cos"3" + Sln 3. The I.C.'s are the same as ln part (a). Applying them to these expressions for x and x', we readily find c 1 = 0, c 2 = O. Thus we obtain the solution of (5) in the form x . 8t = t SIN T. 2. The 20 lb. weight stretches the spring 6 inches; so by Hooke's Law, F = ks, we have 20 = k(;), from which w 205 k = 40 Ib./ft. Also, m = g = 32 = 8 (slugs). Thus the D.E. is 5 " ax' + 40x cos wt + 40x cos wroots r = -: a * I; a 2 - 64. For resonance the system must be 16 2 underdamped, so 25 a - 64 < o. Then the complementary function (2) is of the form (5.52), where 2b = -: a * I; a 2 - 64. For resonance the system must be 16 2 underdamped, so 25 a - 64 < o. Then the complementary function (2) is of the form (5.52), where 2b = -: a * I; a 2 - 64. For resonance the system must be 16 2 underdamped, so 25 a - 64 < o. Then the complementary function (2) is of the form (5.52), where 2b = -: a * I; a 2 - 64. a, A 2 = 64. The resonance frequency is given as 0.5 (cycles/sec.). From t he text, pages 219-220, this is wl/2, where w l = j A 2 - 2b 2 = 64 - a 2. Thus we obtain j 64 32 2 - a 25 2 = 0.5. Thus j 64 - a 2 = r, and from this a 2 = (64 - r 2). Thus recall ing that a > 0, we find a = 5 J 2 - ;.. Section 5.6, Page 232 1. Let i denote the current in amperes at time t. The total electromotive force is 40 V. Using the voltage drops: 1 . across the resistor: E R = Ri = 10i. 2. across the inductor: E L = Li' = 0 . 2 i' . Applying Kirchhoff's Law, we have the D.E. O.2i ' + 10i = 40. (1) 296 Chapter 5 Since the initial current IS 0, the I.C. IS i(O) = 0. The D.E. (1) IS a first order linear D.E. In standard form it is i' + 50 = 50t i' + 50e = 0 this, we obtain Multiplying (2) through by e 50t i' + 50e 50t i = 200e 50t or [e 50t i]' = 200e 0 at t - 0 to this, we find c = -4. Thus we obtain the solution i = 4(1 _ e - 50t). 3. Let i denote the current and let q denote the charge drop laws 1 and 3, we find the following voltage drops: 1. across the resistor: E R = Ri = 10i. 2. across the capacitor: EC = q = (10)4 i Applications of Linear Equations 297 Applying Kirchhoff's Law, we have the equation 10i + (10)4i = 100. (1) Since the charge is initially zero, we have the I.C. q(O) = O. The D.E. (1) IS a first order linear D.E. In standard form it is q' + 500q = 10, (2) and an I. F. IS e f 500 dt = 500t e Multiplying (2) through by this, we obtain 500t, 500 500t 10 500t e q + e q = e or [500t] + 10 500t e q - e. Integrating and simplifying, we find c - 1 = 50. Thus we obtain the solution 298 Chapter 5 q - 1 e- SOOt SO This is the charge. To find the current, we return to 10i + (10)4 i = 100, substitute the expression for q just found, and solve for 1. We find i = -SOOq + 10 and hence 1 - 10e- SOOt . 6. Let i denote the current in amperes at time t. The total 1 . f . 200 -lOOt U . h 1 d e ectromotive orce IS e . sing t e vo tage rop laws 1, 2, and 3, we find the following voltage drops: 1 . across the resistor: E R = Ri - 80i. - 2. across the inductor: E L = Li' = 0.2i' 3. the capacitor: E 1 10 6 q across = -q = S. c Applying Kirchhoff's Law, we have the D.E. 0.2i' + 80i + 10 6 q = 200e-100t. Since 1 = q', this reduces to 0.2q'' + 80q' 10 6 q 200e-100t. (1) + -S - Since the charge q IS initially zero, we have the first I.C. q(0) = 0. Since the current 1 IS initially zero and 1 = q', we have the second I.C. Applications of Linear Equations 299 q'(O) = O. The homogeneous D.E. (1) has the auxiliary equation 1 2 5 r + 80r + 200,000 = 0, and the roots of this are -200 979.8t. Thus the complementary function of D.E. (1) has the auxiliary equation 1 2 5 r + 80r + 200,000 = 0, and the roots of this are -200 979.8t. Thus the complementary function of D.E. (1) has the auxiliary equation 1 2 5 r + 80r + 200,000 = 0, and the roots of this are -200 979.8t. Thus the complementary function of D.E. (1) has the auxiliary equation 1 2 5 r + 80r + 200,000 = 0, and the roots of this are -200 979.8t. write Ae- 100t. q Differentiating twice and substituting into (1), we find A 1 = 970 0.0010. Thus the general solution of D.E. (1) is -200t. e - 100t q = e (c 1 Sln 979.St 10e- 100t + (979.8c 1 - 200c 2) cos 979.St] - 97 (2) Applying the I.C.'s to these expressions for q and q', we have 300 Chapter 5 1 10 c 2 + 970 - 0, 979.8c 1 - 200c 2 - 97 = 0, -10 from these we flnd that c 1 = (97)(979.8) -0.0010 and -1 c 2 = 970 -0.0010. Thus we obtain -200t. q = e (-0.0001 Sln 979.8t) + 0.0010 e - 100t. Since i = q', using formula (2) for q' and the values of c 1 and c 2 determined above, we find -200t. $1 = e(1.0311 \text{ Sln 979.8t} + 0.1031 \cos 979.8t)$ 0.1031e - 100t. Chapter 6 Section 6.1, Page 249 (I) 2. We assume y = L n = 0 Then $y' = L n = 1 \text{ n } c \times n$ (I) $y''
= Ln(n = 2 \text{ obtain } (I) n - 1 \text{ n } c \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ n } c \times n = 2 \text{ n } = 1 \text{ or } (I)$ (I) $n - 2 \text{ n } n(n - 1)c \times n \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or } n(n - 1)c \times n + 3 \text{ or$ 4 L cnx n = 0 n = 0 (I) 4 L cnx n = 0. n=0 (I) 4 L cnx n = 0. n= we have (I) (2c 2 - 4c O) + L [en + 2) (n + 1)C n + 2 + 4(2n - 1)C n [X n - O. n=1 301 302 Chapter 6 Equating to zero the coefficient of each power of x, we obtain 2c 2 - 4c O = 0, (1) (n + 2)(n + 1)C n + 2 + 4(2n - 1)C n [X n - O. n=1 301 302 Chapter 6 Equating to zero the coefficient of each power of x, we obtain 2c 2 - 4c O = 0, (1) (n + 2)(n + 1)C n + 2 + 4(2n - 1)C n [X n - O. n=1 301 302 Chapter 6 Equating to zero the coefficient of each power of x, we obtain 2c - 4c O = 0, (1) (n + 2)(n + 1)C n + 2 + 4(2n - 1)C n [X n - O. n=1 301 302 Chapter 6 Equating to zero the coefficient of each power of x. -, c 4 = -c 2 = -2c O' 2c 1 C s = -c 3 = . Substituting these values into the assumed solution, we have 2 Y = Co + c 1 x + 2c O x 2c 1 3 + x 3 + ... + c 1 x - 3 x + x 5 + ... + c 1 x - 3 x + x + ... + c 1 x - 3 x + x + ... + c 1 x - 3 x + x + ... + c 1 x - 3 x + x + ... + cobtain CD Ln(n n=2 n-2 - l)c x + n CD L n-1 nc x n n=l CD + 3x 2 L cnx n = 0 n=0 Series Solutions of Linear Differential Equations 303 or CD Ln(n n=2 n-2 - l)c x + n CD L n-1 nc x n n=l CD 3 L n+2 + c x = 0. n n=0 We rewrite the summations so that x has the same exponent ln each. We choose n - 2 as this common exponent, leave the first summation alone, and rewrite the second and third accordingly. We have CD CD Ln(n n=2 n-2 - 1)c n $1 \times CD + 3 L c n$ $4 \times n - 2 = 0$. n=4 The common range of these three summations is from 4 to CD. We write out the individual terms in each that do not belong to this common range. Thus we have (2c 2 + c 1) + (6c 3 + 2c 2) + (6c 3 + 2c 2) + (6c 3 + 2c 2))x CD n-2 + [n(n-1)c n + (n-1)c n + (n-1)cthis, we find $c_3 + c_0 c_0 c_1 c_4 = = - - 24 + 44 + 3c_1 1 (- c_0 c_1) 3 c_0 17 c_1 C = - - - - c_1 - - 520 - 5424 20 - 20120$. Substituting these values into the assumed solution, we have $C_1 2 y = C_0 + c_1 x - x C_1 x 3 - (C_0 + C_1) x 4 + 6424 (- c_0 17 c_1) 5 + 20 - 120 x + ... or C O (1 1 4 + 1 - x 5 + ... J y = -x 420 c_1 (x 1 2 1 3 1 4 17 5 + ... J + - - 520 - 5424 20 - 20120)$. x + x - x - 120 x 2 6 24 (I) 5. We assume y = L cnx n + n = 0 CD n - 1 Then y' = ncnx, n = 1 CD y'' = Ln(n = 2 O CD + 2x 2 L cnx n + L cnx n = 0 n = 0 n = 0 Series Solutions of Linear Differential Equations 305 or 00 00 00 Ln(n - 1)C n X n - 2 + Lncnx n = 0 n = 0 n = 0 n = 0 + L 2 c n x n + 2 + Lcnx n - 0. n=2 n=1 n=O n=0 We rewrite the first and third summations so that x has the exponent n in each. Thus we have 00 00 L (n + 2) (n + 1)c n + 2 x n + Lcnx n - 0. n=2 n=0 The common range of these four summations lS from 2 to 00. Ve write out the individual terms in each. that do not belong to this range. Thus we have: $(co + 2c 2) + (2c 1 + 6c 3) \times 00 + [en + 2)(n + 1)c + 2c 2 = 0, n > 2. n + (n + 1)c + n + (1) + 2c 2 = 0, n > 2. n + (2) From (1), we find c 2 find - co = 2' c 3 = -c 3 3 - c 1 3.$ L, cnx n . n=O CD Then y' = L, nc n x n - 1, n=l CD y'' = L, n(n n=2 n-2 l)c x . Substituting into the D.E., we n obtain CD CD L, n(n - 1)C n X n - 2 - x 3 L, nc n x n - 1 n=2 n=l CD n-l 2 ncnx n=l CD - 6X 2 L, cnx n = ° n=O Series Solutions of Linear Differential Equations 307 or CD CD CD Ln(n - 1)C n X n - 2 - x 3 L, nc n x n - 1 n=2 n=l n=l cD n-l 2 ncnx n=l CD - 6X 2 L, cnx n = ° n=O Series Solutions of Linear Differential Equations 307 or CD CD CD Ln(n - 1)C n X n - 2 - x 3 L, nc n x n - 1 n=2 n=l cD n-l 2 ncnx n=l cD n-l 2 ncnx n=l cD - 6X 2 L, cnx n = ° n=O Series Solutions of Linear Differential Equations 307 or CD CD CD Ln(n - 1)C n X n - 2 - x 3 L, nc n x n - 1 n=2 n=l cD n-l 2 ncnx n=l cD n-l CD 6Lc n x n + 2 = 0. n=0 Ve write the first and third summations so that x has the exponent n + 2 in each, as it already has in the second and fourth summations. Ve have CD CD L (n + 3)c 3x n + 2 6 L c n x n + 2 0 - - - n + n = -2 n = 0 The common range of these four summations lS from 1 to CD. We write out the individual terms in each that do not belong to this common range. We have $2(2c_2 - 2c_1) + (6c_3 - 4c_2)x + (12c_4 - 6c_3 - 4c_2)x + (12c_4 - 6c_3)x + (12c_4 - 6c$ 3 - 6c O = 0, (n + 4)(n + 3)c 4 - 2(n + 3)c 3 n + n + - (n + 6)c n = 0, n = 0assumed solution, we have (Co + 5 + 2 2c 1 3 (CO Cl) 4 c 1 x + . x + + 3 x 29C 1) x S + ... 60 y = Co + c 1 x + or C O (l + 4 x 5 5 + ...) y = . - + 2 C l (X 2 2x 3 4 29x S + ...) x + + x + - + - + 60 3 3 CD 10. We assume $y = L cnx n \cdot n = 0$ CD Then y' = Lnc n x n - l, n = 0 CD
Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' = Lnc n x n - l, n = 0 CD Then y' =Solutions of Linear Differential Equations 309 ro ro $Ln(n - 1)C n \times n - 2 - x 2 Lnc n \times n - 1 + L cnx n = 0$ n=l ro ro $Ln(n - 1)C n \times n - 1 + L cnx n = 0$ n=l ro r (n - l)C n l x n n=O n=2 ro ro Lncnx n + Lcnx n = o. n=l n=O The common range of these four summations lS from 2 to roe We write out the individual terms in each that do not belong to this common range. Thus we have (2c 2 + cO) + (6c 3 - c 1 + c 1)x ro + [en + 2)(n + 1)c 2 - (n - 1)c 1 LJ n + n = 2n - (n - 1)c Jx = O. n Equating to zero the coefficients of each power of x, we obtain 310 Chapter 6 2c 2 + Co = 0, 6c 3 = 0, (n + 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n + 2)c n + 2 - (n - 1)c n = 0, n 2. (1) (2) From (1), we have c 2 obtain Co = - and c 3 - o. From (2), we (n - 1)(n - 2)c n = 0, n 3. 6 - C 3 + c 4 10 Co c 1 = -240 + 120. Substituting these values into the assumed solution, we have Co 2 (CO C 1) 4 y = Co + c 1 x x + -24 + 12 x Co x 5 + (Co + 2) x 6 + ...) y = -x 24 x 20 x 240 x 2 c 1 (x 1 4 1 6 + ...) + + 12 x + 120 x CD 11. We assume y = L cnx n . n=O CD n-1 Then y' = ncnx , n=I CD

= Ln(n = 2 obtain n-2 l)c x. Substituting into the D.E., we n Series Solutions of Linear Differential Equations 311 CD CD 2 n-2 n-2 x n(n - 1)c n x + n(n - 1)c n fourth summations so that x has the exponent n in each. We have: CD CD Ln(n - 1)C n x n + L (n + 2) (n + 1)C n + 2 x n n = 2 n = 0 CD CD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD + Lncnx n + L c n = 0 cD)x CD + L {(n n=2 n + c n _ 1 }x + 2)(n + l)c 2 + [n(n - 1) + n]c n + n = 0. 312 Chapter 6 Equating to zero the coefficients of each power of x, we obtain 2c 2 = 0, co + c 1 + 6c 3 = 0, (n + 2)(n + l)c 2 + [n(n - 1) + n]c n + n + c 1 = 0, n 2 n - (1)(2) From (1), we find c 2 = 0, co + c 1 + 6c 3 = 0, (n + 2)(n + l)c 2 + [n(n - 1) + n]c n + n + c 1 = 0, n 2 n - (1)(2) From (1), we find c 2 = 0, co + c 1 + 6c 3 = 0, (n + 2)(n + l)c 2 + [n(n - 1) + n]c n + n + c 1 = 0, n 2 n - (1)(2) From (1), we find c 2 = 0, co + c 1 + 6c 3 = 0, (n + 2)(n + 1)c 2 + [n(n - 1) + n]c n + n + c 1 = 0, n 2 n - (1)(2) From (1), we find c 2 = 0, co + c 1 + 6c 3 = 0, (n + 2)(n + 1)c 2 + [n(n - 1) + n]c n + n + c 1 = 0. > 2. Using this, we find c 4 = -4c 2 + c 1 c 1 = -. 12 12' 9c 3 + c 2 (io) (cO + c 1) · Substituting these c = - = 5 20 values into the assumed solution, we have y = C O (l - x 3 3x 5 "6 + 40 + ...) + c l (x 3 x 6 x 4 3x 5 12 + 40 + ...) + c l (x 3 x y' = n cnx, n = l CD y'' = Ln(n = 2 obtain n - 2 l)c x. Substituting into the D.E., we n Series Solutions of Linear Differential Equations 313 00 00 3 n - 2 n - 2 x n(n - l)c n x - n(n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 00 2 n - 1 n + x cnx + x cnx = 0 n = 1 n = 0 or 00 00 n + 1 n - 2 n (n - l)c n x n = 2 n = 2 00 n + 1 + n cnx + n = 1 00 Lc n x n + 1 = 0. second summation so that x has the exponent n + 1 in it (since this lS the exponent of x 1n all the other summations). Thus we have: 00 n+1 + ncnx + n=1 00 Lc $n \times n + 1 - 0$. n=0 The common range of these four summations lS from 2 to 00. We write out the individual terms in each that do not belong to this range. Thus we have: 314 Chapter $62 - 2c 2 + (cO - 6c 3)x + (2c 1 - 12c 4)x CD + L {-en + 3}(n + 2)C n + 3 n = 2 + [n(n - 1) + n + 1]c 3x n = 0, n 2. (1) (2) From (1), we find$ c 2 = 0, c 3 find Co c 1 = 6' c 4 - 6. From (2), we c n+3 2 (n + 1) c n (n + 2)(n + 3)' n > 2. Using this, we find c 5 5 c 2 = 20 = 0; c 6 - 10c 3 30 = c 3 3 Co = --18' $17c 1 \times 252 + ... + + + or y = C O (1 + x3 + x6 + 20)$...) + c (X + x 4 + 17x 7 + ...). 6 18 1 6 252 Series Solutions of Linear Differential Equations 315 CD 15. \Ie assume y = L cnx n · n = 0 CD Then y' = L nc n x n - 1, n = 1 CD y'' = Ln(n n = 2 obtain n-2 l)c x. Substituting into the D.E., we n CD CD n-2 n-1 - l)c n x - x ncnx n = 1 CD Lcnx n = 0 n=0 Ln(n n=2 or CD CD CD Ln(n - 1)C n x n - 2 + L (-ncn)x n + 1 con x n - 2 + L (-ncn)x n + 1 con x n - 2 + L (-ncn)x n + 1 con x n - 2 + L (-ncn)x n + 1 con x n - 2 con x n - 1 con x n - 2 con x n -L (-cn)x n = 0. n=2 n=1 n=0 We rewrite the first summation so that x has an exponent n. Thus we have: CD CD CD LCn+1)(n + 2)c n + 2 x n + L(-ncn)x n - 0. n=0 n=1 n=0 The common range of these three summations is from 1 to CD. We write out the terms from the 1st and 3rd that do not belong in this range. Thus we have: (2c 2 - cO) CD + [en + 1)(n + 2)c 2 + (-n - 1)c]xn = 0. n + n = 1 Equating the coefficients of each power of x to zero we obtain 316 Chapter 6 2c 2 - Co = 0, (1) (n + 1)(n + 2)c 2 + (-n - 1)c = 0, n 1. (2) n + n From (1), we find c 2 1 = 2 cO. From (2), we find 1 c n + 2 = n + 2 c n ' n > 1. 111 Using this, we find c 3 = 3 c 1; c 4 = 4 c 2 = 8 cO; 1 1 c 5 = 5 c 3 = 15 c 1; c 6 the D.E. is 1 = 6 c 4 = 1 48 co. Thus the G.S. of y = C O(l 1 2 + x 2 1 4 + ... J + ... Jl, n=l m y'' = L, n(n = 2 obtain n-2 l)c x. Substituting into the D.E., we n m m 2 n-2 n-2 x n(n - l)c n x n=2 m m + L, ncn x n + L, 2C n X n + 1 = 0. n=l n=0 or m m L, n(n = 2 m m n-1)c n x n=2 m m + L, n(n - l)c n x n=2 m m +Thus we have: 318 Chapter 6 CD CD Ln(n - 1)C n x n + L (n + 2) (n + 1)C n + 2 x n n = 2 n = 0 CD CD + Lncnx n + L2Cn 1Xn = 0. n = 1 n = 1 The common range of these four summations IS from 2 to m. We write out the individual terms in each that do not belong to this range. Thus we have: CD 2c 2 + (6c 3 +
c 1 + 2c 0)x + L ([n(n - 1) + n] C n n = 2 n + 1)C n x = 0. n = 1 n = 1 The common range of these four summations IS from 2 to m. We write out the individual terms in each that do not belong to this range. Thus we have: CD 2c 2 + (6c 3 + c 1 + 2c 0)x + L ([n(n - 1) + n] C n n = 2 n + 1)C n x = 0. n = 1 n = 1 The common range of these four summations IS from 2 to m. We write out the individual terms in each that do not belong to this range. $(n + 2)(n + 1)c n + 2 + 2c n - 1 \\ x = 0.$ Equating to zero the coefficients of each power of x, we obtain 2c 2 = 0, 2c 0 + c 1 + 6c 3 = 0, $(1) [n(n - 1) + n]c + (n + 2)(n + 1)c 2 + n + 2c n - 1 \\ x = 0, c 3 = find 2c 0 + c 1 \\ x = 0, c 3 =$ 12 c 1 = -6' c - 5 - 9c 3 + 2c 2 20 = 3(2c 0 + c 1) x 6 4 c 1 x 6 5 3(2c 0 + c 1) x 40 + ... + or C 0 (1 3 3x 5 + ...) x Y = - + 20 3 C 1 (X 3 4 + 3: 0 5 + ...) x Y = - + 20 3 C 1 (X 3 4 + ...) x Y = - + 20 3 C 1 (X 3 4 + ...) x Y = - + 20 3 C 1 (X 3 4 + ...) x Y = - + 20 3 C 1 (X 3 4 + ...) x Y = - + 20 3 C 1 (X 3 4 + .this, obtaining Co = 2. Differentiating we find y' Co ($X2 \ 3x \ 4 \dots$) = + + 4 c 1 (1 2 2x 3 3x 4 ...) x + - - + + 2 3 8 Applying the I.C. y'(O) = 3 to this, we have c 1 = 3. Thus we have c 1 = 3. L, cnxn. n=O CD n-l Then y' = ncnx, n=l CD y'' = Ln(n n=2 obtain n-2 l)c x. Substituting into the D.E., we n CD CD 2 n-2 n-2 x n(n - l)c n x n - l + 2 Lcnx n = 0 n=l n=O or CD CD Ln(n - l)c n x n - l + 2 Lcnx n = 0 n=l n=O or CD Ln(n - l)c n x that x has the exponent n, as ln the other three summations. We have: CD CD Ln(n - l) cn x n - L (n + 2) (n + 1) Cn + 2 x n n=2 n=0 CD CD + 4Lnc n x n + 2Lc n x n = 0. n=1 n=0 The common range of these summations IS from 2 to CD. Ve write out the individual terms that do not belong to this common range. We have: Series Solutions of Linear Differential Equations 321(-2c2+2cO) + (-6c3+6c1)x CD + LHn(n - 1) + 4n + 2]c n = 2 (n + 1)(n + 2)c = 0, n > 2. n+2 (1) (2) From (1), we have c 2 = 0, n > 2. n+2 (1) (2) From (Co and c 3 = c 1. From (2), we obtain c 2 = c, n > 2. From this we observe that n + n C 2n = Co and c 2n + 1 = c 1 for all n 1. Thus we obtain the G.S. C O (l + 2 + x 4 + ...) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating
we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c l (x + 3 5 + ...) Y = x + x + x CD CD L 2n L 2n + l = Co x + c 1 x \cdot n = O n = O We now apply the I.C. y(O) = 1 to this, obtaining Co = 1. Then differentiating we have 324 y' = (2x + 1) + c (2 4x + ...) + c 1 (1 + 3x + 5x + ...). Applying the I.C. y'(O) = -1 to this, we find c 1 = -1. Thus we obtain the desired solution CD Y=1-x+x2 x 3 + x 4 x 5 + "=L(-1)n X n. n=O CD Then y' = L nC n (x - 1)n-l, n=l CD y" = Ln(n n=2 n-2 l)c (x - 1). Substituting into the D.E., n we obtain CD CD 2 n-2 n-1 x n(n - l)c n (x - 1) + 3x ncn(x - 1) n = 2 n = l CD LCn(X - l)n = 0 Since the summations involve powers of x - 1, we must express the respective "coefficients" x - 1 n = 2 CD n - 1 + [3(x - 1) + 3] ncn(x - 1) n = l CD LCn(X - l)n = 0. n = 0. Series Solutions of Linear Differential Equations 323 or CD CD Ln(n - l)c n (x - l)n + L 2 n(n - l)c(x - l)n + L2(n + l)ncn + l(x - l)n = 0 n=0 n=0 The common range of these SIX summations IS from 2 to CD. Ve write out the individual terms in each that do not belong to this range. Thus we have: (-co + 3c + 2c 2) + (2c + 1)n + L2(n +10c 2 + 6c 3) (x - 1) CD + LHn(n - 1) + 3n - 1]c n = 2 + [2(n + 1)n + 3(n + 1)]c n + 1 n + (n + 2)(n + 1)c n + 2 } (x - 1) = 0. 324 Chapter 6 Equating to zero the coefficients of each power of x, we obtain -c + 3c 1 + 2c 2 = 0, 2c 1 + 10c 2 + 6c 3 - 0, (1) 0 - 2(2n + 3)(n + 1)c 1 + (n + 2)(n +From (1), we find c 2 1 = c 3 = 2 3 - 5c + 13c 1 0 From (2), find = we 6 c n + 2 = (n 2 + 2n - 1)c + (2n + 3)(n + 1)c 1 n n + (n + 1)(n + 2) n > 2. 7c + 21c 3 Using this, we find c 4 = -2 12 = 7(2c O - 5c 1) 12 Substituting these values into the assumed solution, we have 2 c 1 (x - 1) + (cO - 3c 1)(x - 1) + 6 4 7(2c O - 5c 1) 12 Substituting these values into the assumed solution, we have 2 c 1 (x - 1) + (cO - 3c 1)(x - 1) + 6 4 7(2c O - 5c 1) 12 Substituting these values into the assumed solution, we have 2 c 1 (x - 1) + (cO - 3c 1)(x - 1) + 6 4 7(2c O - 5c 1) 12 Substituting these values into the assumed solution. 1) (x - 1) + 12 + ... or C O [l 2 5 (x - 1) 3 7 (x - 1) 4 + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 (x - 1) 2 + 13 (x - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] y = (x - 1) + + 2 6 6 + c 1 [(X - 1) 3 2 6 35 (x - 1) 4 + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Differential Equations 325 23. Since the initial value is 1, we assume that CD CD Y = ::L, cn(X - 1) A + ...] 12 Series Solutions of Linear Diffe order to obtain n=2 multiples of (x - 1) we rewrite the original D.E. as follows: (x - 1)yH + 1yH + y' + 2y = 0 Substituting into this D.E. we obtain: CD (x - 1) n=2 CD CD + Lncn(x - 1)n-2 n=2 CD n-2 + (n - 1)(n)c n (x - 1) n=2 CD CD + Lncn(x - 1)n-2 n=2 CD CD + ln-l + L 2 C n (X - l)n = O. n = l n = O 326 Chapter 6 We rewrite the first three summations so that (x - 1) has an exponent of n in each one. Thus we have: CD L (n)(n + l)c n + 1 (x - l)n = O. n = O The common range of these summations lS from 1 to CD.We write out the terms that are not in this range separately. Thus we have: CD (2c 2 + c 1 + 2c O) + L [en + 1) (n + 2)C n + 2 n = l n + (n + 1)(n + 2)C n + (n -. From (2), we find 2 2c + (n + 1) c 1 n n + c n + 2 = (n + 1)(n + 2) n > 1. Series Solutions of Linear Differential Equations 327 Using this, c 3 = C 1 + 2c 2 3 2c O c - 3' 4 - C 2 - - 6 3 c 3 4 = C 3 - - 10 4 c 4 5 Co 5 C 1 15. Thus the G.S. of Co c 1 3" + 12 ' c 5 = the D.E. is Co [1 - 2 2 3 1 1)4 y = (x - 1) + - (x - 3 3 1 5 + ...] c 1 [(x - 1) 1 2 + - (x - 1) 2 + - (x - 1) + - (x 1) + 2 (x - 1) 5 1 415 ...] . + 12 (x - 1) - . (x - 1) + 15 Applying the I.C. y(1) = 2 to this, we find Co = 2. Differentiating, we find c1 = 4. Thus we obtain the solution [2 2 3 1 4 y = 2 1 - (x - 1) + 3 (x - 1) + 3 (x - 1) + (x - 1)5 + ...] + 4[(X - 1) - (x - 1)2 + (x - 1)2 + (x - 1)4 - (x - 1)2 + (x - 1)4 - (x (x - 1)5 + ...] 12 15 or 243 Y = 2 + 4(x - 1) - 4(x - 1) + 3 (x 3. We consider x = 0, and the functions formed by the product x + 2xP 1 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x -
3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and 2x x P 2 (x) = x - 3 and x - 3 (x) = x = 3, so x = 3 is a regular singular point. 2. We write the D.E. In the normalized form (6.3) of the text. This IS: "x - 2 Y + y' + x(x + 1) 4 2 x (x + 1) y = 0. x - 2 4 Here P (x) - and P 2 (x) = 21 - x(x + 1) x (x + 1) 4 2 x (x + 1) x (x + 1) this that the singular points are x = 0 and x = -1. We see from We consider x = 0 and x = -1. We see from We consider x = 0 and form the functions defined by the products. Solutions of Linear Differential Equations 329 xP1(x) = x - 2x + 1 and X2P2(X) = 4x + 1. Both of the product functions thus defined are analytic at x = 0; so x = 0 is a regular singular point. We now consider x = -1 and form the functions thus defined are analytic at x = 0; so x = 0 is a regular singular point. We now consider x = -1 and form the functions defined by the products (x + 1)P1(x) = x - 2x + 1 and X2P2(X) = 4x + 1. Both of these product functions thus defined are analytic at x = -1; so x = -1 is a regular point. 4. We write the D.E. In the normalized form (6.3) of the text. This IS: "1, 1 o. y + x(x + 3) this, we see that the singular points are x = 0, x = -3 and x = 2. We consider x = 0, and form the product functions 1 xP1 (x) = (x + 3)(x - 2) and 2 1 x P2 (x) = x(x + 3) P1 (x) = x(x + 3) P2(x) = x + 3 3 . x 330 Chapter 6 Both are defined at x = -3 so x + 3 so -3 IS a regular point. Consider x = 2, and form the product functions: 1 (x - 2)P1(x) = x(x + 3) 2 and (x - 2)2p 2(x) = x - 2 . x (x + 3) Both are defined at x = 2, po in t. so x = 2 IS a regular singular CD n+r 5. We assume y = cnx where Co * o. n=O Then CD Y'=L(n+n=O r)c xn+r-1 and n CD " () () n+r-2 y = n+r n+r - 1 cnx. n=O into the D.E., we obtain Substituting these CD 2 n+r-2 2x (n + r)(n + r - l)c + (n + r)c - c Jx n n n = 0 CD n+r-l + x (n + r cnx n = 0 CD + (x 2 - 1) Lcnx n + r = 0. n = 0 CD + (x 2 - 1) Lcnx n + r = 0. n = 0 CD n+r-l + x (n + r cnx n = 0 CD + (x 2 - 1) Lcnx n + r = 0. n = 0 CD $-c_1 x + (r + 1) CD + L [(2n + 2r + 1)(n + r - 1)c_n n = 2 + c - 2]xn + r = 0$ or $2r_2 - r - 1 = 0$ with roots r = 1 and $r_2 = -1/2$. Since the difference between these roots is not zero or a positive integer, Conclusion 1 of Theorem 6.3 tells us that the D.E. has two linearly independent solutions of the assumed form, one corresponding to each of the roots r 1 and r 2. Equating the coefficients of the higher powers of x in (1) to zero we obtain [2(r + 1)r + r]c 1 = 0 (2) and (2n + 2r + 1)(n + r - 1)c + c 2 = 0, n > 2. (3) n = -32 Chapter 6 Letting r = r 1 = 1 ln (2) we obtain SC 1 = 0 so c 1 = 0. Lettin r = r 1 = 1 In (3) we obtain (2n + 3)(n)c + c - 0, n > 2 n - 2 - n or c n - 2 c = (2n + 3)n' n > 2. n From this c 2 - 1 - 1 = 0, -1 1 = c0' c = 27 c 1 c 4 = 44 c = 616 Co 14 3 2 - 1 0. Using these values, we obtain the C s = 6S c 3 = solution corresponding to the larger root r = 1 : 1 Y1(x) = C O X(l 1 2 1 4 ...). - 14 x + 616 x + Letting r = r 2 = -1/2 In (2), we obtain -c = 0 so c = 1 = 0.1 Letting r = r = 2 = -1/2 In (3) we obtain (n - 3/2) (2n)c + C = 0 n n-2 or C = n C n-2 (2n 3)n' n > 2. F h o 1 rom t IS, c = -2 cO' c =D.E. lS Y = C 1 Y l(x) + C 2 y 2 (x). Series Solutions of Linear Differential Equations 333 CD n+r 7. We assume y = cnx where Co * O. n=O Then CD y' = L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x n = O CD - x L (n = O n+r - 1) c x - 1 c - 1 + c x + n n = 0 CD L n=0 n+r c x n = 0. Simplifying as 1n the solutions of Section 6.1, we write this as CD L[(n n=0 8] n+r + r)(n + r - 1) - (n + r) + 9 cn x CD n+r + cn_2 x = 0 n=2 334 Chapter 6 or [r (r - 1) - r +] cOx r + [(r + 1) r - (r + 1) CD 8] r+1 { [+ 9 c 1 x + (n + r)(n + r - 1) n=2 - (n + r) +] cn + cn_2 x = 0 n=2 334 Chapter 6 or [r (r - 1) - r +] cOx r + [(r + 1) r - (r + 1) CD 8] r+1 { [+ 9 c 1 x + (n + r)(n + r - 1) n=2 - (n + r) +] cn + cn_2 x = 0 n=2 334 Chapter 6 or [r (r - 1) - r +] cOx r + [(r + 1) r - (r + 1) CD 8] r+1 { [+ 9 c 1 x + (n + r)(n + r - 1) n=2 - (n + r) +] cn + cn_2 x = 0 n=2 zero the coefficient of the lowest power of x we have the indicial equation r(r - 1) - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - r + () = 0 with roots r - 1 - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - r + () = 0 or r - 2 - 2r + () = 0 with roots r - 1 - 2r + () = 0 with roots r - 1 - 2r + () = 0 with roots r - 2 each of the roots r 1 and r 2. Equating to zero the coefficients of the higher powers of x in (1), we have [(r + 1)r - (r + 1) + ()]c = 0 (2) and [(n + r)(n + r - 1) - (n + r) +]c n + c 2 = 0, n > 2. n-(3) Letting 4 (2) obtain () c 1 = 0 so o. r = r 1 = ln we c = 3 1 Letting 4 (3) we obtain n(n + 2)c + 0, r = r 1 = ln we c = 3 3 n n - 2 or 3c n - 2 c = n(3n + 2)' n > 2. 2. n Series Solutions of Linear Differential Equations 335. 3c O c 1 From thlS c 2 = - JUf c 3 = - 11 3c 2 = 0, C 4 = -
56 9c O = 896. Using these values, we obtain the solution corresponding 4 to the larger root r 1 = 3 : Y1(x) = 4/3 (1 cOx 3x 2 - + 16 9x 4 - -896 ...). Letting r 2 (2), c 1 O. O. = r 2 = - In we obtain 3 - so c 1 - 3 - , - Letting 2 (3) we obtain $n(n - 2)c + 0r = r^2 = lnc = 33 n n^2$ or $c = n^3c n^2 2 n(3n - 2)' n > 2$. 3c O c 1 From thlS, $c^2 = -8 - 3c^2 - 40 9c O 320$. Using these values, we obtain the solution corresponding 2 to the smaller root $r^2 = 3 : Y^2(X) = C O x^2/3(1 3x^2 - 48 9x^4 ...) 320$. The G.S. of the D.E. IS $Y = C 1 y^1(x) + C^2 y^2(x)$. CD 9. We assume Y L n+r where Co * O. Then = c x n n=O CD y' L(n + r)c x n+r-1 and = n n=O CD CD n+r L n+r+2 + r)c x + c x n n n=O CD CD n+r L n+r+2 + r)c x + c x n n n=O CD 1 n+r 9 cnx = o. n=O + L(n n=O Simplifying, as In the solutions of Section x n + r n=O CD CD n+r L n+r+2 + r)c x + c x n n n=O CD 1 n+r 9 cnx = o. n=O + L(n n=O Simplifying, as In the solutions of Section x n+r n=O CD CD n+r L n+r+2 + r)c x + c x n n n=O CD CD n+r L n+r+2 + r)c x + c x n n n=O CD CD n+r L n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x n n=O CD n+r+2 + r)c x + c x + c x n n=O CD n+ 6.1, we write this as CD L[(n = 0 1] n+r + r)(n + r - 1) + (n + r) -] C n n=2 + c n 2 } x n + r = 0. (1) Equating to zero the coefficient of the lowest power of x we have the indicial equation $r^2 - = 0$ with roots 1 r 1 = 3 ' 1 r 2 = - 3. Since the difference between these roots is not zero or a positive integer, Conclusion 1 of Theorem 6.3 tells us that the D.E. has two linearly Series Solutions of the assumed form, one corresponding to each of the roots r 1 and r 2. Equating to zero the coefficients of the higher powers of x ln (1) we have [(r + 1) r + (r + 1) -] C 1 = 0 (2) and [(n + r) (n + r - 1) + (n + r) -] C n + c 2 = 0, n > 2. n - (3) Letting 1 (3) obtain r = r 1 = ln we 3 n(n +)Cn + c = 0 n-2 or c = n 3 c n 2 n(3n + 2)' n > 2. 3 c O c 1 From thlS, c 2 = -, c 3 = -11 = 0, c 4 3 c 2 = --56 9 c O = 896 0 = 8 c ODD = 0 for n 1. Using these values, we obtain the 1 solution corresponding to the larger root r 1 = 3 : Y1(x) 1/3 [3x2 9x 4 = cOx 1 - + 896 - ...]. Letting r = r 2 = ln (2), we obtain n(n - ;)C n + c n _ 2 = 0 338 Chapter 6 or c = n 3c n _ 2 n(3n - 2)' n > 2... 3c O c 1 3c 2 From thlS, c 2 = - S-' $c 3 = -, c 4 = -40 \ 9c \ O = 320' \ c \ ODD = 0 \ for \ n \ 1.$ Using these values, we obtain the 1 solution corresponding to the smaller root $r 2 = -3 : Y2(X) = C \ Ox - 1/3 \ [1_32 + ; -...]$. The G.S. of the D.E. is $y = C \ 1 \ Y \ 1(x) + C \ 2 \ y \ 2(x)$. ro $n+r \ 12$. We assume Y = cnx where Co * O. n=O Then ro $)n+r \ 1y' = cn(n + r \ x \ and \ n=O \ ro \ n' + r) \ n + r$ -1 x n=0 into the D.E., we obtain Substituting these Series Solutions of Linear Differential Equations 339 ro n+r + c n n + r x n=0 ro 2 L cnx n + r = 0. n=0 Simplifying, as ln the solutions of Section 6.1, we write this as ro L [2(n + r) (n + r - 1) + 5(n + r) - 2]c n x n + r n=0 ro n+r + c n n + r x n=0 ro 1 + r + c n n + r x n=0 ro 2 L cnx n + r = 0. $2 \text{ cn } 1x = 0 \text{ n} = 1 \text{ or } [2r(r-1) + 5r - 2]c \text{ O } xr \text{ ro } + L{[2(n + r)(n + r - 1) + 5(n + r) - 2]c \text{ n} n} + r + 2c \text{ n} - 1}x = 0.$ (1) Equating to zero the coefficient of the lowest power of have the indicial equation 2 0, that x, we 2r + 3r - 2 = 1S(2r - 1)(r + 2) = 0, with roots r + 1 + 2c n - 1 and r = 1 + r + 2c n - 1. integer, Conclusion 1 of Theorem 6.3 tells us that the D.E. has two linearly independent solutions of 340 Chapter 6 the assumed form, one corresponding to each of the roots r 1 and r 2. Equating to zero the coefficients of the higher powers of x in (1), we have [2(n + r)(n + r - 1) + 5(n + r) - 2]c n + 2c 1 = 0, n > 1. n - (2) Letting 1 (2) we obtain 2 5n)c r = r 1 = In (2n + 2n + 2c = 0 or n-1 2c n-1 c = n(2n + 5)' n > 1. n From this c 1 2c O 2c 1 c 1 2c O = --, c -- 2(9) = -- = 63 ' 7 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain the c = -- we 3 33 - 2079. Letting r = r = -2 In (2), we obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain (2n 2 - 5n)c 2 - 9 2c 2 4c O Using these values, obtain n + 2c = 0 so c = 1 = 0 or n - 1 $c = n 2c n_1 (2n - 5)' n > 1$. From this, c = 2c 0 = (-3) 2c 0 3' 2c 2 = -3 3 4c 0 - 9. Using these values, we obtain the solution corresponding to the smaller root r = -2: Series Solutions of Linear Differential Equations 341 - 2 [2 Y2(x) = cOx 1 + 3 x 2 2 + - x 3 4 3 - - x 9 + ...]. The G.S. of the D.E. is y = C 1 Y 1(x) + C 2 y 2(x). ro n+r + 14. We assume Y = cnx where Co f O. n=O Then ro y' = L(n + n=O r)cx n + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn
+ r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''()() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r - 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + r + 1 and n ro''() n+r-2y = n + rn + rn + r + 1 and n ro''() n+r-2y = n + rn + rn + r + 1 and n ro''()n=O n+r+r) c x n ro 10 L cnx n + r = 0. n=O Simplifying, as In the solutions of Section 6.1, we write this as 342 Chapter 6 CD n+r [2(n + r - 1) + (n + r - 1)] c x n + r = 0. n=0 (1 + r - 1) (n + r - 1) + (n + r - 1) (n + r - 1) + (n + r - 1) + (n + r - 1) (n + r - 1) (n + r - 1) + (n + r - 1) + (n + r - 1) (n + r Equating to zero the coefficient of the lowest power of x, we have the indicial equation 2r - r - 10 = 0 or (2r - 5)(r + 2) = 0, with roots r = -2. Since the difference between these roots is not zero or a positive integer, Conclusion 1 of Theorem 6.3 tells us that the D.E. has two linearly independent solutions of the assumed form, one corresponding to each of the roots r 1 and r 2. Equating to zero the coefficients of the higher powers of x in (1), we have [2(n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0, n > 1. n-1 (2) Letting r = r 1 = 5/2 in (2) we obtain n(2n + 9)c n + (n + r) - 10]c n 2 1)c = 0. 9) n) 1. 25c O 49c 1 From this c 1 = - 44 ' c 2 = - 104 1225c O = 4576 · Using these values, we obtain the solution corresponding to the larger root r 1 = 5/2: 5/2 [25x 1225x 2] y = cOx 1 - 44 + 4576 - ... · Letting r = r = -2 in (2), we obtain n(2n - 9) c 2 n 2 + (n - 3) c n - 1 = 0 or c = n 2 (n - 3) c n - 1 n(2n - 9) n 1. From this, 4c O c 1 2c O OC 2 o. and c 1 - c 2 = 35 + c = -7, 10 3 9, hence c = o for all n) 3. The solution corresponding to n the smaller root r 2 = -2 is the finite sum -2 (4x 2X) y(x) = cOx 1 + 1 + 3: The G.S. of the D.E. is y = C 1 Y 1(x) + C 2 Y 2(x). 15. We assume Y ro L n+r = c x, n n=O where Co f o. Then ro y' = L (n + n=O) n+r-1 r c x, n 344 Chapter 6 ro y'' = L Cn n=O into the D.E., we obtain n+r-2 + r (n + r - 1) c x . n Substituting these ro ro n+r-1 (n + r) (n + r - 1) c x + 2(n + r - 1) c x + 2(n + r - 1) c x + 2(n + r - 1) c x - 1 (n + r - 1) c x - 1 (n + r - 1) c x + 2(n + r - 1) c x - 1 (n + r - 1) c l) r = 0, Series Solutions of Linear Differential Equations 345 the assumed form corresponding to the larger root r = 1 = 0, (2) [en + r](n + r - 1) + 2(n + r)]c + c = 0, n - n > 2. (3) Letting r = r = 1 = 0 in (2), we obtain 2c = 1 = 0; so c = 1 = 0. Letting r = r = 1 = 0 in (2), we obtain 2c = 1 = 0; so c = 1 = 0. Letting r = r = 1 = 0 in (2), we obtain 2c = 1 = 0; so c = 1 = 0. Letting r = r = 1 = 0 in (2), we obtain 2c = 1 = 0; so c = 1 = 0. Letting r = r = 1 = 0 in (2), we obtain 2c = 1 = 0; so c = 1 = 0. Letting r = r = 1 = 0. (3), we obtain n(n + 1)c + c = 0 or n n - 2 c n - 2 c = n(n + 1), n > 2. n From this, Co c 1 0, c 2 c 2 = -3! c 3 = -4! = c 4 = -(4)(5) Co Using these values, obtain the solution = 5! we corresponding to the larger root r 1 = 0. Y1(x) - c (1 x 2 + x 4 ...) o 3! 5! = c o x - 1(x - ; + -...) or -1. Y 1 (x) = cOx Sln x. (4) Letting r = r = -1.2 ln (2), we obtain OC 1 = 0; so c 1 IS arbitrary. Letting r = r 2 = -1 in (3), we obtain n(n - 1)c + c 2 = 0 or n n - 346 Chapter 6 c = n c n - 2 n(n - 1)' n) 2. From this, c 2 Co = -2! ' c 3 c 1 = c - 3! ' 4 - c 2 (4)(3) Co = 4! ' c 3 c 5 = -(5)(4) c 1 = 5! ' Using these values, we obtain the solution corresponding to the smaller root r = -1: 2 -1(2 4 ...) Y2(x) x x = cOx 1 - 2! + 4! - -1(3 5 - 1) + 2! + 4! + -1(3 5 - 1) + 2! + \dots) x x + c 1 x x- 3! + Sf - or -1 -1 Y 2 (x) = cOx cos x + c 1 x sin x. (5) The situation here is analogous to that of Example 6.13 of the text (in particular, see text, page 261). In like manner to the case of that example 6.13 of the text (in particular, see text, page 261). In like manner to the case of that example 6.13 of the text (in particular, see text, page 261). In like manner to the case of that example 6.13 of the text (in particular, see text, page 261). In like manner to the case of that example 6.13 of the text (in particular, see text, page 261). In like manner to the case of that example, we see from (4) and (5) that the G.S. of the given D.E. is of the form (24...) -1 x x Y = C 1 x 1 - 2T + 4! - -1(35...) x x + C 2 x x - 3! + Sf - or -1 Y = x (C 1 cos x + C 2 sin x). Series Solutions of Linear Differential Equations 347 16. We assume y CD L n+r = C x, n n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2
n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O CD + x L (n n=O) n+r-1 + r - 1 cn x n=O into the D.E., we obtain Substituting these CD 2 n+r-2 x (n + r)(n + r - 1)c n x n=O into the D.E. (n n=O into the C x n CD + (x 2 - 1/4) L cnxn+r = 0. n = 0 Simplifying, we obtain CD n+r [(n + r) - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 CD n+r + cn 2x = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter 6 or [r(r - 1) + r - 1/4]c n x n = 0 n=2 348 Chapter yields the indicial equation r(r - 1) + r - 1/4 = 0 or r - 1/4 = 0 with roots r = 1/2. However, as in Example 6.13 of the text, if the smaller root r - 1/2. Note that the difference between these roots r = 1/2. However, as in Example 6.13 of the text, if the smaller root r - 1/2. -1/2 has a solution, it will include that of the larger root. Therefore let us try the smaller root first. Letting $r = r^2 = -1/2$ in (1), we obtain [-1/4 + 1/2 - 1/4]c 1 = 0 so c 1 1S arbitrary. Next we obtain two linearly ...) $x + cx x^{-} + 120 - 16$ or $-1/2(124 - ...) x x y = c x^{-} + 2402 + C1 X 1/2 (1 - x 2 + x 5 ...) 6 120$. The second part of this solution is the same expression that r = r 1 = 1/2 will yield. 17. 'We assume y CD L n+r - ex, n n=0 where Co f. o. Then CD y' = L (n + n=0) n+r - 1 c x , n CD II (() n+r - 2 y = n + r) n + r - 1 c nx . n=0 into the D.E., we these CD CD L(n n=O n+r + r)(n + r - 1)c x + n L(n n=O) n+r+3 + r c x n CD CD + L (n + r)cnx n + r - L cnx n + r = 0, n=O n=O 350 Chapter 6 Simplifying, as ln the solutions of Section 6.1, we write this as CD n+r ((n + r)(n + r - 1) + (n + r) - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1) + (n + r)(n + r - 1) + (n + r) - 1]c n x n=O CD + L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1) + (n + r)(n + r - 1) + (n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1) + (n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1) + (n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1) + (n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n n=3 n+r + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 1)c x + n L(n + r - 3)c 3 x = 0 n-or r r+1 [r(r - 1) + r - 1]c O x + [(n + r)(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + r - 3)c 3 x + n L(n + l)r + r]c 1 x + [(r + 2)(r + 1) + r + 1]c 2 x r + 2 CD + L [en + r + 1)(n + r - 1)C n = 3 n + r + (n + r - 3)c 3]x = 0. n - (1) Equating to zero the coefficients of the higher powers of x in (1), we have (r 2 2r)c = 1. Equating to zero the coefficients of the higher powers of x in (1), we have (r 2 2r)c = 1. Equating to zero the coefficients of the higher powers of x in (1), we have (r 2 2r)c = 1. Equating to zero the coefficients of the higher powers of x in (1), we have (r 2 2r)c = 1. Equating to zero the coefficients of the higher powers of x in (1), we have (r 2 2r)c = 1. Equating to zero the coefficients of the higher powers of x in (1) and (1)1 = 0, (2) + (r 2 + 4r + 3)c 2 = 0, (3) (n + r + 1)(n + r - 1)c n + (n + r - 3)c 3 = 0, n > 3. (4) n- Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0, so 1 c 1 = 0. Letting r = r 1 = 1 ln (3), we obtain 3c 1 = 0. Letting r = n(n + 2) n > 3. Co c 1 From this, c 3 = -15 ' c 4 = -12 = 0, c 5 = 3c 2 - 35 = 0, C = -6 C 3 12 Co = |S0' Using these values we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (2), we obtain -c = 0, so c 1 = 0.1 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (2), we obtain -c = 0, so c 1 = 0.1 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (2), we obtain -c = 0, so c 1 = 0.1 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (2), we obtain -c = 0, so c 1 = 0.1 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180
Letting r = r 2 = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r = -1 ln (3), we obtain OC 2 - 0, so c 2 |S - + --15 180 Letting r arbitrary. Letting $r = r 2 = -1 \ln (4)$, we obtain $n(n - 2)c + (n - 4)c n_3 = 0$ or $n(n - 4)c n_3 = 0$ or $n(n - 2)c + (n - 4)c n_3 = 0$ or $n(n - 4)c n_3 = 0$ or ...) (6) + c 2 x x - 15 lS0. 352 Chapter 6 The situation here is analogous to that of Example 6.13 of the text. In like manner to the case of that example, we see from (5) and (6) that the G.S. of the D.E. is of the form C 1 X(1 - : + 6 ...) x y = -- 180 (3 6 + ...). + 1 x x + C x 1 + - 2 3 36 22. We assume y CD L n+r = ex, n n=O where Co f. O. Then CD y' = L (n + n=O) n+r+1 r c x n CD CD + 5 L (n + r)cn x n + r + 2 L c n x n + r + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc n x n + 1 n=O n=O CD + 3Lc Series Solutions of Linear Differential Equations 353 CD CD L(n = 0 n+r + r)(n + r - 1)c 1 x n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + r + 2 L c n 1 x n + r + 3 L cnx n + 1 cnx coefficient of the lowest power of x in (1), we base the indicial equation r + r + 1 (n + r + 3) c + (n + r + 1) (n + r + 3) c + (n + n + 3) c + (n + c n - 1 = 0 and so c - n > 1. From this, - (n 2), n + Co 2c O c 1 2c O and general, <math>c 1 = - = -3!, $c 2 = - - = \ln 3$, 44!, n(-1) 2c O c n = (n + 2)!. Letting r = r 2 = -3 in (2), we obtain (n - 2)(nc n + c n - 1) = 0, n = 0, 1, and so c = n c n - 1 n ' n 1, n f. 2. (3) From (3) with n = 1, we find c 1 = -cO. For n = 2, we go back to (n - 2)(nc + c 1) = 0. Thus c 2 = 4!; and 1n n (-1) 2c 2-c 1 / n = r. From this we obtain the n n-n. solution corresponding to the smaller root r 2 = -3: CD n n (-1) 2c 2 x n! (4) -3 X -3 Y2(x) = cOx (1 - x) + n=2 The situation here is analogous to that of Example 6.13 of the text. In like manner to the case of that example, we see from (3) and (4) that the G.S. of the D.E. is of the form CD -3 -1 (_1) n x n y = C $1 \times (1 - x) + C 2 \times (n + 2)!$. n=O Series Solutions of Linear Differential Equations 355 24. 'We assume y CD L n+r = C x, n n=O where Co f. o. Then CD y' = L (n + n=O) n+r-l r c x, n CD y'' - L (n n=O D.E., we obtain n+r-2 + r)(n + r - l)c x. n Substituting into the CD CD n+r) n+r+2 (n + r)(n + r - l)c x + 2 (n + r cnx n=O n=O CD CD CD C x n + r)(n + r - l)c x + 2 (n + r cnx n=O n=O CD CD CD C x n + r)(n + r - l)c x + 2 (n + r)(n + r - l)c x + 2 (n + r cnx n=O n=O CD CD CD C x n + r)(n + r - l)c x + 2 (n + r)(n + + 2 15 c x n + r = 0. n 4 n n=0 n=0 Simplifying, as ln the solutions of Section 6.1, we obtain CD CD n+r n+r + r)(n + r - 1)c n x + 2 (n + r - 2)c n 2 x n=2 L(n n=0 cr [r(r-1) 1;] c l x r + 1 CD + L {[(n + r)(n + r - 1) - 1;] c l x r + 1 CD + L {[(n + r)(n + r - 1) - 1;] c n n=2 + [2(n + r - 2) - 1]Cn 2}xn+r = Q. (1) 356 Chapter 6 Equating to zero the coefficients of the higher powers of x in (1), we obtain (r 2 + r - 15) c = 0, (2) 4 1 [(n + r)(n + r - 1) 15] c 4 n + [2(n + r - 2) - 1] c n 2 = 0, n > 2. (3) Letting r = r 1 5 (2), we obtain 5c 1 = 0, = ln so 2 = 0. Letting r 5 (3), we obtain n(n + 4)c c 1 = r 1 = ln 2 n 2c n-2 + 2nc n-2 - o and hence c = n + 4' n > 2. From this, n Co c 2 = -3' c 4 = c 2 - - = 4 Co 12' and c 2n + 1 = 0 for n o. Using these values we obtain the solution corresponding to the 5 larger root r 1 = 2 : 5/2(1 2 4 ...). Y1(x) x x (4) = cOx - + - - 3 12 Letting r = 3 (2), we obtain $r 2 = -\ln 2 - 3c = 0$, o. Letting r 3 (3), Since $c 1 = -r = -\ln we 1 - 2 2$ obtain n(n - 4)c n - 2 = 0, n > 2, and hence n c = n 2c n - 2, n = 4, we return to n(n - 4)c n - 2, and hence n c = n 2c n - 2, n = -2, n = -= O. With n = 4, this becomes OC 4 + OC 2 = o. Thus c 4 is independent of c 2 and so IS a second arbitrary constant. Then from (5) with n = 6, c = -c/3. 6 4 and with n = 8, c - -c 6/4 = c 4/12. Also note that 8 - c 2n + 1 = 0 for n o. Using these values we obtain the 3 solution corresponding to the smaller root r 2 = -2 : -3/2 2 Y2(x) = cOx (1 - x) + cOx (1 - x) $x-3/2(x 4 x 6 + x 8 ... J 4 3 12 \cdot (6)$ The situation here is analogous to that of Example 6.13 of the text. In like manner to the case in that example, we see from (4) and (6) that the G.S. of the D.E. is of the form $-3/225/2(24 J y = C1x(1 - x) + Cx1 - + - ... 23 12 \cdot 25$. We assume y CD L n+r = c x, n n=O where Cot O. Then CD y' = L (n + - ... 23 12 \cdot 25)/2(24 J y = C1x(1 - x) + Cx1 - + - ... 23 12 \cdot 25. We assume y CD L n+r = c x, n n=O where Cot O. Then CD y' = L (n + - ... 23 12 \cdot 25)/2(24 J y = C1x(1 - x) + Cx1 - + - ... 23 12 \cdot 25. $CD n+r + cn_1x = 0$ n=l and hence as CD [r(r-1) + r - l]c O + L [en + r + l)(n + r - l]c O + L [en + r + l)(n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L [en + r + l](n + r - l]c O + L
[en + r + l](n + r - l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O + L [en + r + l](n + r + l]c O- l)c n + c n - 1 = 0, n 1. (1) Letting r = r 1 = 1 in (1), we find (n + 2)nc n + c 1 = 0 and hence n- Series Solutions of Linear Differential Equations 359 From this, c 1 = c 2 c 3 - (3)(5) = c = n c n-1 n(n + 2)' n > 1. Co 2c O c 1 2c O (1)(3) = 1 ! 3! ' c = -(2)(4) = 2!4! ' 2 2c O and, in general, 3!5! ' 2c O C - -(_l) n U. h 1 bt. slng t ese va ues, we 0 aln, n n! (n + 2)!. the solution corresponding to the larger root r 1 = 1 and given by y [2x 2x 2 2x 3 ...]. Choosing = cOx 1 - 1!3! + 2!4! - 3!5! + c = 1, we obtain the particular solution denoted by Y1(x) 0 and defined by Y1(x) = X[1 = x[1 2x 2x 2 1!3! + 2!4! + ...] 2x 3 3!5! CD](1) n x n + 2L n!(n+2)! + n=1(2) Letting r = r 2 = -1 ln (1), we find n(n - 2)c + c 1 = r 2 = -1 ln (1), we find n(n - 2)c + c 1 = r 2 = -1 ln (1) n x n + 2L n!(n+2)! + n=1(2) Letting r = r 2 = -1 ln (1), we find n(n - 2)c + c 1 = r 2 = -1 ln (1) n x n + 2L n!(n+2)! + n=1(2) Letting r = -1 ln (1) n x n + 2L n!(n+2)! + n=1(2) Letting r = -1 ln (0 n n- and hence (3) $c = n c n - 1 n(n_2) n 1$, n t 2. (4) For n = 1, (4) gives c 1 = c0. For n = 2, (3) IS OC 2 + c 1 = 0, and hence c 1 = c0 we must have Co = 0. However, we assumed Co f 0 at the start. This contradiction shows there is no solution of the 360 Chapter 6 assumed form with Co f 0 corresponding to the smaller root r 2 = -1. Moreover, use of (4) for n 3 will only lead us to the solution Y1(x). From Theorem 6.3, we know that it is CD of the form l/n*x n - 1 + CY1(x) where c o * f 0 and n=O C t o. We shall use reduction of order to find it. We let Y = y 1(x)v + Y1(x)v ++ [2xy 1'(x) + Y1(x)]w = 0. From this, we have dw [Y 1'(x) 1] - = -2() + -dx, w Y1 x and integrating we obtain the particular solution Series Solutions of Linear Differential Equations 361 w = 1 2. x [y 1(x) J Writing out the first three terms of Y1(x) defined by (2), we have 2 3 x x Y1(x) = x - + 24 + (6) From this, using basic multiplication anddivision, we find 2 [y 1 (x)] = 2 2x 3x - 3 + 7x 4 36 + ... and 1 = 2 [y 1 (x)] 1 2 1 2 + 3x + 4 + x Hence we obtain 1 1 2 1 w = - + - + - + 2 3 3x 2 4x x [y 1 (x)] x Integrating we obtain the desired linearly independent solution of the gIven D.E. We thus find y = x-1(- x + 29x 2 + ...) + Yl(x)lnlxl (8) 2 2 144 4 The general solution is a linear combination of (2) and (S). 362 Chapter 6 28. We assume y CD L n+r = c x, n n=O where Co = 1 = o. Then CD y' = L (n n=O into the D.E., we obtain n+r-2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n + r - 1)c x. n Substituting these CD CD n+r + r)(n - 2 + r)(n r - l) c n x + (n n=0) n+r+1 + r c x n L(n n=0 CD L n=0 n+r c x n = 0. Simplifying, as In the solutions of Section 6.1, we write this as CD L[(n n=0 3] n+r + r)(n + r - 1) - 4 cnx CD + L(n n=1 n+r + r - l)c 1 x n = 0 and hence as CD [r(r - 1) -] coxr + L{[(n + r)(n + r - 1) -] Cn n=1} n+r + (n + r - 1)c n_1 x = 0. Series Solutions of Linear Differential Equations 363 Equating to zero the coefficients of the higher powers of x, we have the indicial equation $r_2 - r_3 4 = 0$, with 3 roots, $r_1 = 2 r_2 - 1 - 2$. Equating to zero the coefficients of the higher powers of x, we have $[(n + r_1)(n + r_2 - 1) - 2 r_3 4 = 0, n > 1.$ (1) Letting $r_1 = 2 r_2 - 1 - 2$. Equating to zero the coefficients of the higher powers of x, we have $[(n + r_1)(n + r_2 - 1) - 2 r_3 4 = 0, n > 1.$ (1) Letting $r_1 = 2 r_2 - 1 - 2$. Equating to zero the coefficients of the higher powers of x, we have $[(n + r_1)(n + r_2 - 1) - 2 r_3 4 = 0, n > 1.$ (1) Letting $r_2 - 1 - 2$. Equating to zero the coefficients of the higher powers of x. and hence $c n = (2n + 1)c n_1 2n (n + 2)$, n > 1. From this, c 1 Co 5c 1 - 2'c 2 = -16 = 5c O c = -32' 3 7c 2 30 7c O = -192' Observe that we can also write <math>c = 1 3c O 2 0 1!3!'c 2 = (3)(5)(7)c O 223!5!, and ln general, (3)(5)(7)...(2n + 1)c O = (-1) n c n n - 1 2 n! (n + 2)! n > 1. Using these values we obtain the solution corresponding 3 to the larger root r 1 = 2 and given by $y = C \circ x 3/2 (1 - +x: -r: + ...)$. Choosing Co = 1, we obtain the particular solution denoted by Y1(x) and defined by 364 Chapter 6 Y 1(X) = x 3/2 (1 - +x: -r: + ...). Choosing Co = 1, we obtain the particular solution denoted by Y1(x) and defined by 364 Chapter 6 Y 1(X) = x 3/2 (1 - +x: -r: + ...). Choosing Co = 1, we obtain the particular solution denoted by Y1(x) and defined by 364 Chapter 6 Y 1(X) = x 3/2 (1 - +x: -r: + ...). n-1 1 2 ln (1), we obtain (n 2 - 2n)c n n(n - 2)c + 2 1 (2n - 3)c 1 = 0 n n - (3) and hence c n = (2n - 3)c n = 1, (4) glves c 1 = -. For n = 2, (3) lS oC 2 c 1 0, and hence o. + = c 1 - - Co must have o. c 1 = -2' we Co = But then, since However, we assumed Co t 0 at the start. This contradiction shows there IS no solution of the assumed form with Co * 0 corresponding to 1 the smaller root r 2 = - 2. Moreover, use of (4) for n > 3 will only lead us to the solution Y1(x). From Theorem 6.3, we know that it is of the form Series Solutions of Linear Differential Equations 365 CD * n-1/2 cnx + CY 1(x)v. From this we obtain y' = Y1 (x)v' + Y1 '(x)v' + Y1 '(after some simplifications, we obtain Y 1 (x)V" + [2y 1 '(x) + Y1(x)]v' = o. (5) Letting w = v', this reduces to dw y 1 (x) dx + [2y 1 '(x) + y 1 (x)] From (2), uSing basic multiplication and division, we find 2 3 [y 1 (x)] = x 4 2 [y 1 '(x) + y 1 (x)] w = 0. dw [2 Y 1
'(x) + y 1 (x)] w = 0. dw [2 Y 1 '(x) + y 1 (x)] w = 0. dw x 9x 5 + --16 11x 6 48 + ... and 12 [y 1 (x)] -3 -2 = x + x + -17x 16 5 + 48 + 366 Chapter 6 -x Thus uSlng e 2 x = 1 - x + - -23 x - + 6 we obtain the particular solution of (5) glven by -2 x (6) v = -- + ... 2 16, where the next nonzero term in this expanSlon IS the term 2 Multiplying (2) by (6), we obtain the desired ln x. linearly independent solution of the given D.E. We thus find (2 ...) Y1 (x)lnlxl Y = X - 1/2 ! + x 5x 2 4 - 64 + 16 (7) The general solution 1S a linear combination of (2) and (7). 31. We assume y CD L n+r = C x, n n=O where Cot o. Then CD Y'=L(n+n=O) n+r-l r c x, n CD " () () n+r-2 y = n + r n + r - 1 cnx. n=O into the D.E., we obtain Substituting these Series Solutions of Linear Differential Equations 367 CD CD n+r + c x + c x = 0. n n n=O n=O Simplifying, as In the solutions of Section 6.1, we write this as CD n+r (n + r)(n + r - 1) - (n + r) + 1]c n x n=O CD n+r + c x + c x = 0. n=2 and hence as r[r(r-1) - r + 1]c O x + [(r+1)r + 1 - (r+1) + 1]c 1 x CD + L ([en + r)(n + r - 1)n=2n+r - (n + r) + 1]c + c 2 x = 0. n n- Equating to zero the coefficients the double root r = 1. Equating to zero of the higher powers of x, we have [(r + 1)r + 1]c + c 2 x = 0. n n- Equating to zero the coefficients the double root r = 1. Equating to zero of the higher powers of x, we have [(r + 1)r + 1]c + c 2 x = 0. (r + 1) + 1 c 1 = 0, (1) [en + r)(n + r - 1) - (n + r) + 1 c n + c 2 = 0, n > 2. n- (2) 368 Chapter 6 Letting r = 1 ln (1), we have c 1 = 0. Letting r = 1 2 ln (2), we obtain n c + c = 0 and hence n n-2 c = n c n-2 2 ' n n > 2. From this, c 2 Co c 1 = -, c 3 = - If = 0, c 4 = c 2 - - 16 Co = 64' Note that all odd coefficients are zero and we can write the general even coefficients as c 2n = (1)n 2' [(2)(4)(6)...(2n)] n > 1. Using these values we obtain the solution of the assumed form corresponding to the double root r = 1 and given by y = C O X(1 246) x x x + 64 - 2304 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + 64 + ... + ... + 64 + ... + ... + ... + ... + ... + ...) n x 2n 2. [(2) (4) (6) \cdots (2n)] (3) = x 1 + L n=1 Since the indicial equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solutions of Linear Differential Equation has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solution has the double root r = 1, by Conclusion 3 of Theorem 6.3, a linearly independent solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solution is of the form CD * n+2 () k c n x + y 1 x In x. n=0 Series Solution is of the form CD * n+2 () k c n x + y 1 x In x + y 1 x + y 1 x In x + y 1 x In x + y 1 x + y 1 obtain y' = y(x) v' + Y'(x)v and 1 1 yH = Yl(x)v' + Y1H(x)v. Substituting these into the given D.E., after some simplifications, we obtain xy 1(x) + Y1(x) + Y1(the particular solution x w = 2- [y 1 (x)] From (3), IlsIng basic multiplication and division, we find 2 [y 1 (x)] 2 = x 4 x - + 2 3x 6 - 32 5x 8 576 + and 1 2 [y 1 (x)] - 2 = x 4 x - + 2 5x 2 23x 4 32 + 576 + ... Thus we obtain the particular solution of (4) glven by 2 5 4 23x 6 v= Inlx1 + x 4 + 1;8 + 3456 + (5) Multiplying (3) by (5) we obtain the desired linearly independent solution is a linear combination of (3) and (6). Section 6.3, Page 280 2. Differentiating y = -128 + 13824 + ... + Y1 (x)lnlxl. (6) The general solution is a linear combination of (3) and (6). Section 6.3, Page 280 2. Differentiating y = -128 + 13824 + ... + Y1 (x)lnlxl. (6) The general solution is a linear combination of (3) and (6). Equations 371 3. The Bessel equation of order 1 p = IS 2 2 "xy' (X2) y = 0.x y + (1) To use the result of Exercise 2, we let y = u and reduce fX (1) to U II + [1 + (-) x12] u = 0 or simply u H + u = 0. The auxiliary equation of this IS 2 m + 1 = 0, with roots m = %i, and the G.S. is u = c 1 s 1 nx + c 2 cosx. Thus, since y = u/fX, the G.S. of (1) IS c 1 Sin x + c 2 cos x y = fX 4. From the series definition (6.124), we at once have (I) (_l)n (k) 2n+p 2n+2p n!(n + p)! 2x · xPJ (kx) P = L n=O We may differentiate this series term-by-term to obtain d: [XPJp(kX)] = f [(_l)n (k 2) 2n+px2n+2P] f...J dx n! (n + p)! n=O (I) _ (-1)n 2 (n + p)! (n + p)! n=O (I) _ (-1)n 2 (P k (k 2) 2n + p - l x 2n + pthe first equation and the operator (D - 4) to the second, obtaining t { (D - 1)(D - 2)x + (D - 4)(D - 1)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first, we obtain [(D - 1)(D - 2)x + (D - 4)(D - 4)y = (D - 4)e Subtracting the second equation from the first [(D - 4)(D - 4we give both here. First, we proceed by returning to the system (1), applying the operator D to the first operator and the operator (D - 2) to the second equation, obtaining { D(D - 2)x + D(D - 4)y = De t, (D - 2)(D - 1)y = (D - 2)e 4t. 373 374 Chapter 7 Subtracting the first equation from the second, we obtain [(D - 2)(D - 1)D(D - 4)Jy = (D - 2)e 4t. 373 374 Chapter 7 Subtracting the first equation from the second equation from the se 2) e 4t - De t or (D + 2)y = 2e 4t = e t. Using undetermined coefficients, we find the G.S. of this D.E. to be -2t 4t t e e (3) y = ke + - 3 \cdot 3 Now the determinant of the operator "coefficients" of x and y in (1) is D - 2 D - 4 = D + 2. D D - 1 Since this is of order 1, only one of the two constants c and k ln (2) and (3) can be independent. To determine the relation which must thus exist between c and k, we substitute x given by (2) and y given by (3) into system (1). Substituting into the second equation of (1), we have -2 t
[+4 e 4t - e t] - 2ce + -2ke - 2t 3 3 - [ke-2t + e: t - e 3 t] = e 4t - 2t or (-2c - 3k)e = o. Thus k 2c = -3. Hence we obtain the G.S. of (1) in the form Systems of Linear Differential Equations 375 -2t x = ce -2t 4t t(4) 2ce e e y - + - - 3 3 3 We now obtain y by the alternative procedure of the text, pages 293-295. We return to system (1) and subtract the second equation from the first, thereby eliminating Dy b d h b .. 2 3 t 4t F ut not y, an t us 0 talnlng - x - y = e - e rom 2 4t t x e e this, y = -3 + 3 - 3. Substituting x -2t 4t 2ce e (2) we at once obtain y = -3 + 3 - this and (2), we again have the G.S. (4) of into this from t e 3. Using the original system. 2. We introduce operator (D - 1) to the first equation and the operator D to the second to obtain { (D - 1)2x + (D - 1)Dy = -2t t 2 . (1) (D 3)x + (D 3 D(D - 3)x + D(D - 1)y = (D - 1)(-2t), Dt 2. Subtracting the second equation from the first, we obtain [(D - 1)2 - D(D - 3)]x = (D - 1)(-2t), Dt 2 or (D + 1)x = -2. The G.S. of this D.E. is -t x = ce -2 (2) 376 Chapter 7 There are two ways to obtain y, and we give both. First, we return to (1) and apply the operator (D - 3)]x = (D - 1)(-2t). 1) to the second equation. We obtain { (D (D 3)(D 1)(D 1)x + (D 3)x + (D 3)Dy = (D - 1)t 2 - (D - 3)(-2t) 1)t 2 - (D - 3)(-2t)(-2t) 1)t 2 - (D - 3)(-2t)(-D = D + 1. D - 3 D - 1 Since this is of order 1, only one of the two constants c and k ln (2) and (3) can be independent. To determine the relation which must thus exist between c and k, we substitute x given by (3) into system (1). Substituting into the first equation of (1), we have -c e -t - c e -t + 2 - k e -t - 2t - 2 = -2t or (2c + k) e -t = -2t or (2c + O. Thus k = -2c. Hence we obtain the G.S. of (1) in the form Systems of Linear Differential Equations 377 -t t : ce 2, -2c e -t - t 2 - 2t + 4. (4) We now obtain y by the alternative procedure of the text, pages 293-295. We return to the system (1) and subtract the second equation from the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2c e -t - t 2 - 2t + 4. (4) We now obtain y by the alternative procedure of the text, pages 293-295. We return to the system (1) and subtract the second equation from the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain y by the alternative procedure of the text, pages 293-295. We return to the system (1) and subtract the second equation from the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain y by the alternative procedure of the text, pages 293-295. We return to the system (1) and subtract the second equation from the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtaining 2 2x + y - 2t + 4. (4) We now obtain the first, thereby eliminating Dy but not y, and thus obtain the first obtain the f = -t - 2t. Substituting x into this from (2) and solving for y, we at once find -t - 2Y = -2c e - t - 2t + 4. Using this and (2), we aga1n have the G.S. (4) of the original system. 4. Ve introduce operator notation and write the system 1n the form r 2D l)x + (D + 2)y = 0, 4e 2t. (1) (3D 2)x + (2D + 1)y - We apply the operator (2D + 1) to the first equation and write the system. the operator (D + 2) to the second, obtaining f (2D + 1)(2D | (D + 2)(3D | x + (2D + 1)(D + 2)y - (2D + 1)(D + 2)y - (2D + 1)(2D +alternative procedure of the text, pages 293-295. Ve subtract the second equation of (1) from 2 times the first, thereby eliminating Dy but not y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve substitute this into (3) and solve for y. Ve find t 3t 2t dx = c 1 e + 3c 2 e + 8e · Ve subs G.S. of (1) IS this t 3t 4 2t x = c 1 e + c 2 e + e, c 1 t 3t 3e 2t. y = -e c 2 e 3 Systems of Linear Differential Equations 379 6. We introduce operator (D - 1) to the first equation and the operator (D - 2) to the second, obtaining f (D - 1)Dx + (D + 2)y = sin t, (1) I)x + (D + 2)y = sin t, (1) I)x + (D + 2)y = sin t, (1) I)x + (D + 2)y = sin t, (2) I = 0 $(D \ l)(D + 2)y = l(D + 2)(D - l)x + (D + 2)(D - l)x + (D + 2)(D - l)x = (D - 1) \text{ sin t }, (D + 2)(D - 1) = (D - 1) \text{ sin t }, (D + 2)(D - l)x = (D - 1) \text{ sin t }, (D - 1) \text{ sin t },$ subtract the second equation of (1) from the first, thereby eliminating Dy but not y. Ve have x + 3y = sin t. Ve substitute x given by (2) into this 1 t 1 and solve for y, obtaining y = -3 ce + 2 sint. The G.S. of (1) is thus t 1. x = ce - 2 sint. The G.S. of (1) is thus t 1. x = ce - 2 sint. The G.S. of (1) is thus t 1. x = ce - 2 sint. The G.S. of (1) is thus t 1. x = ce - 2 sint. 5)x + (D - 1)y = 0, (1)(4D 3)x + Dy = t. Ve apply the operator D to the first equation and the operator (D - 1) to the second, thereby obtaining $\{D(5D 5)x + D(D - 1)y = DO, (D - 1)(4D 3)x + (D - 1)Dy - (D - 1)t, that is, (D 2 + 2D - 3)x = t - 1$. The G.S. of this IS tx = c 1 e-3t + c 2 e ! t 1 3 + 9. (2) Ve find y uSing the alternative procedure of the text, pages 293-295. Ve subtract the second equation of (1) from the first, obtaining Dx - 2x + t. (3) Differentiating (2), we have t Dx = c 1 e - 3t 1 3c 2 e - 3. (4) Systems of Linear Differentiate Equations 381 (1) (D 2)x + (2D 6)y = t. Ve apply the operator (2D - 6) to the first equation and the operator (D - 6) to the second equation, obtaining 3t f(2D - 6)(D - 1)x + (2D - 6)(D - 2)x + (D -= 6t - 1. Using the undetermined coefficients, the G.S. of this D.E. is found to be 382 Chapter 7 J6t - J6t 1 x = c 1 e + c 2 e - t + 6. (2) We find y uSing the alternate procedure of the text, pages 293-295. We multiply the first equation of (1) by 2, obtaining (2D - 2)x + (2D - 12)y - 2e 3t. Ve now subtract the second equation of (1) from this, obtaining 3t Dx - 6y = 2e - t, which involves y but not Dy. From this, y = Dx - 2e - 3t + t - 6 (3) rF / 6t - F - 6 - 3t + 2 - 6 - 3t + 1 - 6 - 6 - 3t + 2 - 6 - 3t + 1 - 6 - 6 - 3t + 2 - 3tthe
form { (2D l)x + (D l)y (D + 2)x + (D + l)y = -t = e t e . (1) We apply the operator (D - 1) to the first equation and the operator (D - 1) to the second, obtaining Systems of Linear Differential Equations 383 { (D + 1)(2D - l)x + (D + l)(D l)y (D - 1)(D + 2)x + (D l)(D + l)y = (D = (D + l)e - t t l)e . Subtracting the second from the first, we obtain [(D + 1)(2D - l)x + (D + l)(D l)y (D - 1)(D + 2)x + (D l)(D + l)y = (D = (D + l)e - t t l)e .1) (2D - 1) - (D - 1)(D + 2)Jx = (D + 1)e + c + (D - 1)e + (D - 1)2y = From (2), Dx = c 1 cost - c 2 sint . Substituting into the preceding expression for y, we get c 1 cost - c 2 sint - 3c 1 sint - 3c 2 cost + tte - ey = -2 + 2 + --2 . 2 The G.S. of system (1) is thus: x = c 1 sint + c 2 cost , 3c 1 + c 2 cost + c 2 sint . Substituting into the preceding expression for y, we get c 1 cost - c 2 sint - 3c 2 . 2 2 12. Ve introduce operator notation and write the system 1n the form r 3D l)x + (2D + 1)y = t 1, (1) (D l)x + Dy = t + 2. 384 Chapter 7 We apply the operator D to the first equation and the operator (2D + 1) to the second, thereby obtaining L2D D(3D l)x + D(2D + 1)y = D(t - 1), + 1)(D l)x + (2D + 1)Dy - (2D + 1)(t + 2). - Ve subtract the second equation from the first, obtaining [D(3D - 1)(2D + 1)(D - 1)]x = D(t - 1) - (2D + 1)(t + 2) or (D 2 + 1)x = -t - 3. The auxiliary equation of this D.E. is m 2 + 1 = 0 with roots % i, and the complementary function 1S Xc = c 1 sin t + c 2 cos t - t - 3. (2) e find y uSlng the alternative procedure of the text, pages 293-295. e multiply the second equation of (1) by 2 to obtain (2D - 2)x + 2Dy = 2t + 4. Then we subtract this from the first equation of (1) to obtain Dx + x + y = -t - 5, which involve Dy. From this, y = -x - Dx - t - 5. (3) Differentiating (2), we have $Dx = c 1 \cos t - c 2 \sin t - 1$. (4) Now substituting x from (2) and Dxfrom (4) into (3), we find $y = -c1 \sin t - c2 \cos t + t + 3 - c1 \cos t + c2 \sin t + 1 - t - 5$ or $y = (c2 - c1) \sin t - (c1 + c2) \cos t - 1$. Systems of Linear Differential Equations 385 The G.S. of system (1) is thus $x = c1 \sin t + c2 \cos t + 1$. Systems of Linear Differential Equations 385 The G.S. of system (2 - c1) sin t - (c1 + c2) cos t - 1. Systems of Linear Differential Equations 385 The G.S. of system (1) is thus $x = c1 \sin t + c2 \cos t + 1$. (2D + 1)x + (D + 5)y = 4t, (1)(D + 2)x + (D + 2)(D + 5)y = (D + 5)(D + 2)x + (D + 2)(D + 2)(D + 2)x + (D + 2)(D + 2)(Dauxiliary equation of this 2 D.E. is m 2m - 8 = 0, that is, (m - 4)(m + 2) = 0, with roots m = 4, -2; and the complementary function is x = c 4t c 1 e + c 2 e - t + 1. (2) Ve find y using the alternative procedure of the text, pages 293-295. We subtract the second equation of (1) from the first to obtain Dx - x + 3y = 4t - 2, which involves y but does not involve Dy. From this, 386 Chapter 7 y =; [x - Dx + 4t - 2]. (3) Differentiating (2), we have 4t Dx = 4c + 2c + 1 + 4t - 2c+ (D + 2)(4D l)y (D t = + 2)3e, (4D - 1)(D + 2)x + (4D - 1)(D + 2)x -2t Xc = c 1 e + c 2 e Using the method of undetermined coefficients, we assume a particular integral of the form t t x = Ate. Ve quickly find A = -1, so x = -t e . Thus, p p the G.S. of the D.E. IS t -2t - t e t (2) x = c 1 e + c 2 e . Ve find y uSlng the alternative procedure of the text, pages 293-295. Ye multiply the second equation of (1) by 4 to obtain (4D) + 8)x + (4D + 8)yt - 4e. Ve then subtract the first equation of (1) from this, obtaining 2 Dx + 7x + 9y = et, which involves y but does not involve Dy. From this 1 t Y = 9 [-7x - 2 Dx + e J. (3) Differentiating (2), we have t Dx = c 1 e 2 - 2t t flnd y = 9 - 7c 1 e - 7c 2 e + 7te - 2clet + 4c2e - 2t + 2tet + 2et + et] 388 Chapter 7tc2 - 2tt1tory = -c1e - 3e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2ttc1)e - (1/3)c2e + te + 3e The G.S. of system (1) is thus x = tc1e - 2tt + c2ete, t-2 to the first equation and the operator D to the second, obtaining rD + 2 (D - 1)Dy = (D - 1)(2t - 8), 8Dx + D(D - 1)y = - Dt 2 . Ve subtract the second equation from the first, obtaining (D 2 + 2D + 5)x = 10. The auxiliary equation from the first, obtaining (D 2 + 2D + 5)x = - t (c l sin 2t + c 2) + $\cos 2t$. A particular integral is x = 2; and the G.S. is p - t x = e (c 1 sin 2t + c 2 cos 2t) + 2. (2) Systems of Linear Differential Equations 389 Ye find y uSlng the alternative procedure of the text, pages 293-295. Ye subtract the second equation of (1) from the first to obtain Dx + 3x + y = t 2 + 2t - 8, which involves y but does not involve Dy. From this, 2 Y = -3x - Dx + t + 2t - 8. (3) Differentiating (2), we find Dx = e - t [(-c1 - 2c2) sin 2t + (2c1 - c2) cos 2t] + t + 2t - 14. The G.S.of the system (1) IS thus $-t + c^2 \cos 2t$ + 2, Y = e (c 1 sin 2t -t [c 1)sin 2t - 2(c 1 + c 2) cos 2t] y = e 2(c 2 - 2 14. + t + 2t - 390 Chapter 7 21. Ye introduce operator notation and write the system ln the form { (2D - 1)x + (D (D + 2)x + (D 1)y = 1, (1) 1)y = t. For this system we merely have to subtract the second equation from the first to eliminate both y and Dy. Ye have [(2D - 1) - (D + 2)]x = 1 - t or (D - 3)x = 1 - t. The G.S. of this IS 3t t 2 x = c 1 e + 3 - 9 . (2) Ve cannot apply the alternate procedure of pages 293-295 to find y here; for, as noted above, elimination of Dy from (1) also results in elimination of Dy from (2) Ve cannot apply the alternate procedure of pages 293-295 to find y here; for, as noted above, elimination of Dy from (1) also results in elimination of Dy from (1) also results in elimination of Dy from (2) Ve cannot apply the alternate procedure of pages 293-295 to find y here; for, as noted above, elimination of Dy from (1) also results in elimination of Dy from (1) also the first equation of (1) and the operator (2D - 1) to the second equation, obtaining { (D + 2)(2D - 1)x + (D + 2)(D - 1)y = (2D - 1)(D - 1) - (D + 2)(D - 1)y = (2D - 1)(D - 1)(D - 1)y = (2D - 1)(D - 1)(Dundetermined coefficients, the G.S. of this D.E. IS found to be t 3t Y = k 1 e + k 2 e t 4 - 3 - 9. (3) The determinant of the operator "coefficients" of x and y in (1) is 2D - 1 D + 2 D + 2 exist among these three constants, we substitute x given by (2) and y given by (3) into system (1). Substituting into the first equation of (1), we have (3t + 1 + 2k + 2 = 1 and k 1 is arbitrary. Thus k 1 is the "second"
arbitrary. constant of the G.S., and we now write it as c 2. Hence we have the G.S. in the form 392 Chapter 7 3t t 2 x = c 1 e + 9 ' 3 5c 1 e 3t t t 4 y = c 2 e - - 9 . 2 3 22. Ye introduce operator D to the second equation, obtaining (D - 1)D 2 x + (D - 1)D y = (D - 1)e 2t, D(D - 1)x + D(D - 1)y = DO. Subtracting the second equation from the first, we obtain [(D - 1)D 2 - D(D - 1)]x = (D - 1)e 2t - 0 or 3 2 2 t (D - 2D + D)x = e The auxiliary equation of this D.E. is m 3 - 2m 2 + m = 0 with roots m = 0, 1, 1 (double root); and the G.S., found by using undetermined coefficients, is t e 2t x = c 1 + (c 2 + c 3 t)e +. (2) Ye find using the alternate procedure of pages 293-295 of the text. Ye subtract the second equations 393 the first, thereby eliminating Dy but not y, and thus obtaining D 2 x - Dx 2t From this, $+x + y = e^{-x} + Dx - D^2 x^2 t$ (3) Y = + e () () t 2t D 2x From 2, we find $Dx = c 2 + c 3 + c 3 t e + e_{t} = t 2t (c 2 + c 3 + c 3 t)e + 2e \cdot Substituting x from (2) and these derivatives into (3), we get <math>y - [t + e 2 2t] - c 2 + c 3 + c 3 t)e + 2e \cdot (c 2 + c 3 + c$ G.S. of system (1). 26. Ve introduce operator notation and write the system 1n the form 2 (D + 9)(4D + 4)y = 0, (1) 2t (D - 1)x + (4D - 4)(D + 9)y = e. 394 Chapter 7 Ye apply the operator (D + 9)(4D + 4)y = 0, (1) 2t (D - 1)x + (4D - 4)(D + 9)y = (4D - 4)(D + 9)y = (4D - 4)(D + 9)(4D + $2t \cdot (4D - 4)(D - 4)(D - 1)[x = 0 - (4D - 4)(D - 1)]x = 0 - (4D - 4)(D - 1)[x = 0 - (4D - 4)(D - 1)]x = 0$, with roots m = -1, m = -2 % 1. Using this information and undetermined coefficients, the G.S. of this D.E. IS found to be -t -2t 4e 2t x = c 1 e + e (c 2 sin t + c 3 cos t) - 51. (2) Ve find y uSing the alternate procedure of pages 293-295 of the text. Ye multiply the second equation of (1) by 4, obtaining (4D - 4)x + (4D + 36)y = 4e 2t. Ye now subtract this from the first equation of (1), obtaining D 2 4D 5 40 4 2t F h o x - x + x - y = - e. rom t IS, y = D 2x - 4Dx + 5x + 4e 2t 40(3) Systems of Linear Differential Equations 395 From (2), t - 2t - 2t Dx = -c 1e + e(-2c 2 - c 3) sin t + e(-4c 3 + 2c 3)+ 3c 3) cost - 51 Y = 40 -t -2t -2t 8e 2t 4 - c 1 e - e (-2c 2 - c 3) sin t + e (c 2 - 2c 3) cos t - 51 40 r -t -2t 4e 2t 1 + :1 cle + e (c 2 - 3c 3 - 4c 2 + 3c 3 - 4c 3 (51) or finally -t c 1 e 4 -2t [(2C2 C3) + e 5 + 5 sin t y = + (_c; + 2:3) cos t + 5:t. (4) The pa1r (2) and (4) constitute the G.S. of system (1). 30. By text, page 286, equations (7.7), we let xl = xl + x 2 = x, x 3 = XH, x 4 = x ffl. From these and the given fourth order D.E., we have x' x' x' "x' ffl = = x 2 ' = x = x 3 ' = x = x 4 ' 1 2 3 x' IV -2tx t 2 x" -2tx 2 $\cos t = x = + + \cos t = + t x 3 + .41$ Systems of Linear Differential Equations 397 Thus we have the system x' = x 2, x' = x 3, x' - x 4, 1 2 3 - x' - 2tx 2 cos t = + t x 3 + .41 Systems of Linear Differential Equations 397 Thus we have the system x' = x 2, x' = x 3, x' - x 4, 1 2 3 - x' - 2tx 2 cos t = + t x 3 + .41 Systems of Linear Differential Equations 397 Thus we have the system x' = x 2, x' = x 3, x' - x 4, 1 2 3 - x' - 2tx 2 cos t = + t x 3 + .41 Systems of Linear Differential Equations 397 Thus we have the system x' = x 2, x' = x 3, x' - x 4, 1 2 3 - x' - 2tx 2 cos t = + t x 3 + .41 Systems of Linear Differential Equations 397 Thus we have the system x' = x 2, x' = x 3, x' - x 4, 1 2 3 - x' - 2tx 2 cos t = + t x 3 + .41 follows: 1. across the resistor R 1: 20i 1. 2. across the inductor L 1: 0.01 i' + 20i 1 = 100. (1) For the right hand loop, the voltage drops are as follows: 1. across the resistor R 1: -20i 1. 2. across the inductor L 2: 0.02i 2. Thus applying the voltage law to this loop, we have the D.E. $o \cdot 02$ i 2 - 20i 1 + 40i 2 = O. (2) For the outside loop, the voltage drops are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 1 : 0.01 i' · 3. across the inductor L 2 : 0.02i 2 · 20i 1 + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 1 : 0.01 i' · 3. across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 + 0.01 i' + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 - 20i 1 + 40i 2 = 100. (3) The three equations are 1 · across the inductor L 2 : 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop, we have the D.E. 0.02 i 2 - 20i 1 + 40i 2 = 0.02i 2 · 398 Chapter 7 Thus applying the voltage law to this loop. not all independent. Equation (3) may be obtained by adding equations (1) and (2). Hence we retain only equations (1) and (2). Ve now apply Kirchhoff's current law to the upper junction point in the figure. Ve have 1 = 11 + 12. Hence we replace i by 11 + i2 in (1), retain (2) as is, and obtain the linear system 0.01 ii + 0.01 i 2 + 20i1 = 100, (4) 0.02 i 2 - 20i 1 + 40i 2 = 0. The initially zero currents g1ve the I.C.'s i 1 (0) = 0, i 2 (0) = 0. Ve introduce operator notation and write (4) In the form (0.01D + 20)i 1 + 0.01Di 2 = 100, -20i 1 + (0.02D + 40)i 2 = 0. (5) We apply the operator 0.02D + 40 i 2 = 0. (5) We apply the operator 0. Linear Differential Equations 399 [(0.01D + 20)(0.02D + 40) + 0.2Djil - (0.02D + 40)+0.2Djil - (0.02D + 40)100 or (.0002D 2
+ D + 800)i l = 20,000,000. (6) The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 5000m + 4,000,000 = 0, with roots m = -1000, -4000. Using this information and the method of undetermined coefficients, we see that the solution of (6) 1S -1000t -4000t 1 1 = c 1 e + c 2 e + 5. (7) Ve use the alternative procedure of page 293 of the text to find i 2. Returning to (5), we multiply the first equation by 2 and then subtract the second equation by 2 and then subtract the second equation from this, obtaining .02Di 1 + 60i 1 - 40i 2 = 200. Note that this contains 1 2 but not Di 2 - 4000t 1 = c 1 e + c 2 e + 5. (7) Ve use the alternative procedure of page 293 of the text to find i 2. Returning to (5), we multiply the first equation by 2 and then subtract the second equation from this, obtaining .02Di 1 + 60i 1 - 40i 2 = 200. Note that this contains 1 2 but not Di 2 - 4000t 1 = c 1 e + c 2 e + 5. (7) Ve use the alternative procedure of page 293 of the text to find i 2. Returning to (5), we multiply the first equation by 2 and then subtract the second equation by 2 and then subtract the second equation from this, obtaining .02Di 1 + 60i 1 - 40i 2 = 200. Note that this contains 1 2 but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 200. Note that this contains 1 a but not Di 2 - 4000t 2 = 2000t From it, we have 3i 1 1 2 = +.0005Di 1 - 5. (8) 400 Chapter 7 From (7), we find Di l = -1000cle-l000t 4000c2e-4000t Now substituting i 1 from (7) and Di l = -1000cle-l000t 1 2 = c 1 e - 4000t c 2 e 2 5 + 2. (9) Finally, applying the I. C. 's to (7) and (9), we have c 1 + c 2 + 5 = 0 and c 1 c 2 5 0, from which we find c 1 - + 2 = 2 10 5 Thus we obtain - 3' c 2 = - 3. 10e-1000t 5e-4000t + 5, 1 1 = 3 3 10e-1000t 5e-4000t 5 1 2 = + + 2. 3 6 6. Let x = the amount of salt in tank X at time t, each measured ln kilograms. Each tank always contains 30 liters of fluid, so the concentration of salt in tank X is 3 x O (kg/liter) and that in tank Y is 10 (kg/liter). Salt enters tank X two ways: (a) 1 kg. of salt per liter of brine enters at the rate of 2 liters/min. from outside the system, and (b) salt ln the brine pumped from tank X at the rate of 2 kg./min.; and by (b), it enters at the rate of lo kg/min. Salt only leaves tank X in the brine flowing from tank X in the rate of 4 liters/min. Thus salt leaves tank X at time t. The D.E. for the amount in tank X at time t. The D.E. for the amount in tank X at time t. The D.E. x' = 2 + lo - (1) for the amount of salt in tank X at the rate of 4 liters/min. Hence we obtain the D.E. x' = 2 + lo - (1) for the amount of salt in tank X at time t. The D.E. for the amount of salt in tank X at time t. Since initially there was 30 kg. of salt in tank X and pure water in tank Y, the I.C.'s are x(O) = 30, y(O) = O. Ve introduce operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (1) and (2) in the forms (D + 1 2 5) x 1 2, 30 Y = (3) 2 (D 2) - x + + 15 y = O. 15 Ve apply the operator notation and write the D.E.'s (+ 125)2 - 25 x = 415402 Chapter 7 or (D 2 + 4 D 3) = 415 + 225 x 15. (4) The auxiliary equation of the corresponding homogeneous D.E. is 2 4m 3 m + 15 + 225 - 0 " th 1 1 Wl roots m = -5' - 15. Using this information and the method of undetermined coefficients, we see that the solution of (4) is -t/5 - t/15 x = c 1 e + c 2 e + 20. (5) To find y, we return to (3), multiply the first equation by ;5 ' apply the operator (D + ;5) to the second, and add. After simplification, we obtain (2 4 3) 4 D + 15 D + 225 y = 15. Comparing this with (4) and its solution is -t/5 - t/15 y = c 3 e + c 4 e + 20. (6) Only two of the four constants c 1, c 2, c 3, c 4 ln (5) and (6) are 2 e + 20. Finally we apply the I.C.'s. We have $c_1 + c_2 + 20 = 30$. $-2c_1 + 2c_2 + 20 = 0$. from which we find $c_1 = 10$. $c_2 = 0$. Thus we obtain the solution x = 10e - t/5 + 20. Section 7.3. Page 317. 3 7t and $t - t_1x = e_1 = e_2$. (a) Let $2e^{-7}t_1 + 2c_2 + 20 = 30$. $-2c_1 + 2c_2 + 20 = 0$. from which we find $c_1 = 10$. $c_2 = 0$. Thus we obtain the solution x = 10e - t/5 + 20. Section 7.3. Page 317. 3 7t and $t - t_1x = e_1 = e_2$. (b) Let $2e^{-7}t_1 + 2c_2 + 20 = 30$. $-2c_1 + 2c_2 + 20 = 0$. From which we find $c_1 = 10$. $c_2 = 0$. Thus we obtain the solution x = 10e - t/5 + 20. Section 7.3. Page 317. 3 7t and $t - t_1x = e_1 = e_2$.) = 15e 7t + 6e 7t = 21e 7t . So = 5x + 3y. Now, = 14e 7t , and 4x + y = 4(3e 7t) + 2e 7t = 12e 7t + 2e 7t + 2e 7t = 12e 7t + 2e 7t + 2e 7+ c 2 e 2c 1 e 7t 2c 2 e -t v - The I.C. f(0) = 0 gives 3c 1 + c - o. and the I.C. 2 - , g(0) = 8 gives 2c 1 - 2c = 8. The unique solution of 2 this system in c 1 and c 2 lS c 1 = 1, c 2 = -3. Thus 3e 7t 3e -t x = , 2e 7t 6e -t y = +, Systems of Linear Differential Equations 405 is the solution of the given linear system that satisfies the two initial conditions. 4. The Vronskian determinant of f 1 and f 2 is Wet) = f 1 (t) gl (t) f 2 (t) = f 1 (t)g2(t) - f 2 (t)gl(t). g2(t) Differentiating, we find V'et) = f 1 (t)g2(t) - f 2 (t)gl(t) - f 2 (t)gl(t solution of system (7.67), we have f 2(t) = a 11(t)f 2(t) + a 12(t)g2(t) and g2(t) = a 21(t)f 2(t) + a 22(t)g2(t). Substituting these derivatives into (1), we find W'(t) = f 1(t) [a 21(t)f 1(t) + a 22(t)g2(t)] + g2(t) [a 11(t)f 1(t) + a 22(t)g2(t22 (t) [f 1 (t)q2(t) - f 2 (t)q](t)] = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q](t) = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) - f 2 (t)q1(t)] = Vet), ql(t) q2(t) so we have proved V'(t) = [a 11 (t) + a 22 (t)] [f 1 (t)q2(t) + a 22 (t)]= 5Ae At 2Be'\t, BAe At = 4Ae At B At e, which leads at once to the algebraic system { (5 - A)B = 0, (4 + (-1 - A)B = 0. (1) For nontrivial solutions of this, we must have 5 - A - 2 = 0. 4 - 1 - A This leads to the characteristic equation (2 - 4) + 3 - 0, whose roots are A = 1, A = 3. Systems of Linear Differential
Equations 407 Setting = 1 = 1 in (1), we obtain 4A - 2B = 0 (twice), a nontrivial solution of which is A = 1, B = 2. With these values of A, B, and , we find the nontrivial solution 3t = 0, y = 2e. (2) Setting z = 3 in (1), we obtain 2A - 2B = 0 and 4A - 4B = 0, a nontrivial solution of which is A = B = 1. With these values of A, B, and , we find the nontrivial solution 3t = 0, y = 2e. (2) Setting z = 2 = 3 in (1), we obtain 2A - 2B = 0 and 4A - 4B = 0, a nontrivial solution of which is A = B = 1. With these values of A, B, and , we find the nontrivial solution 3t = 0, y = 2e. (2) Setting z = 2 = 3 in (1), we obtain 2A - 2B = 0 and 4A - 4B = 0, a nontrivial solution of which is A = 1. (3) The G.S. of the given system is a linear combination t 3t t of (2) and (3). Thus we find x = c 1 e + c 2 e, y = 2c 1 e + c 2 e 2. We assume a solution of the form x = Aet, y = Bet, Substituting into the given system, we obtain 'Aet = 5Aet - Bet, Bet = 3Aet + Bet, which leads at once to the algebraic system { (5 -)A - B = 0, 3A + (1 -)B = 0, (1) 408 Chapter 7 For nontrivial solutions of this, we must have 5 - "-1 = 0. 31 - " This leads to the characteristic equation ... + 8 = 0, whose roots are "1 = 4, "2 = 2. Setting" = "1 = 4 in (1), we obtain A - B = 0, 3A - 3B = 0, a nontrivial solution of which is A = B = 1. With these values of A, B, and ", we find the nontrivial solution 4t 4t x = e y = e (2) Setting" = "2 = 2 in (1), we obtain 3A - B = 0 (twice), a nontrivial solution of which is A = 1, B = 3. Vith these values of A, B, and A, we find the nontrivial solution of (2) and (3). Thus we find 4t x = c e 1 2t + c 2 e 4t 2t Y = c 1 e + 3c 2 e 5. "t "t We assume a Solution of the form x = Ae, y = Be. Substituting into the given system, we obtain Systems of Linear Differential Equations 409 Aet = 3Aet + Bet, Bet = 4Aet + 3Bet, which leads at once to the algebraic system { (3 -)A + B = 0, 4A + (3 -)B = 0, (1) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (1) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (1) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (2) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, we must have 3 - -1 = 0. 4 3 - This leads to the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, (3) For nontrivial solutions of this, (3) For nontrivial solutions of this, (3) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, (3) For nontrivial solutions of this, (3) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (3) For nontrivial solutions of this, (3) For nontrivial solutions of this, (3) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (4) For nontrivial solutions of this, (3) For nontrivial solutions of this, (4) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (4) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (4) For nontrivial solutions of the characteristic equation 2 - 6 + 5 = 0, (4) For non

whose roots are 1 = 1, 2 = 5. Setting $= 1 \ln (1)$, we obtain 2A + B = 0, 1 + A + 2B = 0, a nontrivial solution of which is A = 1, B = 2. With these values of A, B, and , we find the nontrivial solution of which is A = 1, B = 2. With these values of A, B, and A + B = 0, A + B + 0, A + B = 0, A + B + 0, Aand, we find the nontrivial solution 5t x = e y = 2e 5t. (3) 410 Chapter 7 The G.S. of the glven system is a linear combination of (2) and (3). Thus we find t x = c 1 e 5t + c 2 e t Y = -2c e 1 5t + 2c 2 e 6. At We assume a solution of the form x = Ae, y = Substituting into the given system, we obtain BeAt. AAe At = 6Ae At B At e, BAe At = 3Ae At + 2Be At, which leads at once to the algebraic system { (6 - A)A - B = 0, 3A + (2 - A)B = 0, (1) For nontrivial solutions of this, we must have 6 - A - 1 = 0, 3A - 3B = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A - 1 = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A - 3B = 0, a nontrivial solution of which is A = 0, 3A - 1 = 0, 3A -1, B = 1. With these values of A, B, and A, we find the nontrivial solution 5t 5t x = ey = e(2) Systems of Linear Differential Equations 411 Setting = 2 = 3 in (1), we obtain 3A - B = 0 (twice), a nontrivial solution of which is A = 1, B = 3. With these values of A, B, and A, we find the nontrivial solution $3t x = ey - 3e^{-3}t$. (3) The G.S. of the given system IS a linear combination of (2) and (3). Thus we find 5t x = c 1 e 3t + c 2 e 5t Y = c 1 e 3t + 3c 2 e 7. We assume a solution of the form x = Aet, y = Bet. Substituting into the given system, we obtain Aet = 3Aet 4Bet, Bet = 2Aet 3Be At, which leads at once to the algebraic system { (3 - A) A - 4B = 0, 2A + (-3 -)B = 0. (1) For nontrivial solutions of this, we must have 3 - 4 - 0 - 3 - A - 2 This leads to the characteristic equation A - 4B = 0 (twice), a nontrivial solution of which is A = 2, B = 1. With these values of A, B, and , we find the nontrivial solution t t x = 2e, y = e. (2) Setting = 2 = -1 in (1), we obtain 2A - 4B = 0 (twice), a nontrivial solution of which is A = 2, B = 1. With these values of A, B, and , we find the nontrivial solution t t x = 2e, y = e. (2) Setting = 2 = -1 in (1), we obtain 2A - 4B = 0 (twice), a nontrivial solution of which is A = 2, B = 1. With these values of A, B, and , we find the nontrivial solution of which is A = 2, B = 1. With these values of A, B, and the nontrivial solution of which is A = 2, B = 1. given system, we obtain Aet = 2Aet B t e, B' e t t t " = 9Ae + 2Be, which leads at once to the algebraic system { (2 -)A - B = 0, 9A + (2 -)B = 0. (1) Systems of Linear Differential Equations 413 For nontrivial solutions of this, we must have 2 - " - 1 = 0. 9A + (2 -)B = 0. (1) Systems of Linear Differential Equations 413 For nontrivial solutions of this, we must have 2 - " - 1 = 0. 9A + (2 -)B = 0. (1) Systems of Linear Differential Equations 413 For nontrivial solutions of this, we must have 2 - " - 1 = 0. 2 + 3i in (1), we obtain -3iA - B = 0, 9A - 3iB = 0, a nontrivial solution of which is A = 1, B = -3i. With these values of A, B, and ", we find the complex solution (2+3i)t - e, -3 - (2+3i)t y - - le . Using Euler's formula this takes the form x = e 2t (cos $3t + i \sin 3t$), y = e 2t (3 sin $3t - 3i \cos 3t$). Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions $2t \cos 3t + c 2 \sin 3t$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$, $y = e 2t (3c | sin 3t - 3c 2 \cos 3t)$ B' e t Ae t At'' = + Be, which leads at once to the algebraic system { (1 -)A - 4B = 0, A + (1 -)B = 0, (1) For nontrivial solution of which is A = 2i, B = 1. With these values of A, B, and, we find the complex solution Systems of Linear Differential Equations 415 x = 2. (1+2i)t, le y (1+2i)t, $2t 2e 2t \cos 2t x = x = and t \cos 2t + 15$. At We assume a solution of the form x = 2e t (-c l sin $2t + c 2 \cos 2t$), t c 2 sin 2t) $Y = e (c l \cos 2t + 15$. At We assume a solution of the form x = Ae, y = Substituting into the given system, we obtain BeAt. AAe At = 4Ae At - 2Be At , BAe At = 5Ae At + 2Be At , which leads at once to the algebraic system { (4 - A)A - 2B = 0, 5A + (2 - A)B = 0, (1) 416 Chapter 7 For nontrivial solutions of this, we must have 4 - " - 2 = 0, 52 - " This leads to the characteristic equation ,, 2 - 6" + 18 = 0, 5A - (1 + 3i)B = 0, a nontrivial solution of which IS A = 2, B = 1 - 3i. With these values of A, B, and ", we find the complex solutions x = 2e(3+3i)t, y = (1 - 3i)e(3+3i)t. Using Euler's formula this takes the form x = 2e 3t (cos $3t + i \sin 3t$), y = (1 - 3i)e(3+3i)t. Using Euler's formula this takes the form x = 2e 3t (cos $3t + i \sin 3t$), y = (1 - 3i)e(3+3i)t. Using Euler's formula this takes the form x = 2e 3t (cos $3t + i \sin 3t$), y = (1 - 3i)e(3+3i)t. $= e \operatorname{Sln} t \operatorname{3t}(.3 - 3 \cos 3t)$. $Y = e \operatorname{Sln} t$ Systems of Linear Differential Equations 417 The G.S. of the system IS thus $x = 2e \operatorname{3t}(c | \cos 3t + c 2 \sin 3t)$, $y = e \operatorname{3t}(c | \cos 3t + c 2 \sin 3t)$. $Y = e \operatorname{Sln} t$ Substituting into the given system, we obtain Aet = 3Aet 2Be At , B ' e '\t At A = 2Ae + 3Be which leads at once to the algebraic system { (3 - ')B = 0, 2A + (3 - ')B = 0, 2A + (3 - ')B = 0, 2A + (3 - ')B = 0, A = i, B = 1. Vith these values of A, B, and '\, we find the complex solution 418 Chapter 7 x. (3+2i)t = e. Using Euler's formula this takes the form. $3t(2..2)x = e \cot t + 1 \sin 2t$. Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions $3t \sin 2t 3t \cos 2t x = -e x = e$ and $3t \cos 2t x = -e x = e$ and $3t \cos 2t x = -e x = e$. s 2t, 3t sin 2t y = e y = e The G.S. of the system IS thus 3t ($. 2 + c 2 \cos 2t$), x = e -c 1 Sln t 3t c 2 sin 2t), y = e (c 1 cos 2t + 18. At Ve assume a solution of the form x = Ae, y = Substituting into the given system, we obtain BeAt. AAe At = 6Ae At 5Be At , BeAt = Ae At + 2Be At , which leads at once to the algebraic system { (6 - A)A - 5B = 0, A + ($2 + c 2 \cos 2t$), x = e -c 1 Sln t 3t c 2 sin 2t -AB = 0. (1) Systems of Linear Differential Equations 419 For nontrivial solution of this, we must have 6 - 5 = 0. 12 - 7 his leads to the characteristic equation A - 5B = 0, A - (2 + i)B = 0, a nontrivial solution of which is A = -5B = 1 - 2. Vith these values of A, B, and we find, the
complex solution x = 5e(4+i)t, y = (i - 2)e(4+i)t. Using Euler's formula this takes the form 5 4t (..) $x = -e \cos t + 1 \operatorname{Sln} t$. Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions 4t 5 4t. $x = -5e \cos t + 1 \operatorname{Sln} t$. $y = 1 - 2e \cos$ $(-2 \cos t - Y = e (\cos t - 420 \text{ Chapter 7 The G.S. of the system IS thus } x = -5e 4t (c l \cos t + c 2 \sin t), y = e 4t [(c 2 - 2c l) \cos t - (c l + 2c 2) \sin t].$ 19. Ve assume a solution of the At Bet. form x = Ae, y = Substituting into the glven system, we obtain Aet 3Aet B At = e, Bet 4Aet B t = e, Which leads at once to the algebraic system r 3 -)A - B = 0 (1) 4A + c 2 \sin t. (-1 -)B = 0. For nontrivial solutions of this, we must have 3 - 1 = 0. 4 - 1 - This leads to the characteristic equation 2 - 2 + 1 = 0, whose roots are 1, 1 (real and equal). Setting = 1 in (1), we obtain 2A - B = 0, 4A - 2B = 0, a nontrivial solution of which is A = 1, B = 2. Vith these values of A, B, and , we find the nontrivial solution t t x = e, y = 2e. (2) Systems of Linear Differential Equations 421 Ye now seek a second solution of the form x = t t (A 1 t + A 2)e + B 2)-A1 - B2 = 0, (4A1 - 2B1)t + (4A2 - B1 - 2B2) = 0. Thus we must have 2A1 - B = 0, 2A - A1 - B = 0, 2A - A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0. Thus we must have 2A1 - B = 0, 4A2 B1 2B2 = 0.find t t 2c 1 e t + c 2 (2t - 1)e t x = c 1 e + c 2 te, y = . 422 Chapter 7 At Be At . 20. Ye assume a solution of the form x = Ae At + 3Be At , which leads at once to the algebraic system { (7 -)A + 4B = 0, -A + (3 -)B - 0. (1) For nontrivial solutions of this, we must have 7 - 4 = o. -1 3 - This leads to the characteristic equation 2 - 10 + 25 = 0, whose roots are = 5, 5 (real and equal). Setting = 5 in (1), we obtain 2A + 4B = o - A - 2B = , 0, a nontrivial solution of the form <math>x = 5t $5t (A \ 1 \ t + A \ 2)e$, $y = (Bit + B \ 2)e$, $St \ 5t \ 5t \ (5A \ 1 \ t + A \ 2)e + 3(B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ 5t \ (5B \ 1 \ t + B \ 2)e$, $5t \ (5B \$ 1) t + (A 2 + 2B 2 + B i) = 0. Thus we must have 2A 1 + 4B 1 = 0, A 2 + 4B 2 A 1 = 0, A 2 + 2B 2 + B 1 = 0, A 2 + 2B 2 + B 1 = 0, A 2 + 2B 2 + B 1 = 0, A 2 + 2B 2 + B 1 = 0, A 1 + 2B 1 = 0, Athe form 2c 1 e 5t c 2 (2t + 1)e 5t x = +, 5t c 2 t e 5t y = -c e . 1 424 Chapter 7 22. Ye assume a solution of the form x = Ae At, y = Bet. Substituting into the given system { (1 -)A - 2B = 0, 2A + (-3 -)B = 0. (1) For nontrivial solutions of this, we must have 1 - 2 = 0 - 3 - 2 = 0 (twice), a nontrivial solution of which is A = 1, B = 1. With these values of A, B, and , we find the nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B = 0 (twice), a nontrivial solution of the form x = -1 + t (A - 2B - 0 (twice)). 1 + A + 2 = (B + 1 + B + 2)e + (A + 1 + B + 2)e +o. 2 Thus we must have 2A 1 2B 1 = 0, 2A 2 2B 2 A 1 = 0, 2A 2 2B 2 A 1 = 0, 2A 2 - 2B 2 - B 1 = 0, 2A 2 - 2B 2
- B 1 = 0, 2A 2 - 2B 2 - $2c 2 te \cdot 23$. Ve assume a solution of the form x = Aet, y = Bet. Substituting into the given system, we obtain AAe At = 3Ae At B At e = +Be, 426 Chapter 7 which leads at once to the algebraic system { (3 - A)A - B = 0, A + (1 -)B = 0, (1) For nontrivial solutions of this, we must have 3 - 1 = 0, 1 + 2 This leads to the characteristic equation -4 + 4 = 0, whose roots are 2,2 (real and equal). Setting = 2 in (1), we obtain A - B = 0, (twice), a nontrivial solution of the nontrivial solution of the form x = 2t 2t (A 1 t + A 2)e, y = (B 1 t + B 2)e. Substituting these into the given system, we obtain 2t $(2A \ 1 \ t + A \ 2) = 2t (B1 \ t + A \ 2) = 2t (B1 \ t + B \ 2) = 2t (A \ 1 \ t + A \ 2) = 4 (B \ 1 \ t + A \ 2) = 2t (A \ 1 \ t + A \ 2) = 0$, $(A \ 1 \ B \ 1) + (A \ 2 \ B \ 2 \ A \ 1) = 0$, $(A \ 1 \ B \ 1) + (A \ 2 \ B \ 1) = 0$, $(A \ 1 \ B \ 1) + (A \ 2 \ B \ 1) = 0$, $(A \ 1 \ B \ 1)$ 0, A 2 - B 2 - B 1 - o. 1 - A simple nontrivial solution of this IS A 1 = B 1 - A 2 - 1, B 2 = O. This leads to the solution of the form x = Ae At, y = te 2t. (3) The G.S. of the glven system is a linear combination of (2) and (3). Thus we find the G.S. In the form 2t + c 2 (t + 1)e 2t x = c 1 e + c 2 te . 26. Ve assume a solution of the form x = Ae At, y = te 2t. BeAt. Substituting into the given system, we obtain AAe At = 2Ae At 4Be At, BAe At = 2Ae At 4Be At, BAe At = 0, A + (-2 - A)B = 0, (1) 428 Chapter 7 For nontrivial solutions of this, we must have $2 - \sqrt{-4} = 0, 1 - 2 - \sqrt{-4} = 0, 1 - 2 - \sqrt{-4} = 0, 0$ (real and equal). Setting h = 0 in (1), we obtain 2A 4B = 0, A - 2B = 0, a nontrivial solution of which is A - 2, B - 1. With - these values of A, B, and h = 0, A - 2B = 0, a nontrivial solution of the nontrivial solution of the form x = Ot Ot (A 1 t + A 2) e = A 1 t + B 2. Substituting these into the given system, we obtain A 1 = 2(A 1 t + A 2) - 4(B 1 t + B 2), B 1 = (A 1 t + A 1) - 2(B 1 t + B 1). These equations reduce to (2A 1 4B 1)t + (A 2 - 2B 2 - B 1) - O. - Systems of Linear Differential Equations 429 Thus we must have 2A 1 4B 1 = 0, A 2 - 2B 2 - B 1 = 0, A 2 - 2B 2 - B 1) - O. - Systems of Linear Differential Equations 429 Thus we must have 2A 1 4B 1 = 0, A 2 - 2B 2 - B 1 = 0, A 2 - 2B 2 - B 1) - O. - Systems of Linear Differential Equations 429 Thus we must have 2A 1 4B 1 = 0, A 2 - 2B 2 - B 1 = 0, A 2 - 2B 2 - B 1 = 0, A 2 - 2B 2 - B 1 = 0, A 2 - 2B 2 - B 1) - O. - Systems of Linear Differential Equations 429 Thus we must have 2A 1 4B 1 = 0, A 2 - 2B 2 - B 2 - B 2 - B 2 - B 2 - B nontrivial solution of this is A 1 = 2, B 1 = 1, A 2 = 1, B 2 = 0. This leads to the solution x = 2t + 1, y = t. (3) The G.S. of the glven system IS a linear combination of (2) and (3). Thus we find x = 2c 1 + c 2 (2t + 1), (4) y = c 1 + c 2 (2t + 1), (4) y = c 1 + c 2 (2t + 1), (4) y = c 1 + c 2 (2t + 1), (4) y = c 1 + c 2 (2t + 1), (5) z = 0. This leads to the solution x = 2t + 1, y = t. (3) The G.S. of the glven system IS a linear combination of (2) and (3). integrating, x = 2y + k1, where k1 is an arbitrary constant. Now substitute x = 2y + k1. Integrating this, we obtain y = k1 + k2, where k2 is a second arbitrary constant. Then since x = 2y + k1, we have x = 2k + k1. Thus we have found x = 2k + k1. Thus we have found x = 2k + k1. Thus we have found x = 2k + k1. y = k 1 t + k 2, 430 Chapter 7 which is G.S. (4). 27. We assume a solution of the form x = Ae At, y = BeAt. Substituting into the given system, we obtain AAe At = 2Ae At + 7Be At, which leads at once to the algebraic system { (-2 - A)A + 7B = 0, 3A + (2 - A)B = 0. (1) For nontrivial solutions of this, we must have -2 - A 7 = 0.32 - A This leads to the characteristic equation A = 1, B = 1. With these values of A, B, and A, we find the nontrivial solution $5t \ 5t \ x = e \ y = e \ (2)$ Setting $A = A \ 2 = -5$ in (1), we obtain 3A + 7B = 0 (twice), a nontrivial solution of which is A = 7, B = -3. Systems of Linear Differential Equations 431 With these values of A, B, and , we find the nontrivial solution of (2) and (3). Thus we find 5t - 5t x = c 1 e + 7c 2 e, y = 5t c 1 e - 5t 3c 2 e (4) Ve apply the I.C.'s x(O) = 9, y(O) = -1 to (4), obtaining c 1 + 7c - 9, c 1 - 3c 2 = -1. The solution of this is c 1 - 2 - 2, c - 1. Substituting these values back into (4), we 2 - get the desired particular solution 5t -5t x = 2e + 7e, y = 2e - 3e. t Be t. 29. Ve assume a solution of the form x = Ae, y = Substituting into the given system, we obtain Aet = 2Aet 8Bet, Bet = Aet + 6Bet, which leads at once to the algebraic system { (2 -)A - 8B = 0, A + (6 -)B = 0, (1) 432 Chapter 7 For nontrivial solutions of this, we must have 2 - 1, -8 = 0, A + (6 -)B = 0, A + (2 - 2i)B = 0, A + (2 - 2i)BWith these values of A, B, and $\$, we find the complex solutions x = 2(i - 1)e(4+2i)t, y = e + t (cos $2t + i \sin 2t$), y = e + t (cos $2t + i \sin 2t$), y = e + t (cos $2t + i \sin 2t$). Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions $4t \sin 2t$), $y = e + t \cos 2t + i \sin 2t$. $= e \text{ and } 4t - \sin 2t$), $x = 2e (cos 2t 4t \sin 2t . Y = e 3)$ Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. Substituting back into (2), we get = 4, c 1 = 1, c 2 = 3. the desired particular solution 4t = 4e (cos2t - 2sin2t), 4t = 4e (cos2t - 2sin2t), 4t = 6Ae At 4Be At this, we must have 6 - A - 4 = 0. 12 - A 434 Chapter 7 2 This leads to the characteristic equation
A - 8A + 16 = 0, whose roots are 4,4 (real and equal). Setting A = 4 in (1), we obtain 2A + 16 = 0, A - 2B = 0, a nontrivial solution of which is A = 2, B = 1. With these values of A, B, and A, we find the nontrivial solution x = 2e 4t, y = 4t e (2) We now seek a second solution of the form x = 4t 4t (A 1 t + A 2)e - 4(B 1 t + B 2)e + 2(B 1 t + B 2)e + 2(B 1 t + B 2)e + 2(B 1 t + A 2)e + 2(B 1 t + A 2)e + 2(B 1 t + B 2)e + 2(B 1 t +2B 2 - B 1 = 0. Thus we must have 2A 1 4B 1 = 0, 2A 2 4B 2 A 1 - 0, -A 1 - 2B 1 = 0, A - 2B 2 - B = 0. 21 Systems of Linear Differential Equations 435 A simple nontrivial solution of this is A 1 = 2, B 1 = 1, A 2 = 1, B 2 = 0. This leads to the solution x = (2t + 1)e 4t, y = te 4t. (3) The G.S. of the glven system is a linear combination of (2) and (3). Thus we find the G.S. in the form 4t 4t 4t 4t x = 2c 1 e + c 2 (2t + 1)e, y = c 1 e + c 2 te. (4) We apply the I.C.'s x(O) = 2, y(O) = 3 to (4), obtaining 2c 1 + c 2 = 2, c 1 + 0 = 3. Thus c 1 = 3, c 2 = -4. Substituting these values back into (4), we get the desired particular solution x = 6e 4t - 4te 4t; or x = 2e 4t - 5te 4t, y = 3e 4t - 4te 4t; or x = 2e 4t - 5te 4t, y = 3e 4t - 5te 4t, y = 3e 4t - 4te 4t; or x = 2e 4t - 5te 4t, y = 3e 4t - 5te 4t, y = 3e 4t - 4te 4t; or x = 2e 4t - 5te 4t, y = 3e 4t - 5te 4t. 4te 4t. 34. We assume a solution of the form x = Ae At, y = BeAt. Substituting into the given system, we obtain AAe At = Ae At 2Be At, BAe At = 8Ae At 7Be At, which leads at once to the algebraic system f(1 - A)A - 2B = 0, A = Ae At 2Be At, A = Ae At, A = Ae At, A = Ae At, characteristic equation A 2 + 6A + 9 = 0, that IS, (A + 3)2 = 0, with roots A = -3, -3 (real and equal). Setting A = -3 in (1), we obtain 4A - 2B = 0, 8A - 4B = 0, a nontrivial solution of the form x = -3t - 3t (A 1 the nontrivial solution of A, B, and A, we find the nontrivial solution -3t - 3t = 0, A - 4B = 0, a nontrivial solution of the form x = -3t - 3t (A 1 the nontrivial solution -3t - 3t = 0, A - 4B = 0, A - 4B+ A 2)e , y = (B 1 t + B 2)e , (-3B 1 t - 3A 2 + A 1)e - 3t - 3t (A 1 t + A 2)e - 2(B 1 t + B 2)e , (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t - 3B 2 + B 1)e - 3t - 3t (A 1 t + A 2)e - 7(B 1 t + B 2)e . (-3B 1 t + B 2)e 4B B 1 = 0, 2 Thus we must have 4A 1 2B i = 0, 4A 2 2B 2 A i = 0, SA 1 - 4B 1 = 0, SA 2 - 4B 2 - B i = 0. A simple nontrivial solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1, B 1 - 2, A 2 = 1/4, B 2 = 0. This leads to the solution of this is Ai = 1. cie, (4) 2c1 e -3t - 3t Y = + 2c2 te. Ve apply the I.C.'s x(O) = 6, y(O) = 8 to (4), obtaining 1 c 1 + 4 c 2 = 6, 2c 1 = S. Thus ci = 4, c 2 = S. Substituting these values back into (4), we get the desired particular solution x = 4e - 3t + 8(t + 1/4)e where we assume t > o. Then w = In t, and dx dt - (:) (:;) = ():. dx dx Thus t dt = dw; and similarly t dy dt . Substituting into the original system, we obtain the constant coefficient system dx = x + y, dw (1) dy = -3x + 5y. dw AW We assume a solution of (1) of the form x = Ae, y = Be AW. Substituting this into (1), we obtain AAe Aw A AW B AW = e + e, BAe AW = 3Ae AW + 5Be AW, which leads at once to the algebraic system f(1 - A)A + B = 0, l-3A + (5 - A)B = O. (2) For nontrivial solutions of this, we must have 1 - A 1 = 0. -35 - A Systems of Linear Differential Equations 439 This leads to the characteristic equation A 2 - 6A + 8 = 0, whose roots are A 1 = 2, A 2 = 4. Setting A = -6A + 8 = 0, whose roots are A 1 = 2, A 2 = 4. Setting A = -6A + 8 = 0, A = 1 (2), we obtain A + B = 0, A = 1, B = 1. With these values of A, B, and A, we find the nontrivial solution of (1), 2w = 2w = e, y = e Setting A = A = 1, B = 3. With these values of A, B, and A, we find the nontrivial solution of (1), 2w = 2w = e, y = e Setting A = A = 1, B = 3. solution of (1), 4w x = e y = 3e 4w. 2w The G.S. of (1) is thus $x = c 1 e + 4w 3c 2 e \cdot We$ must now return to the original system IS 2 4 x = c 1 t + c 2 t, $y 2 4 - c 1 t + 3c 2 t \cdot 38$. The characteristic equation of the given linear system (S) is A 2 - (a 1 + b 2)A + (a 1 b 2)A-a 2 b l) = 0, and its roots are =a l + b 2 * J (a l - b 2) 2 + 4a 2 b l 2 given by A 440 Chapter 7 2 O bserve that if (a 1 - b 2) 2 + 4a 2 b l 2 given by A 440 Chapter 7 2 O bserve that if (a 1 - b 2) 2 + 4a 2 b l > 0, then j (a l - b 2) 2 + 4a 2 b l > 0, the roots A l and A 2 of the characteristic equation are real and distant. Then by Theorem 7.7, the system has two real linearly independent solutions of the stated form. Now note that if a 2 b l > 0, then (a l - b 2)2 + 4a 2 b l > 0, then (a l - b 2)2 + 4a 2 b l > 0. For example, consider the system (5) Ir which a 1 = 3, b 1 = -1, a 2 = 1, 2 b 2 = 0. Then a 2 b 1 = -1 < 0, but (a 1 - b 2) + 4a 2 b 1 = 5 > 0. Since (a 1 - b 2) + 4a 2 b 1 = 5 > 0. Since (a 1 - b 2) + 4a 2 b 1 > 0 is not necessary for a system to have two solutions of the stated type and form. 40. Suppose there exists a nontrivial solution of the form At At x=Ate, y=Bte. (1) If indeed (1) is a solution of the system, then it must satisfy the equations of the system, we must have (A ' t + A)e At A At b B At A = a 1 t e + 1 t e, (B't + B)e At A t b B At A = a 2 t e + 2 t e. These equations quickly reduce to [(a 1 A)A + b 1 B]t A = 0, [a 2 A + (b 2 - A)B]t - B = 0, contradic- tion. Thus our supposition is invalid, so there exists no nontrivial solution of the stated form. Section 7.5 A, Page 340. Here and following, we shall indicate vectors and matrices by placing a bar over the corresponding letter. Thus, for example, we write A, v, etc. 1. (b) A + B = 2 + 7 - 1 + 2 - 4 + 5 1 - 10 + 4 3 - 5 3 + 6 5 3 - 2 + 19 = 11 0 9 4 $2 - 2 - 1 \quad 442 \text{ Chapter } 7 - 5 + 7 \quad 0 - 2 \quad 4 - 3 \quad 2 - 2 \quad 1 \quad (c) \quad A + B = -2 + 6 - 1 - 3 - 3 + 1 = 4 - 4 - 2 \quad 6 - 2 \quad 2 + 1 \quad 5 - 3 \quad 4 \quad 3 \quad 2 - 4(1) - 4(-3) - 4(5) - 4(1) - 4(-3) - 4(5) - 4(1) - 4(-3)$ -2592-476-688 = + + = -25-3-10-12. 12 4 20 36 (c) Ve have 3 -1 2 -1 2 3 4 2 - - -x + 5x 2 - 2x 3 + 3x = - + 5 - 2 + 3 1 4 - 1 5 0 - 3 4 - 2 6 5 Systems of Linear Differential Equations 443 - 3 -5 - 4 - 3 - 15 - 2 15 - 8 6 11 = + + + = . 1 25 0 - 9 17 - 4 - 10 - 12 15 - 11 4. (b) We have -3 - 572 Ax = 0 4 1 3 - 2 1 3 - 2 - 3(2) - 5(3) + 7(-2) - 35 = 0(2) + 4(3) + 1(-2) = -2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = - + 5 - 2 + 3 + 3x = -
+ 5 - 2 + 3 + 3x = - + 5 + 3 + 3x = - + 5 + 3 + 3x = - 10 - 2(2) + 1(3) + 3(-2) - 7 (c) We have 10 - 3xl + x 2Ax = 2 - 54xl + 2x 2 - 312x - x 32(xl + x 2) + 0 - 3(x - x) 2 3 = 2(x 1 + x 2) + 4(x 2 x 3) - 3(x + x 2) + (x 1 + 2x 2) + 2(x 2 - x) 1 3 444 Chapter 7Xl - 2x + 3x 32 = -3x - 4x 4x 312 - 2x + x - 2x 312 5. Ve have 3 - 12xl YlA = 54 - 3, x = x 2, Y = Y 2, c = 4. Then -512x3Y33 - 12 $x_1 + y_1 = 54 - 3x + 2 + y_2 - 51 + 2x + y_2 + 2x + y_3 + (-5y_1 + y_1) + (-1)(x_2 + y_2) + (-3)(x_3 + y_3) + (-5y_1 + y_2 + 2y_3) + (-5y_1 + 2y_2 + 2y_3) + (-5y_1 + y_2 + 2y_3) + (-5y_1 + 2y_2 +$ 5x 1 + 4x 2 3x 3 + 5Y1 + 4y - 3Y 3 2 - 5x + x 2 + 2x 3 - 5y + Y 2 + 2Y 3 1 1 Systems of Linear Differential Equations 445 3 - 1 2 x 1 3 - 1 2 y1 = 5 4 - 3 x 2 + 5 4 - 3 y2 = A x + A y - 5 1 2 x 3 Y 3 Also, 3 - 1 2 4x 1 12x - 4x 2 + 8x 3 1 A(4x) = 5 4 - 3 x 2 + 5 4 BA = [-1 - :] [: -:] 2 [(-1)(5) + (8)(4) (-1)(-2) + (8)(3)] - (2)(5) + (-5)(4) (2)(-2) + (-5)(3) [27 26] = -10 - 19 = [: -:] [: 3 2 -:] 4. AE 2 -1 [(6)(1) + (-2)(-1) (5)(4) + (-2)(-3) [: 20 11 21]. = 11 12 26 B A is not defined. Systems of Linear Differential Equations 447 3 (1)(3) + (-3)(0) + (4)(5)(2)(1) + (-1)(2) + (-3)(3)(-6)(1) + (0)(2) + (1)(3) = (1)(1) + (-3)(2) + (4)(3) - 2 4 6 0 1 2 7. A B-1 3 5 3 -2 -1 020 542 -5 -6 -3 -4 -6 9 -11 -14 1 (2)(3) + (-1)(1) + (-3)(4) (-6)(2) + (0)(1) + (1)(4)(-2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-1)(2) + (-3)(3) + (-3)(2) + (-3)(3) + rows in A,3, the product B A is not defined. Systems of Linear Differential Equations 449 1 2 -1 11. -2 = A A -2 1 3 A = 2 0 1 (1)(2)+(2)(1)+(-1)(0) (-2)(2)+(1)(1)+(3)(0) (2)(2)+(0)(1)+(1)(0) (-2)(2)+(1)(1)+(2)(2)+(-1)(1) -5 4 4 (-2)(-1)+(1)(2)+(2)(2)+(-1)(2) +(2)(2)(2)+(-1)(2 (4)(0)(2)(2)+(-3)(1)+(8)(0)(4)(2)+(4)(1)+(-1)(0)(4)(1)+(4)(-2)+(-1)(2)-5-6-6(4)(2)+(4)(1)+(-1)(0)=24112(4)(-1)+(4)(3)+(-1)(0)-6128233233-212. T] e have A = A A = 1-2 1 - 2 + (3)(3) + (-1)(-3)(3) + (-1)(-3)(3) + (-1)(-1)(-3)(3) + (-1)(-1)(-3)(3) + (-1)(-1)(-3)(3) + (-1)(-1)(-3)(3) + (-1)(-3)(3) +
(-1)(-3)(3) + (-1)(-3)(4 0 2 - 16 - 10 - 8 14. 13. A = [::]. We have cof A = [::]; adj A = (cof A)T = [::]; and IAI - [::] = -1. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-5 3]. 2 - 1 A - [-1 5] = 2. Thus A - 1 = rlT(ad j A) - [-1 5] = 2. Thus A - 1and A 4 2 (cof A) - 1 1 - 2 = 1 = 2. - -6 - 2 10 2 2 3 4 1 -6 Thus - -1 = (adj A) 1/2 1/2 A - -1 - IAI -3 -1 5 22. We have 7 1 3 1 3 7 0 1 0 1 2 7 1 3 1 3 7 18 -6 2 1 2 0 adj A = -T -6 and A 7 2. (cof A) = 3 -1 = 3 1 = 2 -1 1 1 3 3 9 -3 1 Thus A -1 = (adj A) = -4 3/2 -1/2 IA I 1 -1/2 1/2 Section 7.5C, Page 356. 1. a. Ve must show there exist numbers c 1, c 2, c 3, not all zero, such that - - - This is c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. 3 13 - I 0 c 1 - 1 + c 2 5 + c 3 = 0. 2 - 4 0 454 Chapter 7 This IS equivalent to the homogeneous linear system 3c 1 + 13c + 2c 3 = 0.2 - 4 + 0.2 + 2.2 = 0.2 + 2.2 + 2.2 = 0.2 + 2.2 + 2.2 = 0.2 = 0.2 + 2.2 = 0.2 + 2.2 = 0.2 + 2.2 = 0.2 + 2.2 = 0.2 = 0.2 + 2.2 = 0.2 = 0.2 = 0.23 13 2 - 15 4 - 0. - 2 - 4 - 5 Hence, by Theorem A (text, page 353), the system (1) has a nontrivial solution for c 1, c 2, c 3. For example, one solution IS c 1 - 3, c 2 = -1 c 3 - 2, - - Thus vi' v 2 ' v 3 are linearly dependento 2. Here if - then a. we must show that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0, c 1 = c 2 = c = 0. Thus, we suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 + c 3 v 3 = 0. Thus value are suppose 3 2 1 0 0 c 1 1 + c 2 0 c 3 - 1 - 0 - 0 3 1 0 This lS equivalent to the homogeneous linear system 2c 1 + c 2 - 0, - c c 3 = 0, (1) 1 3 c 2 + c 3 = 0, Systems of Linear Differential Equations 455 the determinant of which is 2 1 0 1 0 - 1 - 5 f- 0. - 0 3 1 Thus by Theorem A (text, page 353), the system (1) has only the trivial solution c 1 = c 2 = c 3 = 0, and so - - v 1, v 2, v 3 are linearly independent. 3. a. The glven vectors are lineraly dependent if and only if there exist numbers c 1 ' c 2 ' c 3 ' not all zero, such that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. (1) This 1S k 1 3 0 c 1 2 + c 2 + 7 c 3 - 0. (2) - c + 2 c 2 8 c 3 = 0, 1 there exist numbers c 1 ' c 2 ' c 3 ' not all zero, such that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. (1) This 1S k 1 3 0 c 1 2 + c 2 + 3 c 3 = 0, 2 c 1 + c 2 + 7 c 3 - 0. (2) - c + 2 c 2 8 c 3 = 0, 1 there exist numbers c 1 ' c 2 ' c 3 ' not all zero, such that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. (1) This 1S k 1 3 0 c 1 2 + c 2 + 3 c 3 = 0, 2 c 1 + c 2 + 7 c 3 - 0. (2) - c + 2 c 2 8 c 3 = 0, 1 there exist numbers c 1 ' c 2 ' c 3 ' not all zero, such that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. (1) This 1S k 1 3 0 c 1 2 + c 2 + 7 c 3 - 0. (2) - c + 2 c 2 8 c 3 = 0, 1 there exist numbers c 1 ' c 2 ' c 3 ' not all zero, such that c 1 v 1 + c 2 v 2 + c 3 v 3 = 0. (1) This 1S k 1 3 0 c 1 2 + c 2 + 3 c 3 = 0. (1) This 1S k 1 determinant of coefficient of which is 456 Chapter 7 k 2 -1 1 3 -1 7 = -6k + 18. 2 -8 (3) By Theorem A (text, page 353), the system (2) has a nontrivial solution for c 1, c 2, c 3, and hence there exist c 1, c 2 ' c 3, not all zero, such that (1) holds, if and only if the determinant (3) is zero. Thus we have -6k + 18 = 0, and hence k = 3. 4. b. Note that 41(t) $-22(t) - 3(t) = 4 \sin t + 4 \cos t 8 \sin t + 4 \cos t 8 \sin t + 2 \cos t 2 \sin t + 4 \cos t 8 \sin t + 2 \cos t 2 \sin t + 4 \cos t 0 = 0$ o Thus there exists the set of three numbers 4, -2, -1, none of which are zero, such that 41(t) + (-2)2(t) + (-1)3(t) = 0 for all t such that a t b, for any a and b. Therefore 1' 2' and 3 are linearly dependent on a t b. Systems of Linear Differential Equations 457 5. b. Suppose there exist numbers c1 and c2 such that c11(t) +
c22(t) = o for all t on a < t < b. -c e = 1 From this, we have both 2c1 + -3t0, 6c1 e 3t + 3c 20, c2 e = = and(1) - 2c + 6c 2 e - 3t0, 3t - 3c0. = c1 e = 12 From the former of (1), 7c 2 e - 3t = 0on a t < b, so 0; and from the latter of (1), 7c1 e 3t 0 a c 2 = = on t b, so c 1 = o. Thus if cll(t) + c22(t) = o for all t on a t b, we must have c 1 = c 2 = 0, and hence ;1 and ;2 are linearly independent on a < t < b, for any a and b. Section 7.5D, Page 367. 1. The characteristic equation IS 1 - (2 = 0, 32 -)(1 + 1)((-4) = 0)Thus, the characteristic values are $\lambda = -1$ and 4. 458 Chapter 7 The characteristic vectors corresponding to A = -1 have components xl and x 2 must satisfy the system xl + 2x 2 = -x 2x 1 + 2x 2 = -x 3x 1 + 3x 3 = -x 3x 3 = characteristic vectors corresponding to = -1 are [_:] for every nonzero real k. The characteristic vectors corresponding to A = 4 have components xl and x 2 must satisfy the system xl + 2x 2 = 4x 1 - 3x + 2x 2 = 0 1 or 3x 1 + 2x 2 = 0 1 or 3x 1 + 2x 2 = 0 1 or 3x 1 + 2x 2 = 4x 2 3x 1 - 3x + 2x 2 = 0 1 or 3x + 2x + 2x + 2 = 0 1 or 3x + 2x + 2x + 2 = 0 1 or 3x + 2x + 2x + 2 = 0 1 or 3x + 2x + 2 = 0 1 or 3x + 2x + 2 = 0 1 or 3x + 2x + 2 = 0 1 or 3x + 2 a + 2 We find xl = 2k, x = 3k for every real k. Hence the characteristic vectors corresponding to A = 4 are [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = 5 and A = -3. The characteristic vectors corresponding to A = 5 have [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = 5 and A = -3. The characteristic vectors corresponding to A = 5 have [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = 5 and A = -3. The characteristic vectors corresponding to A = 5 have [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = 5 have [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = 5 have [2k] for 3k every nonzero real k. 2. The characteristic vectors corresponding to A = -3. The characteristic vectors corresponding to A = -3. components xl and x 2 such that [: -:]:: Xl = 5 X 2 Thus xl and x 2 must satisfy the system 3x 1 + 2x 2 = 5x l - 2x + 2x 2 - 0, 1 - or 6x l - x 2 = 5x 2 x l - x 2 = 0. We find xl = x 2 = k for every real k. Hence the characteristic vectors corresponding to A = -3 have components xl and x 2 such that [: -:1:: Xl = (-3) X 2 Thus xl and x 2 must satisfy the system 3x 1 + 2x 2 = -3x 1. Thus we find x = k, x 2 = -3x 1. Thus we find x = k, x 2 = -3x 1. Thus we find x = k, x 2 = -3x 1. Thus we find x = k, x = -3x 1. Thus we find x = k, x = -3x 1. Thus we find x = k, x = -3x 1. Thus we find x = k, x = -3x 1. Thus we find x = k, x = -3x 1. Thus we find x = k, x = -3x 1. $151 - \sqrt{1 - 123} - \sqrt{-4} = 0, 41 - 4 - \sqrt{1 - 1(2)(+ 3)} = 0.$ Thus, the characteristic values are $\sqrt{-1}(1 - \sqrt{-1})(\sqrt{2})(+ 3) = 0.$ Thus, the characteristic values are $\sqrt{-1}(1 - \sqrt{-1})(\sqrt{$ $1 \times 2 \times 4 = 0, 2 \times 1 + 2 \times 3 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 3 = 0, 2 \times 1 + 2 \times 3 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 3 = 0, 2 \times 1 + 2 \times 4 = 0, 2 \times 1 + 2 \times 1 +$ k, x 3 = k for every real k. Hence the characteristic k vectors corresponding to = 1 are k for every nonzero k real k. The characteristic vectors corresponding to = 2 have components xl' x 2 ' x 3 such that 462 Chapter 7 1 1 -1 xl xl 2 3 -4 x 2 = 2 x 2 4 1 -4 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system xl + x 2 x 3 = 2x 1, -x + x 2 x 3 = 0, 1 2x 1 + 3x 3 = 0, 1 2x 2 4x 3 = 2x 2, 2x 1 + x 2 4x 3 = 0, 4x 1 + x 2 4x 3 = 0, 5x 2 = 2x 3, or 4x 1 + x 2 6x 3 = 0. From the first and second, 3x 1 - 3x 3 = 0, so x 2 = 2x 3. Thus we find x = k, x 2 = 2k, x 3 = k for every real k. Hence the characteristic vectors k corresponding to A = 2 are 2k for every nonzero real k. K The characteristic vectors corresponding to A = -3 have components xl' x 2 ' x 3 such that 1 1 -1 xl xl 2 3 -4 x 2 = -3 x 2 4 1 -4 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system Systems of Linear Differential Equations 463 Xl + x 2 x 3 = 0, 2x 1 + 3x 2 4x 3 = 0, -4x 1 + x 2 4x 3 = 0, -4x 1 + x 2 x 3 = 0. Regarding the first and second equations as two equations ln x 2 and x 3 ' we find x 2 = 7x 1, x 3 = llx 1. Thus we find x 1 = k, x 2 = 7k, x 3 - llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every real k. Hence the k characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every real k. Hence
the k characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every real k. Hence the k characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real k. 8. The characteristic vectors corresponding to $\lambda = -3$ are 7k llk for every nonzero real 2x 3, or -3x + 6x 2 + 4x 3 = 0.11 The first two of these are essentially the same. We write the second and third as Xl + x 2 = -x 3, 3x 1 + 6x 2 = 4x 3, and solve for xl and x 2 in terms of x 3 - 3k, find 10k, = -3x 3 + x 2 = -7k, x 3 = -7k, x = -7k, x3 = 3x 3, or -3x - 6x 2 + 3x 3 = 0. 1 From the second equation, x 3 = -xl; and then the first and third become -x - x 2 = 0, 1 -6x - 6x 2 = -xl. Thus we find xl = k, x 2 = -xl. Thus we find xl = k, x 2 = -xl. Thus we find xl = k, x 2 = -xl. Thus we find xl = k, x 2 = -xl. components xl' x 2 ' x 3 such that 1 -1 -1 xl xl 1 3 1 x 2 = 5 x 2 -3 -6 6 x 3 x 3 466 Chapter 7 Thus xl' x 2 ' x 3 = 5x 1 , -4x x 2 x 3 = 5x 2, x 1 2x 2 + x 3 = 5x 2becomes 3x 1 + x 3 - 0, from which x 3 = -3x 1. Thus we find xl = k, x 2 - -k, x 3 = -3k for every real k. Hence the characteristic equation $lS 1 - A 1 1 - A \circ 1 0 1 - 0$, $-1 - A \circ 1 0 1 - A \circ 1 - A \circ$ (+ 1) = 0. Thus, the characteristic values are A = 1, 2, -1. Systems of Linear Differential Equations 467 The characteristic vectors corresponding to A - 1 have components xl' x 2 ' x 3 must satisfy the system xl + x 2 - xl' x 2 - 0, - xl + x 3 - x 2 ' xl x 2 + x 3 - 0, - x o. - From these, we see at once that x 2 = 0 and then x 3 = -xl. Thus we find xl = k, x 2 = 0, x 3 = -k for every real k. Hence the characteristic vectors corresponding to A - 2 have components xl' x 2 ' x 3 such that 1 1 0 xl xl 1 0 1 x 2 - 2 x 2 - 0 1 1 x 3 x 3 468 Chapter 7 Thus xl' x 2 ' x 3 must satisfy the system xl + x 2 - 2x 1, xl + x 2 = 0 - xl + x 3 = 2x 2, xl 2x 2 + x 3 = 0, x 2 + x 3 2×3 such that $1 \times 2 = -x \times 3 = 0$, $x \times 2 + x \times 3 = 0$, $x \times 2 + x \times 3 = -x \times 3$, or $x \times 2 + x \times 3 = 0$, $x \times 2 + x \times 3 = 0$, $x \times 2 + x \times 3 = 0$, $x \times 2 + x \times 3 = -x \times 2$, $x \times 3 = -x \times 3 = 0$, $x \times 2 + x \times 3 = -x \times 2$, $x \times 3 = -x \times 3 =$ (1) x 2 0 - 1 5 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy xl + 3x 2 6x 3 = xl' 3x 2 6x 3 - 0, - 2x 2 + 2x 3 = x 2 ' x 2 + 2x 3 = 0, -x + 5x 3 = x 3, or -x + 4x 3 = 0. 2 - Thus we find xl - k, x 2 = x 3 = 0 for every real k. Hence - k the characteristic vectors corresponding to A - 1 are 0 o for every nonzero real k. The characteristic vectors corresponding to A = 3 have components xl' x 2 ' x 3 such that 1 3 -6 xl xl 0 2 2 x 2 - 3 x 2 - 0 - 1 5 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system xl + 3x 2 6x 3 = 0, 1 2x 2 + 2x 3 = 3x 2, -x + 2x 3 = 0, 2 - x + 5x 3 - 3x 3, or -x + 2x 3 = 0, 2 - x + 5x 3 - 0. 2 - 2 - The last two equations are identical. From the second, x 2 = 2x 3. Then the first equation becomes -2x l + 6x 3 - 6x 3 = 0, from which xl = 0. Thus we find xl = 0, x 2 = 2k, x 3 = k for every real k. Hence the characteristic vectors o corresponding to A = 3 are 2k for every nonzero real k. k Systems of Linear Differential Equations 471 The characteristic vectors corresponding to A - 4 have components xl' x 2 ' x 3 such that 1 3 -6 xl xl 0 2 2 x 2 = 4 x 2 0 -1 5 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system xl + 3x 2 6x 3 = 0, 2 - x + 5x 3 - 4x 3, or -x + x 3 = 0, 2 - x + 5x 3 - 4x 3, or -x + x 3 = 0, 2 - x + 5x 3 - 4x 3, or -x + x 3 = 0, 2 - x + 5x 3 - 4x 3 Thus we find xl = -k, x 2 = k, x 3 = k for every real k. Hence the characteristic vectors corresponding to A = 4 - k are k for every nonzero real k. k 14. The characteristic vectors corresponding to A = 4 - k are k for every nonzero real k. k 14. The characteristic vectors corresponding to A = 4 - k are k for every nonzero real k. k 14. The characteristic vectors corresponding to A = 4 - k are k for every nonzero real k. k 14. The characteristic vectors corresponding to A = 4 - k are k for every nonzero real k. k 14 - 10 = 0 or (-1) = 0 = -10 = 0. 10x 2 + 29x 3 = x 3 or 5x 1 10x 2 + 28x 3 = 0. Adding four times the first to the second, we find x = 2x 2. Thus we find x = 2x 2. are k for every nonzero real k. o The characteristic vectors corresponding to A - 1 have components xl' x 2 ' x 3 such that -2 6 -18 xl xl 12 -23 66 x 2 = (-1) x 2 5 -10 29 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system -2x + 6x 2 = -x 2, 12x 1 22x 2 + 66x 3 = -x 2, 12x 1 22x 2 + 66x 3 = -x 2, 12x 1 22x 2 + 66x 3 = -x 3, or 5x 110x 2 + 30x 3 = 0. Adding 3/5 the third equation to the first glves 2x 1 = 0, so xl = 0. Then the first equation becomes 6x 2 - 18x 3 = 0, from which x 2 = 3x 3. Thus we find xl = 0, x 2 = 3k, x 3 = k, for every real k. Hence the characteristic ovectors corresponding to A = -1 are 3k for every nonzero k real k. 474 Chapter 7 The characteristic vectors corresponding to A - 4 have components xl' x 2 ' x 3 such that -2 6 -18 xl xl 12 -23 66 x 2 = 4 x 2 5 -10 29 x 3 x 3 Thus xl' x 2 ' x 3 must satisfy the system -2x + 66x 3 - 4x 2, 12x 1 27x 2 + 66x 3 -
4x 2, 12x 1 27x 2 + 66x 3 - 4x 2, 12x 1 27x 2 + 66x 3 - 4x 2, 12x 1 27x 2 + 66x 3 - 4x 2, 123 - 0, 1 - x 1 - 2x 2 + 5x 3 = 0. Regarding these as two equations in xl and x 2 ' we find xl - x 3, x 2 = 2x 3. Letting x 3 - k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every nonzero -k real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every real k. Hence the characteristic, - k vectors corresponding to A - 4 are -2k for every nonzero -k real k. Systems of Linear Differential Equations 475 Section 7.6, Page 377. 1. The characteristic equation of the coefficient matrix [5-2] X = is 4-15-, -2 = 0, -1-, IX - XII = 4 Expanding the determinant and simplifying, this takes the form A 2 - 4A + 3 = 0 with roots A l = 1, A 2 = 3. These are the characteristic values of A. They are distinct (and real), and so Theorem 7.10 of the text applies. We use equation (7.118) of the text to find corresponding characteristic vectors. Vi th A = A 1 = 1 and a = Ii (1) = a 1, (7.118) becomes a 2 [5-2] a 1 4 - 1 a 2 a 1 = a 2, -, or 4a 1 - a 2 = a 2, 2a 1 = a 2, -, or 4a 1 - a 2 = a 2, -, 476 Chapter 7 a (1) = [:]. A solution 1S X = [:] et, that is, t e x = (*) 2 et a 1 = a 2 = 3a 1 a 1 = a 2 = 3a a d a, (7.118) becomes a 2 [5 -2] a 1 4 - 1 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 5a 1 2a 2 = 3a 1 a 1 = a 2 = 3a 2 a 1 = 3a 2 from which we find that a 1 and a 2 must satisfy 5a 1 2a 2 = 3a 1 a 1 = a 2 corresponding to A 2 = 3 is a (2) = [:]. A solution is x = (**) 3t e x = (**) 3t e Systems of Linear Differential Equations 477 By Theorem 7.10 the solution IS t 3t e e x = c 1 2e t + c 2 3t e where c 1 and c 2 are arbitrary constants. In scalar language, t xl = c 1 e t x 2 = 2c 1 e 3t + c 3t + c 2 = 2c 1 e 3t + c 2 e 3t + c 2 e 5. The characteristic equation of the coefficient matrix A = [::] is 3 - A 1 IX - XII - = 0, 4 3 - .. Expanding the determinant and simplifying, this takes the form A 2 - 6A + 5 = 0 with roots A l = 1, A 2 = 5. These are the characteristic values of A. They are distinct (and real), and so Theorem 7.10 of the text applies. We use equation (7.118) of the text to find corresponding characteristic vectors. 478 Chapter 7 With A = A l = 1 and a = :(1) = °1, (7.118) becomes °2 [::] :: °1 = 1 °2 from which we find that a 1 and a 2 must satisfy 3a 1 + a 2 = a 1 2°1 = -a 2, or 4a 1 + 3a 2 = a 2, 4°1 = -2°2. A simple nontrivial solution is a 1 = 1, a 2 = -2, and thus a characteristic vector corresponding to A 1 = 1 is; (1) = L:]. A solution is x = [.:]et, that 1S, te $x = t - 2e a 1 = _a (2) = With A = A 2 = 5 and a$, (7.118) becomes a 2 [::] :: a 1 = 5 a 2 Systems of Linear Differential Equations 479 from which we find that a 1 and a 2 must satisfy 3a 1 + a 2 = 5a 1 2a 1 = a 2, or 4a 1 + 3a 2 = 5a 2, 4a 1 - 2a 2. - A simple nontrivial solution is a 1 = 1, a 2 e 5t x 2 = 1 480 Chapter 7 6. The characteristic equation of the coefficient matrix A = [: -:1 is 6 - A -1 IX - All = = 0, 3 2 - A Expanding the determinant and simplifying, this takes the form A 2 - 8A + 15 = 0 with roots A l = 3, A 2 = 5. These are the characteristic values of A. They are distinct (and real), and so Theorem 7.10 of the text applies. We use equation (7.118) of the text to find corresponding characteristic vectors. With A = A = 3 and a = a(1) = 1 a 1, (7.118) becomes a 2 [6 -1] a 1 3 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1, 3a 1 = a 2, or 3a 1 = a 2, or 3a 1 = a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1, 3a 1 = a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 = 3a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from which we find that a 1 a 2 a 2 a 1 = 3 a 2 from solution is 01 = 1, a = 3, and thus a characteristic vector corresponding to A 1 = 3 is a (1) = [:] A solution is x = [:] 3t, that is, 3t = x = (*) 3e = 3t, (7.118) becomes $2 : -:] :: ^{1} = 5 ^{2}$ from which we find that 01 and 02 must satisfy $6^{\circ}1 ^{2} = 5^{\circ}1 ^{\circ}1 = 2^{\circ}$, or $3^{\circ}1 + 2a = 5^{\circ}2 ^{\circ}1 = 2$. A simple nontrivial solution is 01 = 02 = 1, and thus a characteristic vector corresponding to A 2 = 5 1S a (2) = I:]. A solution is x = [:]e 5t, that 1S, 5t e x = (**) 5t e 482 Chapter 7 By Theorem 7.10 the solutions (*) and (**) are linearly independent, and a general solution IS 3t 5t e e x = c 1 3e 3t + c 2 5t e where c 1 and c 2 are arbitrary constants. In scalar language, 3t xl = c 1e 3t x 2 = 3c 1 e 5t + c 2 e 9. The characteristic equation of the coefficient matrix A = [: -:] is 1 - A -4 II - All = = 0, 1 1 - A Expanding this determinant and simplifying, this takes the form A 2 - 2A + 5 = 0 with roots 1 % 2i. These are the characteristic values of I. They are distinct conjugate complex numbers, and Theorem 7.10 applies. We use equation (7.118) of the text. Systems of Linear Differential Equations 483 W' it h, = , 1 = 1 + 2 i and a = a (1) = a 1, (7.118) a 2 becomes [:-:] :: a 1 = (1 + 2i) a 2 from which we find that a 1 and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 =
2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 1 2i a 1 = -4a, 2 or a 1 + a 2 = (1 + 2i)a 2, a 1 = 2i a 2. A simple nontrivial solution is a 1 = 2i, a 2 = 1, and a 2 must satisfy a 1 4a 2 = (1 + 2i)a 2 a 2 a = (1 + 2i)a 2 a = (1 + 2i)a 2 a 2 a = (1 + 2i)a 2 a = (1 + 2 thus a characteristic a (1) = [2:]. vectors corresponding to Ai = 1 + 2i 18 " [2i] (1+2i)t A 801ut1on 18 x = 1 e, that 18, 2 " (1+2i)t A 801ut1on 18 x = 1 e, that 18, 2 " (1+2i)t a we proceed as in Example 7.18 of Section 7.4C and apply i () Euler's formula $e = \cos(1) + i \sin(2)$ the real and simplifying, this takes the form Xl = 2et (-sin 2t + i cos 2t), x 2 = et(cos 2t + i sin 2t). The real and imaginary parts of this solution are themselves solutions, so we obtain -2e $t \sin 2t$, $t x l = x l = 2e \cos 2t$, and $t \cos 2t$, $t \sin 2t \cdot x 2 = e x 2 = e$ These two solutions are linearly independent, so a G.S. may be written Xl = 2e t (c l cos 2t + c 2 cos 2t), $x 2 = e t (c l \cos 2t + c 2 cos 2t)$, x 2 = e t= [: -:1 is 2 - A -3 IA - All = = 0, 3 2 - A -3 IA - A -3 I becomes [: -:1 :: a 1 = (2 + 3i) , a 2 from which we find that a 1 and a 2 must satisfy 2a 1 3a 2 = (2 + 3i)a 1 , -a = 1 a 1 ' 2 or a 1 = 1 a 2 . 3a 1 + 2a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = 1, and thus a characteristic vector corresponding to '\1 = 2 + 3i is 486 Chapter 7 a (1) = [:]. A solution 1S x = [:]e(2+3i)a 1 , . -a = 1 a 1 ' 2 or a 1 = 1 a 2 . 3a 1 + 2a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = 1, and thus a characteristic vector corresponding to '\1 = 2 + 3i is 486 Chapter 7 a (1) = [:]. A solution 1S x = [:]e(2+3i)a 1 , . -a = 1 a 1 ' 2 or a 1 = 1 a 2 . 3a 1 + 2a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = 1, and thus a characteristic vector corresponding to '\1 = 2 + 3i is 486 Chapter 7 a (1) = [:]. A solution 1S x = [:]e(2+3i)a 1 , . -a = 1 a 1 ' 2 or a 1 = 1 a 2 . 3a 1 + 2a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a 2 , A simple nontrivial solution is a 1 = i, a 2 = (2 + 3i)a x = (*) (2+3i)t e. As in our solution to Exercise 9 just preceding this solution, we now proceed as in Example 7.18 of Section 7.4C. We apply Euler's formula e i () = cos () + i sin () to each component of (*). We have xl = e 2t (-sin 3t + i cos 3t), x 2 = e 2t $(\cos 3t + i \sin 3t)$. The real and imaginary parts of this solution are themselves solutions, so we obtain 2t sin 3t 2t cos 3t, xl = -e x 2 = e Systems of Linear Differential Equations 487 These two solutions are linearly independent, so a G.S. may be written 2t (. 3 3) xl = e -c 1 Sln t + c 2 cos t, X 2 - e 2t (c 1 co s $3t + c 2 \sin 3t$). 12. The characteristic equation of the coefficient matrix A = [: -:] is 5 - , -4 IX - , 13 = 0 with roots 3 2i. These are the characteristic values of X. They are distinct conjugate complex numbers, and Theorem 7.10 applies. We use equation (7.118) of the text. With $a_{i} = a_{i} = 3 + 2i$ and $a = a_{i} = (1) = a_{i} = 1$, (7.118) a 2 becomes [: -:1 :: a 1 = (3 + 2i) a 2 488 Chapter 7 from which we find that a 1 and a 2 must satisfy 5a 1 4a 2 = (3 + 2i) a 2, a 1 = (1 + i) a 2. A simple nontrivial solution is a 1 = 2, a 2 = 1 - 1, and thus a characteristic vector corresponding is a (1) = [1 2 i]. A solution is x = $[1 : to "1 = 3 + 2i](3+2i)t e_{, 1}$ that . 1S, 2 e (3+2i)t x = (*) (1 - i)e(3+2i)t As in our solutions to Exercises 9 and 10, we apply Euler's formula e i (J = cos (J + i sin (J to each component of (*). Ve have Xl 2 (3+2i)t = e_{, x 2} (1 .) (3+2i)t = e_{, x 2} (+ i sin2t), $x 2 = e 3 t[(\cos 2t + \sin 2t) + i(\sin 2t - \cos 2t)]$. Systems of Linear Differential Equations 489 The real and imaginary parts of this solution are themselves solutions, so we obtain $3t 2 3t \cdot 2x = e (\cos 2t x 2 = e (\cos 2$ written XI = 2e 3t (c l cos 2t + c 2 sin 2t), x 2 = e 3t [c l (cos 2t + sin 2t) + c 2 (sin 2t - cos 2t), where c 1 and c 2 are arbitrary constants. 14. The characteristic equation of the coefficient matrix I = [: -:1 is 4 - " -5 II - "II = = 0, 1 6 - " Expanding the determinant and simplifying, this takes the form ,,2 - 10" + 29 = 0 with roots 5 2i. These are the characteristic values of A. They are distinct conjugate complex numbers, and Theorem 7.10 applies. We use equation (7.118) of the text. 490 Chapter 7 With" = "1 = 5 + 2i and a = a (1) = a 1, (7.118) a 2 becomes [:-:] :: a 1 - (5 + 2i) a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (1 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (1 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (1 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (1 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (1 + 2i)a 1 + 5a 2 =
0, or a 1 + 6a 2 = (5 + 2i)a 1, (2 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (2 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (2 + 2i)a 1 + 5a 2 = 0, or a 1 + 6a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 = (5 + 2i)a 1, (3 + 2i)a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 from which we find that a 1 and a 2 must satisfy 4a 1 5a 2 from which we find that a 1 and a + 2i)a 2 , a 1 + (1 - 2i)a 2 = o. A simple nontrivial solution is a 1 = 5, a 2 = -(1 + 2i), and thus a characteristic vector corresponding to Ai = 5 + 2i IS a (1) = [5]. A solution is x = -(1 + 2i) [5] e(5+2i)t, that IS, -(1 + 2i) 5 e(5+2i)t x = (*) (1+2i)e(5+2i)t. As in our solution to Exercises 9, 10, and 12 we apply i () Euler's formula $e = \cos(0) + i \sin(0)$ to each component of (*). We have Systems of Linear Differential Equations 491 Xl 5 (5+2i)t = e, x 2 = (1+2i)e(5+2i)t, and applying the formula and simplifying, this takes the form 5t xl = 5e (cos 2t + i sin 2t), X 2 = e 5t [(cos <math>2t + i sin 2t) + i (2 cos <math>2t + i sin 2t) + i (2 cos <math>2t + i sin 2t). The real and imaginary parts of this solutions, so we obtain 5t xl = 5e (cos <math>2t + i sin 2t) + i (2 cos <math>2t + i sin 2t). $x_1 = 5e \cos 2t$, $X_2 = -e 5 t(\cos 2t - 2 \sin 2t)$, and $5 5t \cdot 2 x_1 - e Sln t$, $X_2 = -e 5 t(2\cos 2t + \sin 2t)$. These two solutions are linearly independent, so a G.S. may be written $5t x_1 = 5e (c \log 2t - 2 \sin 2t) + c 2 (2 \cos 2t + \sin 2t)$. - "-1 IX - III = = 0, 4 -1 - " Expanding the determinant and simplifying, this takes the form A 2 - 2A + 1 = 0 with double root A l = 1. That is the characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equal and so Theorem 7.11 applies. Ve use equation (7.118) to find a characteristic values of A are real and equa $\{31 \text{ satisfies } (A AI)P = a. Thus 73 = \text{satisfies } \{32 [::] - [::] \{31 [:], = \{32 \text{ which reduces to } [::]:= [:], and the "second" solution is fi 1 = 0, fi 2 = -1. Thus we find 13 = [:], and the "second" solution is x = [:]t + [::] \{31 [:], = \{32 \text{ which reduces to } [::]:= [:], and the "second" solution is fi 1 = 0, fi 2 = -1. Thus we find that <math>\{31 \text{ and } \{32 = 1, 4\{31 2\{32 = 2, 494 \text{ Chapter } 7 \text{ A simple nontrivial solution } is fi 1 = 0, fi 2 = -1. Thus we find 13 = [:], and the "second" solution is x = [:]t + [:]t$ Theorem 7.11 the solutions (*)(**) are linearly independent, and a general solution lS t e x = c 1 2 e t te t + c 2 t (2t - l)e where c 1 and c 2 are arbitrary constants. In scalar language, t t x = c 1 e + c 2 t (2t - l)e where c 1 and c 2 are arbitrary constants. In scalar language, t t x = c 1 e + c 2 t (2t - l)e Differential Equations 495 Expanding the determinant and simplifying, this takes the form A 2 - IOA + 25 = 0 with double root A = 5. That is the characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic value which we find that a 1 and a 2 must satisfy 7a 1 + 4a 2 = 5a 1 a 1 = -2a 2, or -a + 3a 2 = 5a 2, a 1 = -2a 2. 1 A simple nontrivial solution is a 1 = -2, a 2 = 1, and thus a characteristic vector corresponding to A = 5 is a = [-:]. A solution 1S X = [-:] e 5t, that 1S, 2e 5t x = (*) 5t e 496 Chapter 7 By Theorem 7.11, a linearly in [1 P] e, ndent solution - At - the form (a t + {3) e, where a = A = 5, and 73 is of {31 satisfies (A AI)/J = a. Thus /J = satisfies {32 [[-::] [::]] {31 [-:1 - 5 = {32 which reduces to [-::-1::: = [-:1. From this we find that {31 and {32 must satisfy 2{31 + 4/3 2 - {31 2{32 = -2 , = 1. A simple nontrivial solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and the "second" solution is {31 = -1, {32 = 0. Thus we find p = [-:], and x = [-:It + [-:] Je 5t, that is, 5t - (2t + 1)e x = (**) te 5t Systems of Linear Differential Equations (*) and (**) are linearly independent, and a general solution 1 S 2e 5t 5t - (2t + 1)e x = (**) te 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In
scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e - c 2 (2t + 1)e, 5t 5t x 2 = c 1 e + c 2, x = c 1 te 5t where c 1 and c 2 are arbitrary constants. In scalar language, 5t 5t x 1 = -2c 1 e + c 2, x = c 1 e + c 2, x = c 1 te 5t where c 1 and c = -2c 1 e + c 2. c 2 te \cdot 19. The characteristic equation of the coefficient matrix I = [: -:1 is 6 -, \ -4 II - ; \ Expanding the determinant and simplifying, this takes the form ; 2 - 8, \ + 16 = 0 with double root ; = 4. That is, the characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. We first use equation (7.118) to - find a characteristic values of A are real and equal and so Theorem 7.11 applies. vector a. 498 Chapter 7 a 1 With A = 4 and a = , (7.118) becomes a 2 [:-:] :: a 1 = 4 a 2 from which we find that a 1 and a 2 must satisfy 6a 1 4a 2 = 4a 2 , or a 1 + 2a 2 = 4a 2 , or a 1 + 2a 2 = 4a 2 , a 1 = 2a 2 - A simple nontrivial solution is x = [:]e 4t, that IS, 2e 4t x = (*) 4t e By Theorem 7.11, a linearly independent solution IS of the form (a t +.8) eAt, where a = [:1,] = 4, and .8 Systems of Linear Differential Equations 499 {31 satisfies (A "1);8 = a. Thus ;8 - satisfies - {32 [[: -:]]; ...] } {31 [:], -4 = {32 which reduces to [: :]: = [:]. From this we find that {31 and {32 must satisfy 2{31 4{32 - 2, {31 2{32 = 1.}}}}} A simple nontrivial solution is $\{31 = 1, \{32 - 0, Thus we find / 3 = [:], and the "second" solution 1S x = [:], t + [:]] e 4t, that IS, (2t + 1)e 4t x = (**) te 4t 500 Chapter 7 By Theorem 7.11 the solutions (*) and (**) are linearly independent, and a general solution 1S 2e 4t (2t + 1)e 4t x = c 1 4t e + c 2, te 4t where c 1 and c 2 are arbitrary constants. In$ scalar language, 2 4t (2t 1)e 4t, $x = c 1 e + c 2 + 4t 4t x 2 = c 1 e + c 2 + 4t 4t x 2 = c 1 e + c 2 te \cdot$ Section 7.7, Page 400. We assume a solution of the characteristic equation of the characteristic equation of the characteristic equation lS 1 - , 1 - 1 23 - 4 = 0.41 - 4 = 0.41 - 4 = 0.41 - 4 = 0.41 - 4 = 0.41 - 4 = 0.41 - 4 = 0.41 - 1.42 = 2.43 = -3. We use A a = A a to find the corresponding characteristic vectors. With A = 1 this is 1 1 -1 a 1 a 1 2 3 -4 a 2 = 1 a 2 4 1 -4 a 3 a 3. Thus a 1 , a 2 , a 3 must satisfy a 1 a 3 = 0, 2a 1 + 2a 2 4a 3 = 0, 4a 1 + a 2 5a 3 = 0. A nontrivial solution of this is a 1 = 1, a 2 = 1, a 3 = 1 (see solution of this is a 1 = 1, a 2 = 1, a 3 = 0, 2a 1 + a 2 5a 3 = 0. A nontrivial solution of this is a 1 = 1, a 2 = 1, a 3 = 1 (see solution of this is a 1 = 1, a 2 = 1, a 3 = 0, 2a 1 + a 2 5a 3 = 0. A nontrivial solution of this is a 1 = 1, a 2 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A nontrivial solution of this is a 1 = 1, a 3 = 0. A n and a corresponding solution lS t 1 e 1 t that is, t e, e 1 t e (*) 502 Chapter 7 With A = 2, Aa = A£1 lS 1 1 - 1 £1 1 a 1 2 3 - 4 £1 2 = 2 a 2 4 1 - 4 £1 3 a 3. Thus ai' a 2, a 3 must satisfy -a + a 2 a 3 = 0, 1 2a 1 + a 2 4£1 3 = 0, 4a 1 + $IS_1 a = 21$ and a corresponding solution $IS_1 2 = 2t_1 that IS_2 te 2e_2 t_2 te (**)$ With A = -3, $A = A a_1 S$ Systems of Linear Differential Equations 503 1 1 -1 a 1 a 1 2 3 -4 f 1 2 = -3 a 2 4 1 -4 a 3 a 3 . Thus a 1 , a 2 , f 1 3 must satisfy $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $4a_1 + a_2 a_3 = 0$, $4a_1 + a_2 a_3 = 0$, $2a_1 + 6a_2 4f_1 3 = 0$, $4a_1 + a_2 a_3 = 0$, $4a_1 + a_2 + a_3 + a_$ 11. Thus a characteristic vector corresponding to A = -3 is 1 a = 7 11 and a corresponding solution IS -3t 1 e 7 -3t that -3t e IS, 7e 11 lle- 3t (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solution IS -3t 1 e 7 -3t that -3t e IS, 7e 11 lle- 3t (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***) By Theorem 7.16, the solutions (*), (**), and (***), and constants. In component form, this is t 2t -3t xl = c 1 e + c 2 e + c 3 e t 2c 2 e 2t 7c 3 e -3t x 2 = c 1 e + +, t 2t llc 3 e -3t x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the solution of the form x = At At At xl = ale, x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the form x = At At At xl = ale, x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the solution of the form x = At At At xl = ale, x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the
solution of the characteristic equation of the solution of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the solution of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the solution of the solution of the characteristic equation of the characteristic equation of the characteristic equation of the solution of the characteristic equation of the solution 6. The characteristic equation is 1 - A - 1 - 1 + 1 + 3 - A + (-1) = 0. (*) -3 - 6 - A + 3 - 4 + (-1) = 0. (*) -3 - 6 - A - 3 - 6 - A + 3 - A + (-1) = 0. (*) -3 - 6 - A - 3 - 6 simplifies to (A - 1)(A - 9A + 24) - (-A + 9) + (-3A + 3) = 0 and hence to A - 3 - 10A - 30 = 0. Thus the characteristic equation of A reduces to the cubic equation of A reduces to the c characteristic equation in the factored form (A - 2)(A - 3)(A - 5) = 0. The roots of this are the characteristic values Ai = 2, A2 = 3, A3 = 5. We have gone through the preceeding evaluation and solution in some detail. In third-order problems it is essential that we carry out the evaluation and solution of the characteristic equation carefully and correctly. Making an error here can be very time-consuming and frustrating. We shall omit the corresponding details from future solutions in this manual. We refer the reader ln need of help to Appendices 1 and 2 of the text. 506 Chapter 7 Going on with the present solution, we use A a to find the corresponding characteristic vectors. = 2, this is = ,\ a With 1 - 1 - 1 = 1 a 1 a 1 1 3 1 a 2 = 2 a 2 - 3 - 6 6 a 3 a 3. Thus ai' a 2 , a 3 must satisfy -a a 2 a 3 = 0, 1 a 1 + a 2 + a 3 = 0, -3a 6a 2 + 4a 3 = -3. Thus a character- istic vector corresponding to $\lambda = 2$ is 10 a = -7 -3 and a corresponding solution lS 10 10e 2t -7 2t that 7e 2t e lS, -3 3e 2t (**) Systems of Linear Differential Equations 507 With A = 3, Aa = Aa lS 1 -1 -1 a 1 a 1 1 3 1 a 2 = 3 a 2 -3 -6 6 a 3 a 3. Thus ai' a 2, a 3 must satisfy -2a - a 2 a 3 = 0, 1 a 1 + a 3 = 0, -3a - 6a 2 + 3a 3 = 0. 1 A nontrivial solution of this is a 1 = 1, a 2 = -1, a 3 = -1. Thus a characteristic vector corresponding to A = 3 is 1 a = -1 -1 and a corresponding to A = -1 and a correspo -3a 6a 2 + a 3 = 0.1 Adding the first two equations, we obtain -3a 1 - 3a 2 = 0, from which a 2 = -3a 1 + 6a 2 = -3a 1 +IS, -e - 3 3e 5t (****) Systems of Linear Differential Equations 509 By Theorem 7.16, the solutions (**), (***), and (****) are linearly independent, and a G.S. IS 10e 2t 3t 5t x = c 1 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 2 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c 3 + c 2 + c 3 + c = c 2 e c 3 e 1 - 3c e 2t 3t 3c 3 e 5t x 3 = c 2 e . 1 6 - t = u e, t hat is, We know that A must Ve assume a solution of the coefficient matrix 1 A = 1 o The characteristic equation is 1 - A 1 1 - A o 1 1 o 1 o 1 1 . 0 1 = O. 1 - 510 Chapter 7 Expanding the determinant and simplifying, it reduces to A 3 - 2A 2 - A + 2 = 0 or (A - 1)(A - 2)(A + 1) = O. Its roots are the characteristic vectors. With A = 1 this is 1 1 0 a 1 a 1 1 0 1 a 2 = 1 a 2 0 1 1 a 3 a 3. Thus ai' a 2 , a 3 must satisfy a 1 = 0, a 1 a 2 + a 3 = 0, a 1 = O. A nontrivial solution of this is a 1 = 1, a 2 = 0, a 3 = -1. Thus a characteristic vector corresponding to A = 1 lS 1 a = 0 - 1 and a corresponding to A = 1 lS 1 a = 0 - 1 and a corresponding to A = 1 lS 1 a = 0 - 1 and a corresponding solution lS Systems of Linear Differential Equations 511 t 1 e 0 t that 0 (*) e , lS - 1 t - e With A = 2, A a - A a lS 1 1 0 a 1 a 1 1 0 1 a 2 = 2 a 2 0 1 1 a 3 a 3. Thus a 1, a 2 , a 3 must satisfy -a + a 2 = 0, 1 a 1 2a 2 + a 3 = 0, a 1 a 3 = 0, a 1 a 3 = 0. A nontrivial solution of this is a 1 = 1, a 2 = 1, a 3 = 1. Thus a characteristic vector corresponding to A = 2 is 1 a = 1 1 and a corresponding solution lS 2t 1 e 1 2t that 2t e lS, e 1 2t e (**) 512 Chapter 7 With A = -1 Aa = Aa is , 1 1 0 a 1 a 1 1 0 1 a 2 = -1 a 2 0 1 1 a 3 a 3. Thus a 1, a 2 = 1, a 3 = 1. Thus a characteristic vector corresponding to A = 2 is 1 a = 1 1 and a corresponding solution lS 2t 1 e 1 2t that 2t e lS, e 1 2t e (**) 512 Chapter 7 With A = -1 Aa = Aa is , 1 1 0 a 1 a 1 1 0 1 a 2 = -1 a 2 0 1 1 a 3 a 3. Thus a 1, a 2 = 1, a 3 = 1. Thus a characteristic vector corresponding solution lS 2t 1 e 1 2t that 2t e lS, e 1 2t e (**) 512 Chapter 7 With A = -1 Aa = Aa is , 1 1 0 a 1 a 1 1 0 1 a 2 = -1 a 2 0 1 1 a 3 a 3. Thus a 1, a 2 = 1, a 3 = 1. Thus a characteristic vector corresponding to A = 2 is 1 a = 1 1 and a corresponding to A = 2 is 1 a = 1 1 and a corresponding to A = 2 is 1 a = 1 1. + a 3 = 0, a 2 + 2a 3 = o. A nontrivial solution of this is a 1 = 1, a 2 = -2, a 3 = 1. Thus a characteristic vector corresponding solution lS 1 -t e -2 e- t 1 that lS, -t -2e (***) -t e By Theorem 7.16, the solutions (*), (**), and (***) are linearly independent, and a G.S. is Systems of Linear Differential Equations 513 t that is, We know that A must coefficient matrix 1 = -20 - 203002. The characteristic equation is 1 - A - 20 - 23A0 = 0. 0 + 2 - 4A - 1 = 0. Its roots are the characteristic values A - 2 + 5. A - 2 - 5. A - 5. A the corresponding characteristic vectors. With A = 2 + this IS 1 - 2 0 a 1 a 1 - 2 3 0 a 2 = (2 +) a 2 0 0 2 a 3 a 3. Thus a 1, a 2, a 3 must satisfy (-1 -) a 1 - 2 a 2 = 0, -2 a + (1 -) a 2 = 0, -2 a + (1 -) a 2 = 0, -2 a + (1 -) a 2 = 0, -2 a + (1 -) a 2 = 0, -2 a + (1
-) a 2 = 0, -2 a + (1 -) a - (1 -) a + (1 -) a + (1 -) a + 15 - 21 + e(2+)t, that o 2e(2+)t(1+)e(2+)t(4) IS, o Systems of Linear Differential Equations 515 With A = 2 - {5, Au = Aa IS 1 - 20 a 1 a 1 - 2 3 0 a 2 = (2 - {5}) a 2 0 0 2 a 3 a 3. Thus ai' a 2, a 3 must satisfy (-1 + {5})a 1 - 2 a 2 = 0, -2 a + (1 + {5})a 2 = 0, 1 {5 a 3 = 0}. A nontrivial solution of this is a 1 = 2, a 2 = -1 + {5, a 3 = 0}. Thus a characteristic vector corresponding to $A = 2 - \{5 \text{ is } 2 - 1 + \{5 \text{ o and a corresponding solution } \text{ls } 2 + \{5 \text{ e}(2 - \{5)\text{t}, \text{that o } 2 \text{t}, \text{tha$ 1. Thus a characteristic vector corresponding to A = 2 is o a = 0.1, and a corresponding solution IS 0 0 0 2t that 0 e IS, 1 2t e (***) By Theorem 7.16, the solutions (*), (**), and (***) are linearly independent, and a G.S. is Systems of Linear Differential Equations 517 2e(2+J1;)t 2e(2-)t x = c l (1 + J1;)e(2+J1;)t + c 2 (-1 + J1;)e(2-J1;)t o o o + c 3 o 2t ewhere c 1 ' c 2 ' c 3 are arbitrary constants. In component form, this is = 2 $(2+J_1)t 2 (2-J_1)t x - c 1 e + c 2 e$, x 2 = a 2 e, x a = a a e b e a solution of the characteristic equation of the -t = a e that is, We know that must coefficientmatrix 37 -a A - 12 - 2 - 16 - 2. The characteristic equation is a - 7 -a 12 A - 2 = 0. 16 - 2 - A 518 Chapter 7 Expanding the determinant and simplifying, it reduces to 3 32 + 4 - 12 = 0, or (-3)(2 + 4) = 0. Its roots are the characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 0. To the characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = 2i, 3 = -2i. We use A a = a to find the corresponding characteristic values 1 = 3, 2 = -3 a 1 a 1 1 2 -2 a 2 = 3 a 2 1 6 -2 a 3 a 3. Thus a 1, a 2, a 3 must satisfy 7a 2 3a 3 = 0, a 1 a 2 2a 3 = 0, a 1 a + 6a 2 5a 3 = Equations 519 With A = 2i, Aa = Aa IS 37-3 a 1 a 1 1 2 - 2 a 2 = 2i a 2 1 6 - 2 a 3 a 3. Thus ai' a 2, a 3 must satisfy (3 2i)a 1 + 7a 2 3a 3 = 0, a 1 + (2 - 2i)a 3 = 0, a 1 + (2 - 2 1 - 2i. Then a 1 = (-2 + 2i)a 2 + 2a 3 = (-2 + 2i)(1) + 2(1 - 2i) = -2i. Thus a characteristic vector corresponding to A = 2i lS - 2i a = 1 1 - 2i and a corresponding to A = 2i lS - 2i a = 1 1 - 2i 2i t e, that lS, 2i t e (1 2)e2i t Proceeding as in Section 7.6, we apply Euler's formula i () () ... () h t u bt. th e = cos + 1 sln to eac componen. we 0 aln e solution - 2 i (cos 2t + i sin 2t) cos 2t + i sin 2t) cos 2t + i sin 2t (1 - 2i) (cos 2t + i sin 2t), which simplifies to 2 sin 2t - 2 cos 2t + sin 2t) real and imaginary parts of this solution are themselves solutions, so we obtain the solutions 2 sin 2t cos 2t + 2 sin 2t - 2 cos 2t + sin 2t (**) The solution (*) and the two solutions (**) are linearly independent, so a G.S. is Systems of Linear Differential Equations 521 17e 3t 2 cos 2t + c 3 sin 2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t - 2 cos 2t + c 3 sin
2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t - 2 cos 2t + c 3 sin 2t sin 2t sin 2t sin 2t - 2 cos 2t + c 3 sin 2t $\cos 2t + c 3 \sin 2t$, x 2 = +7c 1 e 3t c 2 ($\cos 2t + 2 \sin 2t$) x 3 = ++c 3 ($\sin 2t - 2 \cos 2t$). 13. Ve assume a solution of the form x t t t x 1 - ale, x 2 = a 2 e x 3 = a 3 e be a solution of the -t = ae, that is, Ve know that must coefficient matrix 1 - 3 9 A = 0 - 5 18 0 - 3 10. The characteristic equation is 1 - - 3 0 - 5 - 0 - 3 9 18 = O. 10 - A 522 Chapter 7 Expanding this determinant and simplifying, it reduces to (-1)(2 - 5 + 4) = 0 or (-1)(2 - 5 + 4) = 0 or (-1)(2 - 5 + 4) = 0 or (-1)(2 - 5 + 4) = 0. Its roots are the characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Yith = 4 this is 1 - 39 = 1. Ye use A = a to find the corresponding characteristic vectors. Ye use A = a to find the corresponding characteristic vectors. Ye use A = a to find the corresponding characteristic vectors. Ye use A = a to find the correspond to the correspondence of the correspond 3 = 0, 19a 2 + 18a 3 = 0, 3a 2 + 6a 3 = 0. From either the second or third equation of this is a 1 = 1, a = 2, a 3 = 1. Thus 2 a characteristic vector corresponding to = 4 lS 1 a = 2 1 Systems of Linear Differential Equations 523 and a corresponding solution lS 1 2 = 4t + 1 + 1 (*) With = 1, Aa = a = 0, 2 - 3a + 9a = 0, 2 - 3a + 3a = 0, 2 - 3a + 3observe that a 1 lS arbitrary. Two linearly independent solutions of (**) are a 1 = 0, a = 3, a - 1 and a 1 = 1, a 2 = 3, a 3 = 1. That lS, 2 3 - 524 Chapter 7 corresponding to the double characteristic value = 1, we have the two linearly independent characteristic value = 1, we have the two linearly independent characteristic value = 1. that is, 0 t e 3e t and 3e t (***) t t e e The three solutions. by (*) and (***) glven are linearly independent, and a G.S. IS 4t t 0 e e x = c 1 2e 4t + c 2 3e t + c 3 3e t 4t t t e e e where c 1, c 2, c 3 are arbitrary constants. In component form, this is Systems of Linear Differential Equations 525 4t t xl = c 1 e + c 2 e, 2c 1 e 4t + ac 2 e t + acae t x 2 = , 4t t t xa = c 1 e + c 2 e + cae. 15. Ye assume a solution of the form x t t t xl = ale x 2 = a 2 e, xa = aa e be a solution of the characteristic equation of the - t = a e, that is, Ye know that must coefficient matrix 11 6 A = 98 - 9 - 6 18 18 - 16. The characteristic equation of the equation of the characteristic equation of the characteristic equation of the characteristic equation of the characteristic equation of the reduces to a 2 (+1)(2)2 = 0. Its - a + 4 = 0 or roots are the characteristic vectors. Yith = -1 this is 11 6 18 a 1 a 1 9 8 18 a 2 = (-1) a 2 -9 -6 -16 aa aa. 526 Chapter 7 Thus a 1, a 2, a 3 must satisfy 12a 1 + 6a 2 + 18a 3 = 0, 9a 1 + 9a 2 + 18a 3 = 0, -9a 6a 2 15a 6 18 a 1 a 1 9 8 18 a 2 = 2 a 2 - 9 - 6 - 16 a 3 a 3. Thus ai' a 2, a 3 must satisfy 9a 1 + 6a 2 + 18a 3 = 0, 9a 1 + 6a2, a 2 = 0, a 3 = -1 and a 1 = 0, a 2 = 3, a 3 = -1. That is, corresponding to the double characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent characteristic value A = 2, we have the two linearly independent chara

(**) are linearly independent, and a G.S. IS -t 2e 2t 0 e -t 0 3e 2t x = c 1 e + c 2 + c 3 -t 2t 2t -e -e where c 1, c 2, c 3 are arbitrary constants. In component form, this is -t 2c 2 e 2t x 1 = c 1 e + , -t 3c 3 e 2t x 2 = c 1 e + , -t 3c 3 e is, Ye know that A must Systems of Linear Differential Equations 529 be a solution of the characteristic equation lS 1 - , 99019, 18 = 0.0910 - , Expanding the determinant and simplifying, it reduces to <math>, 3 - 30, 2 + 57, -28 = 0 or (, -1)2(, -28) = 0. Its roots are the characteristic equation of the coefficient matrix 1 A = 0 o 9 19 9 8 18 10. The characteristic equation lS 1 - , 99019, 18 = 0.0910 - , Expanding the determinant and simplifying, it reduces to <math>, 3 - 30, 2 + 57, -28 = 0 or (, -1)2(, -28) = 0. characteristic vectors: 1 = 28, 2 = 3 = 1. We use A a - , a to find the corresponding characteris - tic vectors. With = 28 this is 1 9 9 a 1 a 1 0 19 18 a 2 = 28 a 2 0 9 10 a 3 a 3. Thus ai a 2 = 28 a 2 0 9 10 a 3 a 3. Thus ai a 2 = 28 a 2 0 9 10 a 3 a 3. Thus ai a 2 = 2, a 3 = 1. Thus a characteristic vector corresponding to A = 28 is 1 a - 2 1 and a corresponding solution IS 1 2 e 28t 1 28t e that IS, 2e 28t 28t e (*) With A = 1, Aa = Aa is 1 9 9 a 1 a 1 0 19 18 a 2 = 1 a 2 0 9 10 a 3 a 3. Thus a 1, a 2, a 3 must satisfy 9a 2 + 9a 3 = 0, 18a 2 + 9a 3 = 0, 9a 2 + 9a 3 = 0, 2a 2 + 9a 3 = a 2 + a 3 = 0, Systems of Linear Differential Equations 531 and this the only relationship which a 1 = 1, a 2 = a 3 = 0, and a 1 = 0, a 2 = 1, a 3 = -1. That is, corresponding to the double characteristic value A = 1, we have the two linearly independent solutions of it are a 1 = 1, a 2 = a 3 = 0, and a 1 = 0, a 2 = 1, a 3 = -1. That is, corresponding to the double characteristic value A = 1, we have the two linearly independent solutions of it are a 1 = 1, a 2 = a 3 = 0, and a 1 = 0, a 2 = 1, a 3 = -1. That is, corresponding to the double characteristic value A = 1, we have the two linearly independent solutions of it are a 1 = 0, a 2 = 1, a 3 = -1. independent characteristic vectors 1 o o and o 1 -1. Respective corresponding solutions are 1 0 0 t and 1 t e, e, 0 -1 that IS, t 0 e 0 and t (**) e 0 t -e The three solutions given by (*) and (**) e 0 t -e The three solutions given by (*) and (**) e 0 t -e The three solutions given by (*) and t (**) e 0 t -e The three solutions given by (*) and (**) e 0 t -e The three solutions given by (*) and t (**) e 0 t -e The three solutions given by (*) and (**) e 0 t -e The three solutions given by (*) and t (**) e 0 t -e The three solutions given by (*) and (**) e 0 t -e The three solutions given by (*) e 0 t -e The th component form, this is 28t t x l = c 1 e + c 2 e, 2c 1 e 28t t x 2 = + c 3 e, 28t t x 3 = c 1 e c 3 e. 17. We assume a solution of the characteristic equation ls -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8. The characteristic equation lS -5 - At = a e, t $-12 \ 6 \ 1 \ 5 - A \ -1 = 0$. $-7 \ -10 \ 8 - A \ -1 = 0$, $-7 \ -10 \ 8 - A \ -12 = 0$, that is, $(A \ 1)(A - 3)(A - 4) = 0$. Its roots are $A \ 1 = 1$, $A \ 2 = 3$, $A \ 3 = 4$. We use $A \ a = A \ a$ to find the corresponding characteri s- tic vectors. With A = 1 this is Systems of Linear Differential Equations 533 - 5 - 12 \ 6 \ a \ 1 \ a \ 1 \ 5 - 12 \ 6 \ a \ 1 \ a \ 2 \ -7 \ -10 \ 8 \ a \ 3 \ a \ 3 \ -12 \ 6 \ a \ 3 \ a \ 3 \ -12 \ -1 a 2, a 3 must satisfy -8a 12a
2 + 6a 3 = 0, 1 a 1 + 2a 2 a 3 = 0, -7a 10a 2 + 5a 3 - 0. 1 - A nontrivial solution of this is a 1 = 0, a 2 = 1, a 3 = 2. Thus a characteristic vector corresponding to A = 3 lS o a = 1 2 and a corresponding to A = 3 lS o a = 1 2 and a corresponding solution lS o 0 1 e 3t 2 that lS, 3t e (**) 2e 3t · With A = 4, Aa - Aa lS - 5 - 12 6 a 1 a 1 1 5 - 1 a 2 = 4 a 2 - 7 - 10 8 a 3 a 3. Thus ai' a 2, a 3 must satisfy Systems of Linear Differential Equations 535 -9a 12a 2 + 6a 3 = 0, 1 a 1 + a 2 a 3 = 0, -7a 10a 2 + 4a 3 = 0, 1 a 1 + a 2 a 3 = 0, -7a 10a 2 + 4a 3 = 0, 1 a 1 + a 2 a 3 = 0, -7a 10a 2 + 4a the solutions (*), (**), and (**) are linearly independent, and a G.S. is t 0 2e 4t e 0 3t 4t x = c 1 + c 2 e + c 3 - e t 2e 3t 4t e e where c 1, c 2, c 3 are arbitrary constants. In component form, this is 536 Chapter 7t 4t xl = c 1 e + 2c 3 e 3t x 2 = c 2 e t x 3 = c 1 e 4t - c 3 e 3t + 2c 2 e 4t + c 3 e 19. We assume a solution of the form x t t t xl = ale x 2 = a 2 $e x 3 = a 3 e be a solution of the characteristic equation of the - \t = a e, t hat is, We know that \must coefficient matrix -5 - 3 - 3 A = 8 5 7 - 2 - 1 - 3 - (\ Expanding the determinant and simplifying, it reduces to 3 2 ((\ 1)(\ + 2)2 Its roots (\ + 3) - 4 = 0, or - = o. are the characteristic values \1$ = 1, 12 = 1, 2 = -2. We use A a = , a to find the corresponding characteris - tic vectors. With, = 1 this is -5 - 3 - 3 a 1 a 1 8 5 7 a 2 = (1) a 2 - 2 - 1 - 3 a 3 a 3 . Systems of Linear Differential Equations 537 Thus ai' a 2 , a':) must satisfy v -6a 3a 2 3a 3 = 0, 1 8a 1 + 4a 2 + 7a 3 = 0, -2a a 2 4a 3 = 0, equation, we find -9a 3 = 0, from which a 3 = 0. With a 3 = 0, each equation is equivalent to a 3 = -2a 1. Hence a nontrivial solution of the algebraic system IS a i = 1, a 2 = -2, a 3 = 0. Thus a characteristic vector corresponding to = -2a 1. Hence a nontrivial solution of the algebraic system IS a i = 1, a 2 = -2, a 3 = 0. Thus a characteristic vector corresponding to = -2a 1. Hence = -2a 1 + 2a = -2a + 2a = -2a + 2a = -2a = -2ai 8 5 7 a 2 = -2 a 2 - 2 - 1 - 3 a 3 a 3. 538 Chapter 7 Thus a 1 = 0, each equation to (-3) times the third, we find 3a 1 = 0, from which a 1 = 0, from which a 1 = 0, each equation to (-3) times the third, we find 3a 1 = 0, a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3539 Multiplying this out yields -38 3{32 3{33 = 0, 1 8{31 + 7{32 + 7{33 = 1, -2/3 / 3 2 {33 = -1, 1 a solution of which IS {31 = 1, {32 = -1, {33 = 0. Thus we obtain 1 13 = -1 0 and the second solution (at 13)e -2t + lS -2t 0 1 e 1 -1 -2t that (t - l)e -2t + e IS, -1 0 -2t + e 0 - 2t e = t - 2t (t l) e - 2t x = c - 1 - 2e + c - 2equation of the - t = a e, that is, We know that must coefficient matrix 3 - 2 - 1 = 0. We use A a = a to find the corresponding characteristic equation is 3 - 2 - 1 = 0. We use A a = a to find the corresponding characteristic equation is 3 - 2 - 1 = 0. We use A a = a to find the corresponding characteristic equation is 3 - 2 - 1 = 0. We use A a = a to find the corresponding characteristic equation is 3 - 2 - 1 = 0. We use A a = a to find the corresponding characteristic equation is 3 - 2 - 1 = 0. a 2 5 -3 -3 a 3 a 3. Systems of Linear Differential Equations 541 Thus ai' a 2, a 3 must satisfy a 1 - 2a 2 a 3 = 0, 4a 1 + 4a 3 = 0, 5a 1 - 3a 2 5a 3 = 0. A nontrivial solution of this is a 1 = 1, a 2 = 0, a 3 = 1. Thus a characteristic vector corresponding to = 2 IS 1 a = 0 1 and a corresponding solution IS 2t 1 e 0 2t that 0 e IS, 1 2t e (*) We now consider Systems of Linear Differential Equations 543 Multiplying this out yields 3,81 2,82 + 4,83 = 4, 15,81 3,82 3,83 = 1. Multiplying the first equation by 4, the second by 3, and adding, we find -2,82 + 4,83 = 24 and so ,82 = 4,83 - 12. The first equation by 4, the second by 3, and adding, we find -2,82 + 4,83 = 4, 15,81 3,82 3,83 = 1. Multiplying the first equation by 4, the second by 3, and adding, we find -2,82 + 4,83 = 24 and so ,82 = 4,83 - 12. The first equation gives f] l = f] 2 + f] 3 + 1. Then substituting ,82 = 4,83 - 12 into this, we have ,81 = 3,83 - 7. One quickly checks that the pair ,81 = 3,83 - 7, 82 = 4,83 - 12 satisfies the third equation for arbitrary ,83 = 3. Thus we obtain 2 -0.3 - ,8 and the second solution at +1S.32 at +2.4t + 0 that 1S, 4t (***) 1.3t + 3. The three solutions (*), (**), and (**) are of the characteristic equation of the - At = a e, that is, We know that A must coefficient matrix 7 4 4 A = -6 -4 -7 -2 -1 2 - A It reduces to A 3 5A 2 + 3, +9 = 0, that lS, (, +1) $(, 3)^2 = 0$. Its roots are the characteristic values 1 = -1 + 2 = -3 = 3. Systems of Linear Differential Equations 545 We use A a = 1 + a to find the corresponding characteristic vectors. With = -1 this is 7 4 4 a 1 a 1 -6 -4 -7 a 2 = -1 a 2 -2 -1 2 a 3 a 3 . Thus a 1 -6 -4 -7 a 2 = -1 a 2 -2 -1 2 a 3 a 3 . Thus a 1 -6 -4 -7 a 2 = -1 a 2 -2 -1 2 a 3 a 3 . Thus a 1 - 4 a 3 = 0, 1 -2 a 3 a 3 = -2 o and the corresponding solution is -t 1 e -2 -t that -t e IS, -2e 0 0 (*) We now consider the repeated characteristic value '\2 = '\3 = 3. With, = 3, A a = '\a becomes 546 Chapter 7 7 4 4 a 1 a 1 -6 -4 -7 a 2 = 3 a 2 -2 -1 2 a 3 a 3. Thus ai' a 2, a 3 must satisfy 4a 1 + 4a 2 + 4a 3 = 0, -6a 7a 2 7a 3 = 0, 1 -2a a 2 a 3 = 0. 1 A solution of this is a 1 = 0, a 2 = 1, a 3 = -1. Thus a characteristic vector corresponding to = 3 is of the - - 3t - form (at +) e, where satisfies (A - 3I)P = a. This IS Systems of Linear Differential Equations 547 4 4 { 31 0 - 6 - 7 - 7 { 32 = 1 - 2 - 1 - 1 { 33 - 1 { 33 - 1 } } } Multiplying this out yields $4{31 + 4{32 = 0}, -6{3 - 7{3 - 7{3 - 1}}}$ and the second solution (at ${3)e 3t + 1S 3t 0 1 e 1 t + -1 3t that (t - 1)e 3t (***) e 1S, -1 0 te 3t The three solutions (*), (**), and (***) are linearly independent. Thus a G.S. is -t 0$ 3t e e - t 3t (t - 1)e 3t x = c 1 - 2e + c 2 e + c 3 0 3t te 3t - e 548 Chapter 7 where c 1, c 2, and c 3 are arbitrary constants. In component form, this IS - t 3t x 3 = -c e c 3 te + c 2 e, 1 3t 3t x 3 = -c e c 3 te + c 2 e, 1 3t 3t x 3 = -c e c 3 te + c 2 e, 1 3t 3t x 3 = -c e c 3 te + c 2 e, 1 3t 3t x 3 = -c e c 3 te + c 3 e, 1 3t 3t x 3 = -c e c 3 te + c 3 e, 2 - c e - t 3t + c 3 (t - 1)e 3t x 2 = + c 2 e, 1 3t 3t x 3 = -c e c 3 te + c 3 e, 1 3t 3t 3t x 3 = -c e c 3 te + c 3 e, 1 3t 3t 3t x 3 = -c e c 3 te + c 3 e, 1 3t 3t 3t 3t 3 = -c e c 3 te + c 3 e, 1 3t 3t 3t 3t 3t 3 = -c e c 3 te + c 3 e, 1 3t 3tequation of the - At = a e, that is, Ve know that must coefficient matrix 4 - 1 - 1 = 0. 2 - 1 - 1 = 0. 2 - 1 - 1 = 0. 2 - 1 - 1 = 0. 2 - 1 - 1 = 0. Its roots are the characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. Systems of Linear Differential Equations 549 4 -1 -1 a 1 a 1 2 1 -1 a 2 = 2 a 2 2 -1 1 a 3 a 3. Each of the three resulting relationships in a 1, a 2 = 2, a 3 = 0 and a 1 = 1, a 2 = 2, a 3 = 0. Both a 1 = 1, a 2 = 2, a 3 = 0 and a 1 = 1, a 2 = 2, a 3 = 0. Both a 1 = 1, a 2 = 2, a 3 = 0. Both a 1 = 1, a 2 = 2, a 3 = 0. Both a 1 = 1, a 2 = 2, a 3 = 0. Both a 1 = 1, a 2 = 0, a 3 = 2 are distinct solutions of this. Thus we obtain the characteristic vectors 1 a (1) = 2 o 1 and a (2) = 0 2 and the corresponding solutions a (1) e 2t and a (2) e 2t, that is 2t 2t e e 2e 2t and 0 (*) 0 2e 2t. A third solution corresponding to = 2 is of the form (at + p)e 2t, where a satisfies (A - 2I)P = a. Ye apply this last equation with 1 1 k 1 + k 2 - (1) + k a (2) k 1 2 + k 2 0 2k 1 a = k 1 a = = 2 0 2 2k 2 550 Chapter 7 Thus we have 2 - 1 - 1 31 + k + 2 = 2k + 1 = 2k + 2 = 2k + 2and we obtain 1 {3 - 0 0. Systems of Linear Differential Equations 551 Thus the third solution (at + P)e 2t ls 2 1 (2t + 1)e 2t 2 t + 0 2t that 2t e 2t e ls, 2 0 2t e 2t x = c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2, c 2, c 2t + 0 2t that 2t e 2t e ls, 2 0 2t e 2t x = c 1 + c 2 + c 3 0
2e 2t 2t e 2t where c 1, c 2, c 2t + 0 2t that 2t e 2t e ls, 2 0 2t e 2t x = c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2, c 2t + 0 2t that 2t e 2t e ls, 2 0 2t e 2t x = c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2 + c 3 0 2e 2t 2t e 2t where c 1, c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t where c 1 + c 2 + c 3 0 2e 2t 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t e 2t + 0 2t that 2t e 2t + 0 2t + 0 2t that 2t e 2t + 0 2t 3 are arbitrary constants. In component form, this is 2t + c = 3 (2t + 1) e = 2t + 2c = 2t-2 - 8 - 2 - 8 - 3 = 0, that is, $(-2)^3 = 0$. Its roots are the characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. We use A = a to find the corresponding characteristic values 1 = 2 = 3 = 2. a 1, a 2, a 3 are 2a 1 + 6a 2 a 3 = 0, -a 4a 2 + a 3 = 0, 1 - 2a 8a 2 + 2a 3 = 0, 1 - 2a 8a 2 + 2a 3 = 0, 1 - 2a 8a 2 + 2a 3 = 0, 1 - 2a 8a 2 + 2a 3 = 0, 1 - 2a 8a 2 + 2a 3 = -1, a 3 = -2 is a nontrivial solution of these. Thus we obtain the characteristic vector 2 a = -1 - 2. Systems of Linear Differential Equations 553. - 2t and the corresponding solution ae , that IS, 2e 2t 2t - e (*) _2e 2t . A second solution corresponding to = 2 is of the - - 2t - form (at + p)e, where P satisfies (A - 2I)P = a. This lS 2 6 - 1 Pi 2 - 1 - 4 1 P 2 = -1 - 2 - 8 2 P 3 - 2. Multiplying this out yields 2P 1 + 6P 2 P 3 = -2 1, a solution of which lS Pi = 1, P 2 = P 3 = 0. Thus we obtain 1 {3 = 0 0 and the second solution (at {3)e 2t + lS 554 Chapter 7 (2t + 1)e 2t 2 1 - 1 t + 0 2t that -t e 2t (**) e IS, -2 0 - 2t e 2t (- 2 -) 2t A third solution if of the form at (It -+ + 1 e, 2 where, satisfies (A - 21)1 = (3. This IS 2 6 - 1 '1 1 - 1 - 4 1 '2 = 0 - 2 - 8 2 0 . '3 Multiplying this out yields 2'1 + 6'2 13 = 1, -, 4'2 + '3 = 0, 1 - 2, 8'2 + 2'3 = 0. 1 A solution of this IS '1 = 1, '2 - 0, 13 = 1; and we obtain 1, = 0 1. Thus the third solution (a; 2 + fit + 1)e 2t 1S Systems of Linear Differential Equations 555 2 1 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t - + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t - + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t - + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t - + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t - + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t x = c 1 - e + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t x = c 1 - e + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t x = c 1 - e + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t x = c 1 - e + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t that (1/2)t 2 e 2t x = c 1 - e + e IS, 2 - 2 0 1 (t 2 + t + 1)e 2t -1 t 2 0 t + 0 2t c 2 + c 3 2e 2t - 2t e 2t (t 2 + 1)e 2t 2t 2 2t x 3 = 2c 2 te + c 3 (+ t + 1)e + cdifferential equation y' = xy determines the family of isoclines c = xy; these are the hyperbolas for $c = \frac{1}{10}$, $\frac{3}{10}$, $\frac{$ Figure 8.1B-2A 556 Approximate Methods 557 Now, on each of the isoclines drawn in our first graph, we draw several line element on the fixed isocline y = cO/x 1S cO; the corresponding angle of inclination is then a O = arctan cO. We obtain the graph found in Figure 8.1B-2B. y Figure 8.1B-2B c = 1/10; Q60 c = 3/10; Q170 c = 1/2; a 270 c = 6; Q 810 c = 6; Q 810 c = 6; Q 810 c = 4; Q 76° c = -6; a -810 c = -2; Q -630 c -3; Q 720 c = -2; Q -630 c -3; Q 720 c = -3/10; Q 770 c = -1/10; Q-60 558 Chapter 8 The line element configuration seen in the second graph indicates how to draw the approximate integral curves. We sketch several. In particular, we show those curves which have y intercepts -1, -0.1, -0.001, 1/16, 1/8, 1/4, 1/2, 1, and 2, respectively. See Figure 8.1B-2C. y x Figure 8.1B-2C. y x Figure 8.1B-2C. y a figure 8.1B-2C. y a figure 8.1B-2C approximate integral curves appear without the integral curves appear without the integral curves appear without the integral curves. We show the integral curves appear without the integral curves appear without the integral curves. the background of isoclines and the corresponding line elements. See Figure 8.1B-2D 560 Chapter 8 Exercise 11. We separate the solution into four graphs. The differential equation y' = y Sln x determines the family of isoclines c = y Sln x, or equivalently, y = c csc x. Our first graph is the result of drawing these isoclines for c = corresponding angle of inclination is then a O = arctan cO. We obtain the graph found in Figure 8.1B-11B. y x Figure 8.1B-11B 562 Chapter 8 The line element configuration seen in the second graph indicates how to draw the approximate integral curves. We sketch several, including one whose y intercept is. See Figure 8.1B-11C. y x Figure 8.1B-11C is the approximate integral curves. Approximate Methods 563 Our last graph just shows how the integral curves appear without the background of isoclines and the corresponding line elements. See Figure 8.1B-11D. y x Figure 8.1B-11D. y = y(O) + y'(O) + y'(O)D.E. glves y'(O) = 21 + (0)(2) = 1. Differentiating the D.E., we obtain "2 Y = 2xyy' + y, (2) " 2 " 2 () 2 4 y = xyy + x y' + yy', (3) IV III " () 2 y = 2xyy + 6xy' + 6y' + (4)(2)(1) = 8. Substituting x = 0, y = 2, y' = 1, yH = 4, y''' = 8 into (4), we obtain yIV(0) = (2)(0)(1)(8) + (6)(2)(4) +-+34b. We assume 2y = Co + c1x + c2x3 + c3x4 + c4x + ... (5) Differentiating this, we find 23y' = c1 + 2c2x + 3c3x + 4c4x + (6) Since the initial y value $1S_2$, we must express y_2 in the D.E. in powers of y - 2. Then the D.E. takes the form 2y' = 1 + x[(y-2) + 4(y-2) + 4]. Now substituting (5) and (6) into this, we obtain 566 Chapter 8 2 3 c 1 + 2c 2 x + 3c 3 x + 4c 4 x + ... 22 = 1 + x[(c 1 x + c 2 x + 3c 3 x + 4c 4 x + ...) + 4] or 2 3 c 1 + 2c 2 x + 3c 3 x + 4c 4 x + ... 22 = 1 + 4x + 4c 1 x + (c 1 + 4c 2)x + ... From this, 1, 2c 2 4, 3c 3 4c 1, 4c 4 2 4c 2; c 1 = + 4x + 4c 1 x + (c 1 + 4c 2)x + ... + 4]= = c 1 + and from these, 1, 2, 4c 1 4 c 1 = c 2 = c 3 = 3 3 2 c 1 + 4c 2 4 9 = 4. Substituting these into (5) we again c 4 = obtain 2 4x 3 9x 4 y = 2 + x + 2x + 3 + 4 + 4. a. We have 2 3 y(x) = y(O) + y'(O)x + y''(O) ;! 4 Y 1V (0) x + 4! + (1) The I.C. states that y(O) = 3, and the D.E. glves y'(O) = 3 3 (0) + (3) = 27. Differentiating the D.E., we obtain Approximate Methods 567 " 2 2 (2) y = 3x + 3y y I, ", 3 2 " 2 (3) y = 6x + y y + 6y(y), IV 6 + 3 2 "," (3 (4) y - y y + 18yy/y + 6 y' - 529 into (3), we obtain ym(O) = 6(0) + 3(3)2(729) + 6(3)(27)2 = 32,805. Substituting y = 3, y' = 27, y'' = 729, y'' = 729+ ... 228 568 Chapter 8 b. 2 Ye assume y = Co + c1 x + c2 x 3 + c3 x 4 + c4 x + ... To satisfy the I.C. y(O) = 3, we must have Co = 3, and hence 2 Y = 3 + c1 + 2c2 x + 3c3 x + 4c4 x + (6) Since the initial y value IS 3, we must express y3 in the D.E. in powers of y - 3. Then the D.E. takes the form y' = x 3 + (y - 3) 3 + 9(y - 3) 2 + 27(y - 3) + 27. Now substituting (5) and (6) into
this, we obtain 2 3 c 1 + 2c 2 x + ...) + 9(c 1 x + c 2 x + ...) + 27 or 2 3 c 1 + 2c 2 x + 3c 3 x + 4c 4 x + ... 2 2 = 27 + 27c 1 x + (9c 1 + 27c 2) x 3 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c 2) x 3 + (1 + c 1 + 27c + l8c l c 2 + 27c 3)x + From this, c l = 27, 2c 2 = 27c l, 3c 3 2 = 9c l + 27c 2, Approximate Methods 569 4c 4 = 1 + 3 27c 3; and from these, 27, c 1 + 18c 1 c 2 + c 1 = 27c 3 c 1 + c 4 = 4 1 + 19,683 + 177, 147 + 295,245 2 688,907 = 4 8 Substituting these into (5), we again obtain 729x 2 3 4 3 + 27x + 10,935x 688,907x y = + + + ... 2 2 8 5. a. 'We have y(x) y(O) = 0, and the D.E. gives y'(O) = 0, and the D.E. gives y'(O) = 0, and the D.E., we obtain $y'' = 1 + (\cos y)y'$, III () "(.)() 2 y = cos y y - Sln y y' . IV Y = $(\cos y)y'' - 3(\sin y)y''' - 3(\sin y)(y_{,})^2 - 6(\cos y)(y_{,})^2 + (\sin y)(y_{,})^2 - 6(\cos y)$ $c_1 x + c_2 x_3 + c_3 x_4 + c_4 x_5 + c_5 x + ...$ The I.C. y(0) = 0 gives Co = 0, so $2Y = c_1 x + c_2 x_3 + c_3 x_4 + c_4 x_5 + c_5 x + ...$ (1) Differentiating, we find $234 y' = c_1 + 2c_2 x + 3c_3 x + 4c_4 x + ScSx + (2)$ Also, the Maclaurin Series for slny 13 = y - Y + 6 + ... Approximate Methods 571 Thus the D.E. takes the form 1 3 $y' - X + y - y + 12 + 12 + 2c_2 x + 3c_3 x + 4c_4 x + ScSx + (2)$ Also, the Maclaurin Series for slny 13 = y - Y + 6 + 6 Now substituting (1) and (2) into this, we obtain 234 c 1 + 2c 2 x + 3c 3 x + 4c 4 x + 5c 5 x + ... 234 = x + [c 1 x + c 2 x + c 3 x + 4c 4 x + 5c 5 x + ... 2 = (1 + c 1)x + c 2 x + (c 3 133 6 c 1)x 124 + (c 4 - 2 c 1 c 2)x + From this, c 1 = 0, 2c 2 = 1 + c 1 ' 3c 3 = c 2 ' 1 3 1 (c + 1)x + c 2 x + (c 3 133 6 c 1)x 124 + (c 4 - 2 c 1 c 2)x + From this, c 1 = 0, 2c 2 = 1 + c 1 ' 3c 3 = c 2 ' 1 3 1 (c + 1)x + c 2 x + (c 3 133 6 c 1)x 124 + (c 4 - 2 c 1 c 2)x + From this, c 1 = 0, 2c 2 = 1 + c 1 ' 3c 3 = c 2 ' 1 3 1 (c + 1)x + c 2 x + (c + 1)x + (c 2 4c 4 = c 3 - 6 c 1 + 5 c 5 = c 4 - 2 c 1 c 2 + and from these, c 1 1 = 0, c 2 = 2 + c 3 1 = 3 c 2 1 = 6 + 1 c 4 = 4 c 3 = 1 1 24; C s = 5 1 c 4 - 120. Substituting this into (1), we again obtain 1 2 y = -x 2 1 3 + -x 6 1 4 + 24 x 1 S + 120 x + ... 572 Chapter 8 8. a. 'We have y(x) y(0) + y'(0)x + y''(0) 2 y''(0) 3 = 2! x + 3! x y V(0) 4 + x + ... 4! The I.C. x 4 + ... b. 'We assume 2 Y = Co + c 1 x + c 2 x 3 + c 3 x 4 + c 4 x + ... (2) Since the initial y value ls 1, we express y4 in the D.E. in powers of y - 1. Then the D.E. takes the form y' - x 4 + 1 + 4(y - 1) + 6(y - 1)2 + 4(y - 1) + 6(y - 1)2 + 4(y - 1)3 + (y - 1)3 + (y - 1)4. Substituting (1) and (2) into this, we obtain 234 c 1 + 2c 2 x + 3c 3 x + 4c 4 x + 5c 5 x + 4 234 = x + 1 + 4(c 1 x + 2c 1 c 2 x + ...] 3 3 2 4 4 4 + 4[c 1 x + 3c 1 c 2 x + ...] 4 (2 1 c 3 + c 2 x + ...] + (2 c 1 c 3 + ...] + (2 c 1 x + ... 2 2 3 3 = 1 + 4c 1 x + (4c 2 + 6c 1) x + (4c 3 + 12c 1 c 2 + 4c 1) x 2 244 + (1 + 4c 4 + 12c 1 c 3 + 6c 2 + 12c 1 c 2 + c 1) x + From these, c 1 = 1, c 2 = 4c 1, 3c 3 2 = 4c 2 + 6c 1, 4c 4 3 = 4c 3 + 12c 1 c 2 + 4c 1, ...; 574 Chapter 8 and from these, c 1 = 1, c 2 = 2c = 2, 1 4 2 14 3c 1 c 2 3 35 c 3 = 3 c 2 + 2c 1 = 3' c 4 = c 3 + + c 1 + 3'Substituting these into (1), we agaln obtain y = 1 + x + 2x + 4x + 3x + 3x + 3x + 4 + ... 12. a. 'We have 2y(x) = y(1) + y'(1)(x + 1) + y $(\sin y) y \text{III} - 3(\cos y)y''y' \text{I} = + (\sin y)(y/2, (4) \text{Approximate Methods 575 } yV = -(\sin y)y'' (y,)2 + 3(\sin y)y''(y,)2 + 3(\sin y)y'''(y,)2 + 3(\sin y)y'''(y,)2 + 3(\sin y)y'''(y,)2 + 3(\sin y)y'''(y,)2$ 0, yH = 1, yIIt = 0 into (4), we obtain $yIV(1) = -(\sin 11^{"})(0) - (3\cos)(1)(0) + (\sin)(0)2 = 0$. S. "", IV O. ubstluting y = , y' = 0, y = 1, Y = 1, Y = 0, Y = 1, Y = 1,2(x-1)5Y = + + 3 + ... 2!5!(x-1)2(x-1)5 = 11" + + + ... 240 576 Chapter 8 (b) 'We assume y = Co + c1(x 2 3 1) + c2(x - 1) + c3(x - 1) + c3(x - 1) + c5(x - 1)= c 1 + 2c 2 (x 2 1) + 3c 3 (x - 1) + 4c 4 (x 1) 3 4 + 5c 5 (x - 1) + (7) Since the initial y value is , we must express cosy In the D.E. in powers of y -. By Taylor's Theorem, CD ; (n) () n n! (y -) ; (y) = L n = 0 with ; (y) = cos y, we obtain cos y = -1 + (y -) 2 (y -) 4 2! 4! + ... Approximate Methods 577 We also express x ln powers of x - 1, and then the D.E. takes the form y' = [(x - 1) + 1] + [-1 + (y - r) 2 4 [x - 1) + 1] + [-1 + (y - r) 2 4 (x - 1) + ...] 24 + ... + Cx - 1] + [c] [cx - 1] 24y'(l)(x - 1) + y 2!(x - 1) y'''(1) 3 + 3!(x - 1) + The I.C. states that y(l) = 2, and the D.E. glves y'(l) = e + 2. Differentiating the D.E., we obtain "() x y = y' + x + 1 e, III "(x + 3)e x y - y + -, IV III (x +
3)e x y - y + -, IV III (x + 3is y = 2 + (e + 2)(x + 1) + (e + 1)(x + 1)2 + (e + 1)(x + 1)(x + 1)2 + (e + 1)(x + 1)(x + 1)2 + (e + 1)(x + 1(1) 3 + + c 4 x - x We also need to expand xe ln powers of x - 1. (1) (2) We have x [(1) J e (X - 1) + 2T (x - 1) + 2 ve obtain Cl + 2c 2 (x - 1) + 3c 3 (x - 1) + 4c 4 (x - 1) + ... 2 3 = 2 + C 1 (x - 1) + c 2 (x - 1) + c 3 (x - 1) + ... 32 3 + e + 2e(x - 1) + 3 e(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1) + (e + c 2)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)(x - 1)(x - 1)(x - 1) + ... = (e + 2) + (2e + c 1)(x - 1)function defined by ; O(x) = 0 for all x. The nth approximation n for n = 1, 2, 3, successively we obtain 'l(x) = j(X {1 + t[O(t)]2} dt o = j(X o (1 + 0) dt - x, '2(x) x x = j({1 + t[l(t)]2} dt o 0 4 x = x + 4' (1 + t[O(t)]2 dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o 0 4 x = x + 4' (1 + t[O(t)]2) dt o 0 4 x = x + 4' (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = 1, 2, 3, successively we obtain 'l(x) = j(X (1 + t[O(t)]2) dt o = 1, 2, 3, successively we obtain 'l(x) = 1, 3, 3, successively we o $(3(x) = j(X \{1 + t[2(t)]2\} dt o = X [1 + t (t + t 4 4)2] dt X (1 + t 3 t 6 t 9) = + - + 16 dt 2 4 7 10 x x x = x + 4 + 14 + 160$. 582 Chapter 8 5. Since the initial y value is 0, we choose the zeroth approximation; for n > 1 is given by n, (x) = n Yo + IX Xo f[t, ;n-l(t)] dt x = 0 + Jl {e t (x)} + J(x) + J($+ [n l(t)]_2 dt, n > 1. o Using this formula for n = 1, 2, 3, successively we obtain; l(x) = rX tt J (e + 0) dt = e o x x = e - 1. o; 2(X) = IX ott 2 [e + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e 2t - e t + 1) dt o 2t e = --2 x 2x e x = --e 2 1 + x + 2' te + to 3(X) = J: X [e t + (e - 1)] dt x - I (e - 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 1 + x + 2' te + 1) dt a = -2 x 2x e x = --e 2 x 2x e x = --e 2 x 2x e x =$ e 4t e 3t 2t (2t - 1) 3e2t 16 3 + e 4 + 4tt 3t 2 + t] x - 2e (t - 1) + 3 + 2 4 0 4x 3x 2x 2x e e xe e = --+ + - 16 3 2 2 3 2 107 2xe x x x x + 2e + -- + -- 48 . 3 2 4 7. As in the solutions of Exercises 4 and 5, we choose ; o(x) = 0 for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; $n(x) = YO + Ix f[t, ;n-1(t)] dt Xo x = 0 + j({2t + [n l(t)]2} dt. o Using this solutions of Exercises 4 and 5, we choose ; <math>o(x) = 0$ for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n; n(x) = 0 for all x. For n 1, ; (x) IS given by n formula for n = 1, 2, 3, successively we obtain 584 Chapter 8 x; 1 (x) = i [2t + (0)3] dt = o 2 x, x 7; 2(x) = i [2t + (t 2)3] dt = x 2 + x 7' o; 3(x) - J:X [2t + [t 2 + t;)3] dt]: x [2t 6 3t 11 3t 16 t 21) = + t + + + 343 dt 7 49 2 7 12 3x 17 22 x x x = x + - + + + 7546. 7 28 833 NOTE In the solutions of Sections 8.4 through 8.7, the exact solutions of the problems in Exercises 1, 4, 5,8,9, and 12 are required. The D.E.'s In Exercises 5 and 8 are both linear and separable. We list the exact solution of each of these problems here: 1.1-2x v = 4 (2x - 1 + 5e). 4.1 (-2x - 1 3 2 2x) y = 4 + e e . 2 5. y x /2 - 2x + 2 = e . Approximate lethods 585 8. 1 1 - cos x y = - e 2 9. Y = 1:. j 25x 2 + 1 . 5 12. Y = y3 - 2 cos x. Section 8.4, Page 447. General Information: For each problems n were done maintaining 11 to 12 digit accuracy from step to step; however, we've rounded the results to fewer places (usually four to six) as we tabulate them below. 1. Let h = 0.2 and $f(x,y) = x - 2y \ln (8.53)$; we have $X_0 = 0$ and $Y_0 = 1.0000 + (0.2)f(0.0, 1.0000) = 1$ -2(1.0000) = 0.6000. (b) x 2 = xl + h = 0.2 + 0.2 = 0.4. To find Y2' we use (8.53) with n = 1: Y2 = Y1 + hf(x 1 'Y1) = 0.6000 + (0.2)f(0.2, 0.6000) = 0.4000 + (0.2)f(0.2, 0.6000) = 0. (0.4 - 2(0.4000)) = 0.3200. Proceeding in this manner, uSlng (8.53) with n = 3 and n = 4, we successively obtain Y4 = 0.3120 corresponding to x 5 = 1.0. All results and errors are summarized in
Table 8.4.1. The exact solution of the differential equation IS given in the pages immediately preceding this section of this manual. TABLE 8.4.1 Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Euler Error % ReI X n Solution Method Error 0.2 0.687900 0.106493 24.969419 0.8 0.402371 0.312000 0.090371 22.459553 1.0 0.419169 0.347200 $0.071969 \ 17.169468 \ 4.$ Let h = 0.2 and $f(x,y) = x + 2y \ln (8.53)$; we have Xo = -1 and Yo = 1. (a) xl = x + h = -0.8 + 0.2 = -0.6. To find Y1' we use (8.53) with n = 0: Y1 = yO + hf(xO'YO) = 1.0000 + (0.2)f(-1.0, 1.0000) = 1.0000 + (0.(8.53) with n = 1: Y 2 = Y 1 + hf(x 1, y 1) = 1.2000 + (0.2)f(-0.8, 1.2000) = 1.5200 + (0.2)(-0.8 + 2(1.2000)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8 + 2(1.200)) = 1.5200 + (0.2)(-0.8)(-0.8)(-0.8)(-0.8)(-0.8)(successively obtain Y4 = 2.7312 corresponding to $x \ 5 = 0.0$. All results and errors are summarized in Table 8.4.4. The exact solution of the differential equation is given in the pages immediately preceding this section of this manual. TABLE 8.4.4 Euler Method for y' = x + 2y, y(-1) = 1, with h = 0.2. Exact Euler Error % ReI X n Solution Method Error -0.8 1.268869 1.200000 0.068869 5.427554 -0.6 1. 719156 1.520000 0.199156 11. 584506 -0.4 2.440088 2.008000 0.432088 17.707876 -0.2 3.564774 2.731200 0.833574 23.383649 0.0 5.291792 3.783680 1.508112 28.499080 588 Chapter 8 5. Let h = 0.1 and f(x,y) = xy - 2y ln (8.53); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl we use (8.53) with n = 0. Y1 = Y + -. 0 hf(x0'YO) = 1.0000 + (0.1)((2.0)(1.0000) = 1.0000 + (0.1)((2.0)(1.000)) = 1.0000 + ((0.1)((2.1)(1.0000) - 2(1.0000)) = 1.0000 + (0.1)(0.1000) = 1.0100. (c) x 3 = x 2 + h = 2.2 + 0.1 = 2.3. To find Y3' we use (8.53) with n = 2: Y3 = Y 2 + hf(x 2, y 2) = 1.0100 + (0.1)((2.2)(1.0100) - 2(1.0100)) = 1.0100 + (0.1)(0.2020) = 1.0302. Proceeding in this manner, uSlng (8.53) with n = 3 and n = 4, we successively obtain Y4 = 1.0611 corresponding to x 4 = 2.4, and Y5 = 1.1036 corresponding to x 5 = 2.5. All results and errors are summarized in Table 8.4.5. The exact solution of this manual. Approximate Methods 589 TABLE 8.4.5 Euler Method for y' = xy - 2y, y(2) = 1, with h 0.1 Exact Euler Error % Rei X n Solution Method Error 2.1 1.005013 1.000000 0.005013 0.498752 2.2 1.020201 1.010000 0.010201 0.999934 2.3 1.046028 1.030200 0.015828 1.513139 2.4 1.083287 1.061106 0.022181 2.047571 2.5 1.133148 1.103550 0.029598 2.612033 8. Let h = 0.2 and f(x,y) - y Sln x ln (8.53); - we have Xo = 0 and YO - 0.5. - (a) xl = x + h = 0.0 + 0.2 = 0.2. 0 To find Yl' we use (8.53) with n = 0: Y1 = Y0 + hf(x 0'y0) = 0.5000 + (0.2)f(0.0, 0.5000) = 0.5000 + (0.2)f(0.2, 0.500) = 0.5000 + (0.2)f(0.2, 0.500) $= 0.5000 + (0.2)(0.0993) = 0.5199.(c) \times 3 = \times 2 + h = 0.4 + 0.2 = 0.6$. To find Y3' we use (8.53) with n = 2: Y3 = Y2 + hf(x 2 'Y2) = 0.5199 + (0.2)(0.2024) = 0.5604. 590 Chapter 8 Proceeding in this manner, uSlng (8.53) with n = 3 and n = 4, we successively obtain Y4 = 0.6236 corresponding to x 4 = 0.8, and Y5 = 0.7131 corresponding to x 5 = 1.0. All results and errors are summarized in Table 8.4.8. The exact solution of this manual. TABLE 8.4.8 Euler Method for y' = y Slnx, y(O) = 0.5, with h = 0.2. Exact Euler Error % Rei X n Solution Method Error 0.2 0.510067 0.500000 0.010067 1.973607 0.4 0.541069 0.519867 0.021202 3.918579 0.6 0.595423 0.560356 0.035067 5.889452 0.8 0.677156 0.623636 0.053520 7.903582 1.0 0.791798 0.713110 0.078687 9.937829 9. Let h = 0.2 and f(x,y) = x/y ln (8.53); we have Xo = 0 and Yo = 0.2. (a) xl = Xo + h = 0.0 + 0.2 = 0.2. To find Yl' we use (8.53) with n = 0: Y1 = Y0 + hf(x0'YO) = 0.2000 + (0.2)f(0.0, 0.2000) = 0.2000 + (0.2)(0.0/0.2000) = 0.2000 + (0.2)f(0.2, 0.2000) = 0.2000 + (0.2)f(0we use (8.53) with n = 2: Y3 hf(x 2 'Y2) = 0.4000 + (0.2)f(0.4, 0.4000) = 0.6000. = Y + 2 0.4000 + Proceeding in this manner, uSlng (8.53) with n = 3, n = 4, ..., n = 9, we successively obtain Y4 = 0.8000 corresponding to x 4 = 0.8, Y 5 = 1.000 corresponding to x 10 = 2.0000 corresponding to x 10 = 2.00000 corresponding to x 10 = 2.000000 corresponding to x 10 = 2.0000000000 corresponding to x 10 = 2.00000Results for all x and the errors are n summarized in Table 8.4.9. The exact solution of the differential equation is given in the pages immediately preceding this section At a solution Method Error 0.2 0.282843 0.200000 0.082843 29.289322 0.4 $0.447214\ 0.400000\ 0.047214\ 10.557281\ 0.6\ 0.632456\ 0.600000\ 0.032456\ 5.131670\ 0.8\ 0.824621\ 0.800000\ 0.0146553\ 1.\ 360608\ 1.4\ 1.414214\ 1.400000\ 0.014214\ 1.005051\ 1.6\ 1.612452\ 1.600000\ 0.012452\ 0.772212\ 1.8\ 1.811077\ 1.800000\ 0.011077\ 0.611627\ 2.0$ $2.009975 \ 2.000000 \ 0.009975 \ 0.496281 \ 592 \ Chapter 8 \ 12.$ Let h = 0.2 and $f(x,y) = (sinx)/y \ln (8.53)$; we have $X_0 = 0$ and $Y_0 = 1$. (a) $x_1 = X_0 + h = 0.2 + 0.2 = 0.2$. To find YI' we use (8.53) with n = 0: YI = YO + hf(xo'Y O) = 1.0000 + (0.2)((sin(0.0)/1.0000) = 1.0000 + (0.2)((sin(0.0)/1.000)) = 1.0000 + 0.4. To find Y 2, we use (8.53) with n = 1: Y 2 = Y 1 + hf(x 1, y 1) = 1.0000 + (0.2)f(0.2, 1.0000) = 1.0000 + (0.2)(sin(0.2)/1.0000) = 1.0397 + (0.2)f(0.4, 1.0397) = 1.0397 + (0.2)f(0.4, 1.0397) = 1.0397 + (0.2)(sin(0.4)/1.0397) = 1.0397 + (0 (0.3745) = 1.1146. Proceeding in this manner, uSlng (8.53) with n = 3, n = 4, ..., n = 9, we successively obtain Y4 = 1.2160 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 10 = 2.0. Results for all x n and corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 10 = 2.0. Results for all x n and corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to
x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 10 = 2.0. Results for all x n and corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 4 = 0.8, Y5 = 1.3339 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 corresponding to x 5 = 1.0, ..., and Y10 = 1.9352 correspo exact solution of the differential equation is given in the pages immediately preceding this section of this manual. TABLE 8.4.12 Euler Method for y' = (sinx)/y, y(O) = 1, with h = 0.2. Exact Euler Error % Rei X n Solution Method Error 0.2 1.019739 1.000000 0.019739 1.935654 0.4 1.076047 1.039734 0.036314 3.374715 0.6 1.161606 1.114641 0.046965 4.043104 0.8 1.267512 1.215955 0.051557 4.067577 1.0 1. 385422 1.333946 0.051477 3.715614 1.2 1.508405 1.460108 0.048296 3.201819 1.4 1.630971 1.587775 0.043195 2.648449 1.6 1.748828 1.711905 0.036923 2.111296 1.8 1.858603 1.828684 0.029919 1.609740 2.0 1.957624 1.935192 0.022432 1.145899 Section 8.5, Page 454 General Information: For each problem below, we show the first three calculations in detail, while summarizing calculations at each value of x ln a table. The problems n were done maintaining 11 to 12 digit accuracy from step to step; however, we've rounded the results to fewer places (usually four to six) as we tabulate them below. 1. Let h = 0.2 and f(x,y) = x - 2y in (8.59) and (8.60); we have xo = 0 and Yo = 1. (a) xl - x + h = 0.0 + 0.2 = 0.2. 0 To find Y1' we use (8.59) and (8.60) with n = 0. First using (8.60) with this value of Y1 we obtain f(xo'YO) + f(x + 1) = 0.2 + 0.2 = 0.2. 0 To find Y1' = YO + hf(xo, yo) = 0.60000. Now using (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. $Y_1 = Y_0 + h_2 = 1.0000 + (0.2) f(0.0, 1.0000) ; f(0.2, 0.6000) = 1.0000 + (0.2) -2.0000 ; 1.0000 = 0.7000 = 0.7000 ; 1.0000 = 0.7000 ; 1.0000 = 0.7000 ; 1.0000 = 0.7000 ; 1.0000 = 0.7000 ; 1.0000 = 0.7000 ;$ Y2 we obtain f(x 1, y 1) + f(x 2, y 2) Y 2 = Y1 + h 2 = 0.7000 + (0.2) f(0.2, 0.7000); f(0.4, 0.4600) - 0.7000 + (0.2) - 1.2000; 0.5200 - 0.5280. Approximate Methods 595 (c) x 3 - x + h = 0.4 + 0.2 = 0.6. - 2 To find Y3' we use (8.59) and (8.60) with n = 2. First using (8.59), we find "Y3 = Y2 + hf(x 2, y 2) = 0.5280 + (0.2)f(0.4, 0.5280) = 0.5280 + (0.2)f(0.4, (0.2)(-0.6560) = 0.3968. "- Now uSlng (8.60) with this value of Y 3 we obtain f(x 2, y 2) + f(x 3 'Y3) Y3 = Y 2 + h 2 = 0.5280 + (0.2) f(0.6, 0.3968) = 0.5280 + (0.2) f(0.4, 0.5280); f(0.6, 0.5280); f(0. equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.5.1 Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, with h = 0.2. Exact Improved Euler Method for y' = x - 2y, y(O) = 1, $0.417267 \ 0.014897 \ 3.7022 \ 1.0 \ 0.419169 \ 0.431742 \ 0.012573 \ 2.9994 \ 596 \ Chapter \ 8 \ 4. \ Let \ h = 0.2 \ and \ f(x,y) - x + 2y \ ln \ (8.59) \ and \ (8.60) \ ; - we \ have \ x = -1 \ and \ YO = 1 \ . \ 0 \ (a) \ xl = x + h = -1.0 + 0.2 = -0.8. \ 0 \ To \ find \ Yl \ we \ use \ (8.59) \ and \ (8.59), we \ find \ Yl = YO + hf(xo'YO) = 1.0000 + (0.2)f(-1.0, \ 1.0000) = 1.0000 + (0.2)f(-1.0, \ 1.000) = 1.0000 + ($ (1.0000) = 1.2000. Now uSing (8.60) with this value of Y1 we obtain f(xo'YO) + f(x 1, y 1) Y1 = YO + h 2 = 1.0000 + (0.2) f(-1.0, 1.0000) + f(-0.8, 1.2000) + f(-0.8, 1.200 (0.2)f(-0.8, 1.2600) = 1.2600 + (0.2)(1.7200) = 1.6040. Now uSing (8.60) with this value of Y 2 we obtain Approximate liethods 597 f(x 1 'Y1) + f(x 2 'Y2) Y 2 = Y 1 + h 2 = 1.2600 + (0.2) f(-0.8, 1.2600) + f = 2. First uSlng (8.59), we find Y 3 = Y 2 + hf(x 2, y 2) = 1.6928 + (0.2)f(-0.6, 1.6928) = 1.6928 + (0.2)f(-0.6, 1.692 remaining x 's, as well as n the errors are summarized in Table 8.5.4. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. 598 Chapter 8 TABLE 8.5.4 Improved Euler Method for y' = x + 2y, y(-1) = 1, with h = 0.2. Exact Improved Error % ReI X n Solution Euler Error -0.8 1.268869 1 $260000 \ 0.008869 \ 0.6989 \ -0.6 \ 1. \ 719156 \ 1.692800 \ 0.026356 \ 1.5331 \ -0.4 \ 2.440088 \ 2.381344 \ 0.058744 \ 2.4074 \ -0.2 \ 3.564774 \ 3.448389 \ 0.116385 \ 3.2649 \ 0.0 \ 5.291792 \ 5.075616 \ 0.216176 \ 4.0851 \ 5.$ Let h = 0.1 and f(x,y) = xy - 2y ln (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 2.0 + 0.1 = 2.1. 0 To find Yl' we use (8.59) and (8.60); we have Xo = 2 and YO = 1. (a) xl = x + h = 0. First using (8.59), we find A Yl = YO + hf(xo'YO) = 1.0000 +
(0.1)f(2.0, 1.0000) = 1.00 (8.59) and (8.60) with n = 1. Approximate Methods 599 First using (8.59), we find Y2 = Y1 + hf(x 1, y 1) = 1.0050 + (0.1)f(2.1, 1.0050) = 1.0050 + (0. (c) x 3 = x 2 + h = 2.2 + 0.1 = 2.3. To find Y 3, we use (8.59) and (8.60) with n = 2. First using (8.59), we find Y 3 = Y 2 + hf(x 2, y 2) = 1.0202 + (0.1)f(2.2, 1.0202) = 1.0202 + (+ (0.1) 0.2040 + 0.3122 1.0460. = 2 600 Chapter 8 These results and those for the remaining x '8, as well as n the errors are summarized in Table 8.5.5. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.5.5 Improved Euler Method for y' = xy - 2y, y(2) = 1, with h = 0.1. Exact Improved Error % ReI X n Solution Euler Error 2.1 1.005013 1.005000 0.000013 0.0012 2.2 1.020201 1.020175 0.000026 0.0025 2.3 1.046028 1.045986 0.000042 0.0040 2.4 1.083287 1.083223 0.000064 0.0059 2.5 1.133148 1.133051 0.000097 0.0086 8. Let h = 0.2 and $f(x,y) = y \operatorname{Sln} x \ln (8.59)$ and (8.60); we have Xo = 0 and Yo = 0.5. (a) xi = 0.2 x + h = 0.0 + 0.2 = 0.2.0 To find Yl' we use (8.59) and (8.60) with n = 0. First using (8.59), we find Yl = yO + hf(xo'YO) = 0.5000 + (0.2)f(0.0, 0.5000) = 0.5000 + (0.2)f(0.0, 0.500) 0.5000 + (0.2) 0.0000; 0.0993 = 0.5099. (b) x 2 = xl + h = 0.2 + 0.2 = 0.4. To find Y 2 = yl + hf(x 1, y 1) + f(x 2, y 2) Y2 = Y1 + h2 = 0.5099 + (0.2)f(0.2, 0.5099) = 0.5099 + (0.2 (0.5099); f{(0.4, 0.5302) = 0.5099 + (0.2) 0.1013; (0.2065 - 0.5407, (c) x 3 = x + h = 0.4 + 0.2 = 0.6.2 To find Y 3, we use (8.59) and (8.60) with n - 2. - First uSlng (8.60) with n - 2. - F 3'Y3)Y3 = Y2 + h2 = 0.5407 + (0.2) f(0.4, 0.5407); feO.6, 0.5828) = 0.5407 + (0.2) 0.2106; 0.3291 = 0.5947. These results and those for the remaining x's, as well n as the errors are summarized in Table 8.5.8. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.5.8 Improved Euler Method for y' = y sin x, y(O) = 0.5, with h = 0.2. Exact Improved Error % ReI X n Solution Euler Error 0.2 0.510067 0.509933 0.000133 0.0261 0.4 0.541069 0.540711 0.000358 0.0662 0.6 0.595423 0.594676 0.000747 0.1255 0.8 0.677156 0.675731 0.001425 0.2104 1.0 0.791798 0.789224 0.002574 0.3251 9. Let h = 0.2 and f(x,y) = 0.5, with h = 0.2. Exact Improved Error % ReI X n Solution Euler Error 0.2 0.510067 0.509933 0.000133 0.0261 0.4 0.541069 0.540711 0.000358 0.0662 0.6 0.595423 0.594676 0.000747 0.1255 0.8 0.677156 0.675731 0.001425 0.2104 1.0 0.791798 0.789224 0.002574 0.3251 9. Let h = 0.2 and f(x,y) = 0.5, with h = 0.2. Exact Improved Error % ReI X n Solution Euler Error 0.2 0.510067 0.509933 0.000133 0.0261 0.4 0.541069 0.540711 0.000358 0.0662 0.6 0.595423 0.594676 0.000747 0.1255 0.8 0.677156 0.675731 0.001425 0.2104 1.0 0.791798 0.789224 0.002574 0.3251 9. Let h = 0.2 and f(x,y) = 0.5, with h = 0.2. Exact Improved Error % ReI X n Solution Euler Error 0.2 0.510067 0.509933 0.000133 0.0261 0.4 0.541069 0.540711 0.000358 0.0662 0.6 0.595423 0.594676 0.000747 0.1255 0.8 0.677156 0.675731 0.001425 0.2104 1.0 0.791798 0.789224 0.002574 0.3251 9. Let h = 0.2 and f(x,y) = 0.5, with h = 0.2 and f(x,y) = 0.5, with h = 0.5, w $x/y \ln (8.59)$ and (8.60); we have Xo = 0 and Yo = 0.2. (a) xl - x + h - 0.0 + 0.2 = 0.2. - 0 - To find Yl' we use (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. First using (8.59) and (8.60) with n = 0. = YO + h 2 = 0.2000 + (0.2) f(0.0, 0.2000) = 0.2000 + (0.2) f(0.2, 0.2000) = 0.2000 + (0.2) f(0.2, 0.2000) = 0.3000 + (0.2) f(0.2, 0.3000) = 0.3000 + (0.2)f(x 1, y 1) + f(x 2, y 2) Y 2 = Y 1 + h 2 = 0.3000 + (0.2) f(0.4, 0.4333) = 0.3000 + (0.2) 0.6667; 0.9231 = 0.4590 + (0.2)(0.8715) - 0.6333.Now using (8.60) with this value of Y3 we obtain f(x 2, y 2) + f(x 3'Y3) Y 3 = Y 2 + h 2 = 0.4590 + (0.2) f(0.4, 0.4590); f(0.6, 0.6333) = 0.4590 + (0.2) f(0.4, 0.4590); f(immediately preceding Section 8.4 of this manual. Approximate Methods 605 TABLE 8.5.9 Improved Euler Method for y' = x/y, y(O) = 0.2, with h = 0.2. Exact Improved Euler Methods 605 TABLE 8.5.9 Improved Euler Method for y' = x/y, y(O) = 0.2, with h = 0.2. Exact Improved Euler Method for y' = x/y, y(O) = 0.2, with h = 0.2. Exact Improved Euler Method for y' = x/y, y(O) = 0.2, with h = 0.2. Exact Improved Euler Method for y' = x/y, y(O) = 0.2, with h = 0.2. Exact Improved Euler Method for y' = x/y, y(O) = 0.2, $0.831098 \ 0.006477 \ 0.7854 \ 1.0 \ 1.019804 \ 1.025049 \ 0.005245 \ 0.5144 \ 1.2 \ 1.216553 \ 1.220953 \ 0.004401 \ 0.3617 \ 1.4 \ 1.414214 \ 1.414201 \ 0.003787 \ 0.2678 \ 1.6 \ 1.612452 \ 1.615774 \ 0.003323 \ 0.2061 \ 1.8 \ 1.811077 \ 1.814036 \ 0.002959 \ 0.1634 \ 2.0 \ 2.009975 \ 2.012642 \ 0.002667 \ 0.1327 \ 12.$ Let h = 0.2 and f(x,y) = (sinx)/y ln (8.59) and (8.60); we have Xo = 0 and YO = 1. (a) xl - x + h = 0.0 + 0.2 = 0.2. - 0 To find Yl' we use (8.59) and (8.60) with n = 0. First uSlng (8.59), we find Yl = YO + hf(xo'YO) = 1.0000 + (0.2)f(0.0, 1.0000) = 1.0 + 0.1987 = 1.0000 + (0.2) 2 = 1.0199. 606 Chapter 8 (b) x 2 = x + h = 0.2 + 0.2 = 0.4.1 To find Y2' we use (8.59) and (8.60) with n = 1. First using (8.59), we find Y 2 = Y 1 + hf(x 1, y 1) = 1.0199 + (0.2)f(0.2, 1.0199) = 1.0199 + (0.2)f(1.0199 + (0.2) f(0.2, 1.0199); f(0.4, 1.0588) = 1.0199 + (0.2) (0.3619) = 1.0761 + (0.2)(0.361Approximate Methods 607 f(x 2 'Y2) + f(x 3 'Y3) Y 3 = Y 2 + h 2 = 1.0761 + (0.2) f(0.4, 1.0761) + f(0.6, 1.1485) 2 = 1.0761 + (0.2) 0.3619; 0.4916 = 1.1615. These results and those for the remaining x 's, as n well as the errors are summarized in Table 8.5.12. The exact solution of the differential equation IS given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.5.12 Improved Euler Method for y' = (sin x) /y, y(O) = 1, with h = 0.2. Exact Improved Error % Rei X n Solution Euler Error 0.2 1.019739 1.019867 0.000128 0.0126 0.4 1.076047 1.076125 0.000078 0.0072 0.6 1.161406 1.161476 0.000130 0.0112 0.8 1.267512 1.267082 0.000430 0.0340 1.0 1.385422 1.384659 0.000764 0.0551 1.2 1.508405 1.507310 0.001095 0.0726 1.4 1.630971 1.629565 0.001405 0.0862 1.6 1.748828 1.747140 0.001688 0.0965 1.8 1.858603 1.856666 0.001937 0.1042 2.0 1.957624 1.955473 0.002152 0.1099 Section 8.6, Page 462. General Information: For each problem below, we show the first two calculations In detail, while summarizing calculations at each value of x ln a table. The problems n were done maintaining 11 to 12 digit accuracy from step to 608 Chapter 8 step; however, we've rounded the results to fewer places as we tabulate them below. 1. Let h = 0.2 and $f(x,y) = x - 2y \ln (8.59)$ and (S.60); we have Xo = 0 and YO = 1. k = 1 = hf(xo'YO) = 0.2f(0.0, 1.0000000)= 0.2(-2.0000000) = -0.4000000, k2 = hf(x O + h/2, YO + k 1/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k3 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k
2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S000000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00000) = -0.3200000, k4 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 0.S00(-0.4000000 - 0.6400000 - 0.6400000 - 0.6400000 - 0.2320000)/6 = -0.3120000, and so the approximate value of the solution at xl = 0.2 lS Y1 = 1.0000000 - 0.3120000 = 0.6880000. Approximate Methods 609 Now using the above value Y1' we calculate successively new k1, k2, k3, k4, and then K. Ye first find k1 = hf(x1, y1) = 0.2f(0.2, 0.6880000) = 0.6880000 = 0.6880000 = 0.6400000 - 0.3120000 = 0.6880000. $0.2(-1.1760000) = -0.2352000, k_2 = hf(x_1 + h/2, Y_1 + k_1/2) = 0.2f(0.3, 0.5704000) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_2/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_2/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.6039200) = -0.1815680, k_4 = hf(x_1 + h, Y_1 + k_3/2) = 0.2f(0.3, 0.603920) = -0.1815680, k_4 = hf(x_1 + h, Y_1 +$ 0.3363200 - 0.3631360 - 0.1225728)/6 = -0.1762048, and so the approximate value of the solution at x 2 = 0.4 lS Y 2 = 0.6880000 - 0.1762048 = 0.5117952. We summarize the calculations for these and the remaining y 's in Table 8.6.1. The exact solution of the n 610 Chapter 8 differential equation 1S given 1n the pages immediately preceding Section 8.4 of this manual. TABLE 8.6.1 Runge-Kutta Method for x - 2y, y(O) = 1, with h = 0.2. k 1 k 2 k3 k4 K Exact Runge- Error % ReI X n Solution Kutta Error 0.2 -0.4000 -0.3200 --0.0888 - 0.0492 - 0.0852 0.426493 0.426628 0.000135 0.031592 0.8 - 0.0507 - 0.0205 - 0.0265 0.0000 - 0.0241 0.402371 0.402371 0.402491 0.000101 0.024075 4. Let h = 0.2 and f(x,y) = x + 2y 1n (8.59) and (8.60); we have xo = -1 and YO = 1. k 1 = hf(xo'YO) = 0.2f(-1.0, 1.0000000 = 0.2(1.0000000) = 0.2(1.0000000) = 0.2(1.7440000) =
0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.7440000) = 0.2(1.744 $1 + 2k^2 + 2k^3 + k^4$ //6 (0.2000000 + 0.5200000 + 0.5200000 + 0.5440000 + 0.3488000)/6 0.2688000, and so the approximate value of the solution at xl = -0.8 lS Y1 = 1.0000000 + 0.2688000 = 1.2688000 = 1.2688000 = 0.2680 $0.2(1.7376000) = 0.3475200, k_2 = hf(x_1 + h/2, Y_1 + k_1/2) = 0.2f(-0.7, 1.4425600) = 0.2(2.1851200) = 0.4370240, k_3 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.2f(-0.7, 1.4873120) = 0.2(2.2746240) =$ (0.3475200 + 0.8740480 + 0.9098496 + 0.5694899)/6 = 0.4501513, and so the approximate value of the solution at x 2 = -0.6 1S Y2 = 1.2688000 + 0.4501513 = 1.7189513. We summarize the calculations for these and the remaining y 's in Table 8.6.4. The exact solution of the n differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.6.4 Runge-Kutta Method for y' = x + 2y, y(-1) = 1, with h = 0.2. $k \mid k \mid 2$ ks k4 K Exact Runge-Error % ReI X n Solution Kutta Error -0.8 0.2000 0.2600 0.2720 0.3488 0.2688 1.268869 1.26889 1.268869 1.26889 1.26889 1.26889 1.2688 $0.7278 \ 0.8987 \ 0.7207 \ 2.440088 \ 2.439630 \ 0.000457 \ 0.018748 \ -0.2 \ 0.8959 \ 1.0950 \ 1.1349 \ 1.3898 \ 1.1242 \ 3.563864 \ 0.000910 \ 0.025526 \ 0.0 \ 1.3855 \ 1.6827 \ 1.7421 \ 2.1224 \ 1.7262 \ 5.291792 \ 5.290095 \ 0.001697 \ 0.032064 \ Approximate Methods \ 613 \ 5. Let h = 0.1 and f(x,y) = xy - 2y \ In \ (8.59) and \ (8.60); we have Xo = 2 and YO = 1. k \ 1 = hf(xo'YO)$ = 0.1f(2.0, 1.0000000) = 0.1(0.0000000) = 0.1(0.0000000) = 0.0000000, k2 = hf(x O + h/2, YO + k 1/2) = 0.1f(2.05, 1.0000000) = 0.1(0.0501250) = 0.0050125, k4 = hf(x O + h, YO + k 3) = 0.1f(2.10, 1.0050125) = 0.01005012, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.1f(2.05, 1.0000000) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) = 0.0050125, k4 = hf(x O + h/2, YO + k 2/2) =00 + 0.0100000 + 0.0100250 + 0.0100501/6 0.0050125, and so the approximate value of the solution at xl = 2.1 lS Y1 = 1.0000000 + 0.0050125 = 1.0050125. 614 Chapter 8 Now using the above value Y1' we calculate successively new k 1, k 2, k 3, k 4, and then K. Ye first find k 1 - hf(x 1 'Y1) = 0.1f(2.1, 1.0050125 = 1.0050125) 0.1(0.1005013) = 0.0100501, $k_2 = hf(x_1 + h/2, Y_1 + k_1/2) = 0.1f(2.15, 1.0100376) = 0.1(0.1515056) = 0.0151506$, $k_3 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878)
= 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.01515888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.1518882) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(2.15, 1.0125878) = 0.1(0.151888) = 0.0151888$, $k_4 = hf(x_1 + h/2, Y_1 + k_3/2) = 0.1f(x_1 + h/2, Y_1 + h/2, Y_1 + k_3/2) = 0.1f(x_1 + h/2, Y_1 + h/2, Y_1 + h/2, Y_1 + h/2) = 0.1f(x_1 + h/2, Y_1 + h/2, Y_1 + h/2) = 0.1f(x_1 + h/2, Y_1 + h/2, Y_1 + h/2) = 0.1f(x_1 + h/2, Y_1 + h/2) = 0.1f(x_1 +$ 0.0303011 + 0.0303776 + 0.0204040)/6 0.0151888, and so the approximate value of the solution at x 2 = 2.2 lS Y 2 = 1.0050125 + 0.0151888 = 1.0202013. Approximate Methods 615 We summarize the calculations for these and the remaining y 's in Table 8.6.5. The exact solution of the n differential equation lS given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.6.5 Runge-Kutta Method for y' = xy - 2y, y(2) = 1, with h = 0.1. k 1 k 2 ks k4 K Exact Runge- Error % Rei X n Solution Kutta Error 2.1 0.0000 0.0050 0.0050 1.005013 1.005013 1.005013 0.000000 2.2 0.0101 0.0152 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.020201 1.020201 1.020201 1.020201 0.000000 2.3 0.0204 0.0152 1.020201 1.0 $0.0258\ 0.0258\ 0.0314\ 0.0258\ 1.046028\ 1.046028\ 0.000000\ 0.000000\ 2.4\ 0.0314\ 0.0372\ 0.0373\ 0.0433\ 0.0373\ 1.083287\ 1.083287\ 0.000000\ 0.000000\ 2.5\ 0.0433\ 0.0499\ 0.0567\ 0.0499\ 1.133148\$ 0.5000000 = 0.2(0.0000000) = 0.2(0.0000000) = 0.2(0.0000000) = 0.2(0.1013379) = 0.2(0.101 $2k_3 + k_4$)/6 (0.0000000 + 0.0199667 + 0.0201660 + 0.0202676)/6 0.0100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667 = 0.5100667, and so the approximate value of the solution at xl = 0.2 IS Yl = 0.5000000 + 0.0100667, and so the approximate value of the solution at xl = 0.5000000 + 0.0100667, and so the approximate value of the solution at xl = 0.5000000 + 0.0100667, and so the approximate value $0.0202669, k_2 = hf(x_1 + h/2, Y_1 + k_1/2) = 0.2f(0.3, 0.5202002) = 0.2(0.1537297) = 0.0307459, k_3 = hf(x_1 + h/2, Y_1 + k_2/2) = 0.2f(0.3, 0.5254397) = 0.2(0.1552780) = 0.0210556, k_4 =
hf(x_1 + h, Y_1 + k_3) = 0.2f(0.4, 0.5411223) = 0.2f(0.4, 0.5$ + 0.0614919 + 0.0621112 + 0.0421446)/6 = 0.0310024, and so the approximate value of the solution at x 2 = 0.4 1S Y2 = 0.5100667 + 0.0310024 = 0.5410691. We summarize the calculations for these and the remaining y 's in Table 8.6.8. The exact solution of the n differential equation IS given in the pages immediately preceding Section 8.4 of this solution at x 2 = 0.4 1S Y2 = 0.5100667 + 0.0310024 = 0.5410691. We summarize the calculations for these and the remaining y 's in Table 8.6.8. The exact solution of the n differential equation IS given in the pages immediately preceding Section 8.4 of this solution at x 2 = 0.4 1S Y2 = 0.5100667 + 0.0310024 = 0.5410691. manual. TABLE 8.6.8 Runge-Kutta Method for y' = y Sln x, y(O) = 0.5, with h = 0.2. k 1 k 2 k3 k. K Exact Runge- Error % Rei z" Solution Kutta Error 0.2 0.0000 0.0101 0.0203 0.0101 0.510067 0.510067 0.00000 0.4 0.0203 0.0307 0.0311 0.0421 0.0310 0.541069 0.541069 0.541069 0.00000 0.6 0.0421 0.0539 0.0545 0.06 73 0.0545 0.595423 0.595423 0.000000 0.000016 0.8 0.0672 0.0810 0.0972 0.0817 0.677156 0.677155 0.000000 0.000055 1.0 0.0972 0.1137 0.1150 0.1333 0.1146 0.791798 0.791796 0.00001 0.000139 618 Chapter 8 9. Let h = 0.2 and f(x,y) = x/y In (8.59) and (8.60); we have Xo = 0 and YO = 0.2. k 1 = hf(xo'YO) = 0.2f(0.0, 0.2000000) = 0.2f(0.0, 0.200000) = 0.2f(0.0, 0.2000000) = 0.2f(0.0, 0.200000) = 0.2f(0.0, 0.20000) 0.2000000 + 0.1600000 + 0.1428571)/6 = 0.0838095, and so the approximate value of the solution at xl = 0.2 lS Y2 = 0.2000000 + 0.0838095 = 0.2838095. Now using the above value Yl' we calculate successively new k 1, k 2, k 3, k 4, and then K. Ve first find Approximate Methods 619 k 1 = hf(x 1 'Y1) = 0.2f(0.2, 0.2838095) - 0.2(0.7046980) = 0.2838095 = 0.2838095. 0.1409396, k 2 = hf(x 1 + h/2, Y 1 + k1/2) = 0.2f(0.3, 0.3542793) = 0.2(0.8467895) = 0.1693579, k 3 = hf(x 1 + h/2, Y 1 + k 2/2) = 0.2f(0.3, 0.3684885) = 0.2(0.8467895) = 0.2(0.847895) =+ 0.1791164)/6 0.1640711, and so the approximate value of the solution at x 2 = 0.4 lS Y2 = 0.2838095 + 0.1640711 = 0.4478806. Ve summarlze the calculations for these and the remaining y 's in Table 8.6.9. The exact solution of the n differential equation lS given in the pages immediately preceding Section 8.4 of this manual. 620 Chapter 8 TABLE 8.6.9 Runge-Kutta Method for y' = x/y, y(O) = 0.2, with h = 0.2. k 1 k 2 k3 k. K Exact Runge- Error % Rei X n Solution Kutta Error 0.2 0.0000 0.1000 0.0800 0.1429 0.0838 0.282843 0.283810 0.000967 0.341819 0.4 0.1694 0.1628 0.1791 0.1641 0.447214 0.447881 0.000667 0.149149 0.6 0.1786 0.1862 0.1849 0.1897 0.1850 0.632456 0.632930 0.000474 0.074943 0.8 0.1896 0.1924 0.1920 0.1940 0.1921 0.824621 0.824985 0.000364 0.044113 1.0 0.1939 0.1952 0.1951 0.1961 0.1951 1.019804 1.020098 0.000294 0.028848 1.2 0.1967 0.1972 0.1967 1.216553 1.216799 0.00024 7 0.020273 1.4 0.1972 0.1977 0.1976 0.1980 0.1976 1.414214 1.414426 0.000212 0.015002 1.6 0.1980 0.1982 0.1982 0.1982 0.1982 0.1982 1.612452 1.612638 0.000186 0.011541 1.8 0.1984 0.1986 0.1986 0.1986 0.1986 0.1986 0.1986 0.1988 0.1989 0.1989 0.1989 0.1989 0.1989 0.1989 0.1990 0.1989 2.009975 2.010124 0.000149 0.007427 12. Let h = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f(x,y) = (sinx)/y in (8.59) and (8.60); we have Xo = 0 and YO = 1. k 1 = 0.2 and f hf(xo'YO) = 0.2f(0.0, 1.0000000) = 0.2(0.0000000) = 0.2(0.0000000) = 0.2(0.0000000) = 0.2(0.098334) = 0.0199667, k3 = hf(x O + h/2, YO + k 2/2) = 0.2f(0.1, 1.0000000) = 0.2(0.0988366) = 0.0197693, k4 = hf(x O + h, YO + k 3) = 0.2f(0.2, 1.0197693) = 0.2(0.1948179) = 0.0389636. Approximate Methods 621 Therefore K = (k 1 + 2k 2 + 2k3 + k 4)/6 - (0.0000000 + 0.0399334 + 0.0399334 + 0.0399334 + 0.0399334 + 0.0399334 + 0.0399334 + 0.0399334 + 0.0197393. Now using the above value Y 1, we calculate successively new k 1, k 2, k 3, k 4, and then K. We first find k 1 = hf(x 1 'Yl) = 0.2f(0.2, b) 1.0197393 = 0.2(0.1948237) = 0.0389647, k = hf(x 1 + h/2, Y1 + k 2/2) = 0.2f(0.3, 1.0392216) = 0.2(0.2843669) = 0.0568734, k = hf(x 1 + h/2, Y1 + k 2/2) = 0.2f(0.3, 1.0481760) = 0.2(0.2819376) = 0.02f(0.3, 1.0481760) = 0.2f(0.3, 1.0481760) = 0.2f+ k 4)/6 (0.0389647 + 0.1137467 + 0.1127750 + 0.0723741)/6 0.0563101, and so the approximate value of the solution at x 2 = 0.4 lS Y2 = 1.0197393 + 0.0563101 = 1.0760494. We summarize the calculations for these and the remaining y 's in Table 8.6.12. The exact solution of the n differential equation IS given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.6.12 Runge-Kutta Method for y' = (sin x)/y, y(O) = 1, with h = 0.2. k 1 k 2 k 3 k. K Exact Runge-Error % Rei X n Solution Kutta Error 0.2 0.0000 0.0198 0.0390 0.0197 1.019739 1.019739 0.00001 0.000064 0.4 0.0390 0.0569 0.0564 0.0724 0.0563 1.076047 1.076049 0.00002 0.000182 0.6 0.0724 0.0862 0.0857 0.0972 0.0856 1.161609 0.000003 0.000248 0.8 0.0972 0.1065 0.1061 0.1132 0.1236 0.1230 1.267512
1.267512 1.267512 1.267512 1.267512 1.267512 1.20.1208 0.1226 1.630971 1.630974 0.000003 0.000173 1.6 0.1208 0.1179 0.1181 0.1143 0.1179 1. 748828 1.748831 0.000003 0.000156 1.8 0.1143 0.1098 0.1000 0.0992 0.0992 0.0992 0.0992 0.0992 0.0990 1.957624 1.957627 0.000003 0.000137 Approximate Methods 623 Section 8.7, Page 468 General Information: For each problem below, we show the first two calculations in detail, while summarizing calculations at each value of x in a table. The problems n were done maintaining 11 to 12 digit accuracy from step to step; however, we've rounded the results to fewer places as we tabulate them below. 1. Let h = 0.2 and $f(x,y) = x - 2y \ln x$ $x_1 = 0.2, Y_1 = 0.6880000000, x_2 = 0.4, Y_2 = 0.5117952000, x_3 = 0.6, Y_3 = 0.4266275021, and we set x_4 = 0.8.$ Now we find Yo = f(x 2 'Y2) = f(0.4, 0.5117952) = -0.6235904, Y_3 = f(x 3 'Y3) = f(0.6, 0.4266275) = -0.2532550. 624 Chapter 8 We now use (8.67) with n = 3 and h "" 0.2 to determine Y4. We have Y4 = Y3 + $^{\circ}242$ (55Y 3 - 59Y2 + 37Yt - 9y O) = 0.4266275 + (-13.9290252 + 36.7918336 - 43.5120000 + 18.0000000) 120.0 = 0.4045509. Having thus determined Y4' we use (8.68) with n = 3 to find Y4. We obtain Y 4 = f(x 4 'Y4) = f(0.8, 0.4045509) = -0.0091018. Ve use this value of Y4 ln (8.69) with n = 3 and h = 0.2 to finally obtain Y4 as follows: Y4 = Y 3 + 0.2 (9"" + 19y' - 5y' + y/) 24 Y4 321 = 0.4266275 (-0.0819163 - 4.8118451 + 3.1179520 - 1.1760000) + 120.0 = 0.4020291. Now we set n = 4 in order to calculate Yn+1 = Y5. Using the value we just found for Y 4, we first calculate Y4 = f(x 4 'Y4) = f(0.80, 0.4020291) = 10.4020291. 0.4206124 = 0.1587751. We use this value of Y S ln (8.69) with n = 4 and h = 0.2 to finally obtain Y5 as follows: 0.2 (9 A, 19 I 5 I I) Y5 = Y4 + 24 Y 5 + Y4 - Y 3 + Y 2 = 0.4020291 + (1.4289763 - 0.0771054 + 1.2662750 - 0.6235904) 120.0 = 0.4186504. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. 626 Chapter 8 TABLE 8.7.1! ABAM Method (Calculations Summary) for y' = X - 2y, y(O) = 1, with h = 0.2. y'n A AI n X n Yn Xn + 1 Yn + 1 $0.420612\ 0.158775\ 0.418650\ 5\ 1.0\ 0.418650\ 5\ 1.0\ 0.418650\ 5\ 1.0\ 0.418650\ 5\ 1.0\ 0.418650\ 0.000519\ 0.123749\ 4.$ Let h = 0.2 and f(x,y) = x + 2y in (8.67), (8.68) and (8.69); we have $X_0 = -1$ and $Y_0 = 1$. Before we can begin uSing the ABAM method, we need to have values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of these is given by the initial condition y(-1) = 1.0, while values for Yo' Yl' Y2' and Y3. The first of the you was set of the you wa = -0.6, Y2 = 1.7189512533, x 3 = -0.4, Y3 = 2.4396302163, Approximate Methods 627 and we set x 4 = -0.2. Now we find Yo = f(x0'YO) = f(-1.0, 1.0000000) = 1.0000000, yi = f(x 2, y 2) = f(-0.6, 1.7189513) = 2.8379025 Y 3 = f(x 3, y 3) - f(-0.4, 2.4396302) = 4.4792604. We now use (8.67) with n - 3 = -0.6, Y2 = 1.7189512533, x 3 = -0.4, Y3 = 2.4396302163, Approximate Methods 627 and we set x 4 = -0.2. Now we find Yo = f(x0'YO) = f(-1.0, 1.0000000) = 1.0000000, yi = f(x 1'YI) - f(-0.8, 1.2688000) - 1.7376000, Y2 = f(x 2, y 2) = f(-0.6, 1.7189513) = 2.8379025 Y 3 = f(x 3, y 3) - f(-0.4, 2.4396302) = 4.4792604. We now use (8.67) with n - 3 = -0.4, Y3 = -0.4and h = 0.2 to determine Y4. We have Y4 = Y3 + °ii (55Y 3 - 59Y2 + 37Yi - 9y O) = 2.4396302 (246.3593238 - 167.4362479 + 64.2912000 - 9.0000000) + 120.0 = 3.5580825) - 6.9161650. e use this value of Y4 ln (8.69) with n = 3 and h = 0.2 to finally obtain Y4 as follows: 628 Chapter $8 \ 0.2$ (" + 19y' - 5y' + y,) Y4 = Y3 + 24 9Y4 321 = 2.4396302 + (62.2454853 + 85.1059482 - 14.1895125 + 1.7376000) 120.0 - 3.5637929. - Now we set n = 4 in order to calculate Yn+1 = Y5. Using the value we just found for Y4' we first calculate Y 4 = f(x 4 'Y4) - f(-0.20, 3.5637929) = 6.9275858. Ve next use (8.67) with n - 4 and h = 0.20 to determine "Y5. 'We find Y5 = Y4 + °ii (55Y 4 - 59Y 3 + 37Y2 - 9y t) = 3.5637929 + (381.0172180 - 264.2763655 + 105.0023927 - 15.6384000) 120.0 = 5.2813333, "Having thus determined Y 5 we use (8.68) with n - 4 to find ", Y 5. 'We obtain Y s = f(x 5, y 5) = f(0.0, 5.2813333) = 10.5626665. We use this value of YS ln (8.69) with n = 4 and h = 0.2 to finally obtain Y 5 as follows: Approximate Methods 629 0.2 (A) Y5 = Y4 + 24 9yS + 19Y4 - 5Y 3 + Y 2 = 3.5637929 + (95.0639988 + 131.6241299 - 22.3963022 + 2.8379025) 120.0 = 5.2898740. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.7.41 ABAM Method (Calculations Summary) for y' = x + 2y, y(-l) = 1, with h = 0.2. n X n Yn y'n ... "'1 Yn+l Xn+l Yn+l Y n +l 0 -1.0 1.000000 1 -0.8 1.268800 1.737600 2 -0.6 1.718951 2.837903 3 -0.4 2.439630 4.479260 -0.2 3.558083 6.916165 3.563793 4 -0.2 5.289874 TABLE 8.7.4B ABAM Method (Comparisons and Errors) for y' = x + 2y, y(-1) = 1, with h = 0.2. Exact ABAM X n Solution Method Error % Rel Error -0.2 3.564774 3.563793 0.000981 0.027531 0.0 5.291792 5.289874 0.001918 0.036247 5. Let h = 0.1 and f(x,y) = xy - 2y in (8.67), (8.68) and (8.69); we have Xo = 2 and Yo = 1. Before we can 1.0202013398, x 3 = 2.3, Y 3 = 1.0460278589, and we set x 4 = 2.4. Now we find y' = f(x 2 Y2) = f(2.1, 1.0050125) = 0.1005013, 1 y' = f(x 2 Y2) = f(2.2, 1.0202013) = 0.2040403, 2 y' = f(x 3, y 3) = f(2.3, 1.0460279) = 0.3138084. 3 We now use (8.67) with n = 3 and h - 0.1 to determine Y4. We have $Y_4 = Y_3 + ^{\circ}i1(55Y_3 - 59Y_2 + 37Y_i - 9y_0) = 1.0460279(17.2594597 - 12.0383758 + 3.7185463 - 0.0000000) + 240.0 = 1.0832763$. Having thus determined Y4 we use (8.68) with n = 3 to find Y4. We obtain Approximate Methods 631 Y4 = f(x 4 'Y4) = f(x obtain Y4 as follows: Y4 = Y 3 + 0.1 (9"" + 19y' - 5y' + y') 24 Y4 321 = 1.0460279 + (3.8997947 + 5.9623588 - 1.0202013 + 0.1005013) 240.0 = 1.0832881. Now we set n = 4 in order to calculate Yn+1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = 1.0460279 + (3.8997947 + 5.9623588 - 1.0202013 + 0.1005013) 240.0 = 1.0832881. Now we set n = 4 in order to calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = 1.0460279 + (3.8997947 + 5.9623588 - 1.0202013 + 0.1005013) 240.0 = 1.0832881. Now we set n = 4 in order to calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = 1.0460279 + (3.8997947 + 5.9623588 - 1.0202013 + 0.1005013) 240.0 = 1.0832881. Now we set n = 4 in order to calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = 1.0460279 + (3.8997947 + 5.9623588 - 1.0202013 + 0.1005013) 240.0 = 1.0832881. Now we set n = 4 in order to calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just
found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. Using the value we just found for Y4' we first calculate Y1 = Y 5. h = 0.10 to determine "Y 5. We find Y s = Y4 + °ii (SSY 4 - S9Y 3 + 37Y2 - 9Yi) = 1.0832881 + (23.8323378 - 18.5146931 + 7.5494899 - 0.9045113) 240.0 = 1.1331323. Having thus determined Y5' we use (8.68) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. We use this value of Y s ln (8.69) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. We use this value of Y s ln (8.69) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. We use this value of Y s ln (8.69) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. We use this value of Y s ln (8.69) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. We use this value of Y s ln (8.69) with n = 4 to find YS. Ye obtain 632 Chapter 8 Y s = f(x 5, y 5) = f(2.5, 1.1331323) = 0.5665662. = 4 and h = 0.1 to finally obtain Y5 as follows: 0.1 (9 A, 19 ' 5 ') Y5 - Y4 + 24 Y 5 + Y4 - Y 3 + Y 2 = 1.0832881 + (5.0990956 + 8.2329894 - 1.5690418 + 0.2040403) 240.0 = 1.1331509. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.7.5A ABAM Method (Calculations Summary) for y' = xy - 2y, y(2) = 1, with h = 0.1. y'n A AI Yn+l n X n Yn Xn+l Yn+l Y n +l 0 2.0 1.000000 0.000000 1 2.1 1.005013 0.100501 2 2.2 1.020201 0.204040 3 2.3 1.046028 0.313808 2.4 1.083276 0.433311 1.083288 4 2.4 1.083276 0.433315 2.5 1.133132 0.566566 1.133151 5 2.5 1.133151 TABLE 8.7.5B ABAM Method (Comparisons and Errors) for y' = xy - 2y, y(2) = 1, with h = 0.1. Exact X n Solution 2.4 1.083287 2.5 1.133148 ABAM Method Error % Rei Error 1.083288 0.000001 0.000094 1.133151 0.000002 0.000219 A.pproximate Methods 633 8. Let h = 0.2 and f(x, y) = y S 1 n x 1 n (8. 67), (8. 68) and (8.69); we have Xo = 0 and YO = 0.5. Before we can begin uSing the ABAM method, we need to have values for YO' Y1' Y2' and Y3. The first of these IS given by the initial condition y(O) = 0.5, while values for the other three are supplied by the text and were found using the Runge-Kutta Method. Ye have Xo = 0.0, YO = 0.5000000000, xl = 0.2, Y1 = 0.5100667118, x 2 = 0.4, Y2 = 0.5410691444, x 3 = 0.6, Y3 = 0.6 0.5954231442, and we set x 4 = 0.8. Now we find y' = f(x0'YO) = f(0.0, 0.5000000) = 0.0000000, 0 y' = f(x 1 'Y1) - f(0.2, 0.5100667) = 0.1013346, -1 y' = f(x 3 'Y3) - f(0.6, 0.5954231) - 0.3362012. - 3 We now use (8.67) with n = 3 and h = 0.2 to determine Y4. We have 634 Chapter 8 A 0.2 (55 '

59y' + 37y' - 9y' Y4 = Y3 + 24 y 3 210 = 0.5954231 (18.4910658 - 12.4314327 + 3.7493807 - 0.0000000) + 120.0 = 0.6771649. Having thus determined Y4' we use this value of Y 4 ln (8.69) with n = 3 and h - 0.2 to finally obtain Y4 as follows: Y4 = Y3 + 24 y 3 210 = 0.5954231 (18.4910658 - 12.4314327 + 3.7493807 - 0.0000000) + 120.0 = 0.6771649. Having thus determined Y4' we use this value of Y 4 ln (8.69) with n = 3 and h - 0.2 to finally obtain Y4 as follows: Y4 = Y3 + 24 y 3 210 = 0.5954231 (18.4910658 - 12.4314327 + 3.7493807 - 0.0000000) + 120.0 = 0.6771649. $0.2 (9 \text{ A}_{+} + 19y' - 5y' + y_{+}) 24 \text{ Y4} 3 21 = 0.5954231 + (4.3719155 + 6.3878227 - 1.0535112 + 0.1013346) 120.0 = 0.6771528$. Now we set n = 4 in order to calculate Y₄ = f(x 4 'Y₄) = f(0.80, 0.6771528) = 0.4857597. We next use (8.67) with n = 4 and h = 0.20 to determine A Y5. We find Approximate Methods 635 A 0.2 (55 I - 59y' + 37yl - 9y') Y5 = Y 4 + 24 y 4 321 = 0.6771528 (26.7167836 - 19.8358706 + 7.7959832 - 0.9120115) + 120.0 = 0.7918602) = 0.6663274. We use this value of Y S ln (8.69) with n = 4 and h = 0.2 to finally obtain Y5 as follows: Y5 = Y4 + 0.2 (9 A, + 19 y 1 - 5y' + y') 24 Y5 432 = 0.6771528 + (5.9969464 + 9.2294344 - 1.6810060 + 0.2107022) 120.0 = 0.7917868. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. 636 Chapter 8 TABLE 8.7.8A ABAM Method (Calculations Summary) for y' = y s in x, y (0) = 0.5, with h = 0.2. n X n Yn y'n Xn+1 A AI Yn+1 Y n + 1 Yn+1 0 0.0 0.500000 0.0000000 1 0.2 0.510067 0.101335 2 0.4 0.541069 0.210702 3 0.6 0.595423 0.336201 0.8 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677153 4 0.8 0.677153 0.485768 0.677158 0.485788 0.485788 0.485788 0.485788 0.485788 0.485788 0.485788 0. (Comparisons and Errors) for $y' = y \sin x$, y(O) = 0.5, with h = 0.2. X n Exact ABAM Solution Method Error % ReI Error 0.8 0.677156 0.677153 0.000011 0.001363 9. Let h = 0.2 and $f(x,y) = x/y \ln (8.67)$, (8.68) and (8.69); we have Xo = 0 and YO = 0.2. Before we can begin USlng the ABAM method, we need to have values for YO' Y1' Y2' and Y3. The first of these IS given by the initial condition y(O) = 0.2, while values for the other three are supplied by the text and were found using the Runge-Kutta method. We have Xo = 0.0, YO = 0.2000000000, xl = 0.2, Yl = 0.2838095238, x 2 = 0.4, Y2 = 0.4478806080, x 3 = 0.6, Y3 = 0.6329295155, Approximate Methods 637 and we set x 4 = 0.8. Now we find y' = $f(x_0'Y_0) = f(0.0, 0.2000000) = 0.0000000, 0 y' = f(x_1'Y_1) = f(0.2, 0.2838095) = 0.7046980, 1 y' = f(x_1'Y_1) = f(x_1$ 37Yi - 9Yb = 0.6329295 + (52.1385070 - 52.6926140 + 26.0738255 - 0.0000000) 120.0 = 0.8455938. A Having thus determined Y4' we use (8.68) with n = 3 to find Y4. Ve obtain Y4 = f(x 4, y 4) = f(0.8, 0.8455938) = 0.9460807. Ve use this value of Y4 In (8.69) with n = 3 and h = 0.2 to finally obtain Y4 as follows: 638 Chapter 8 0.2 (" + 19y' - 5y' y,) Y4 = Y3 + 24 9y 4321 = 0.6329295 + (8.5147262 + 18.0114843 - 4.4654758 + 0.7046980) 120.0 = 0.8226415. Now we set n = 4 in order to calculate Y1 = f(x 4, y 4) = f(0.80, 0.8226415) = 0.9724771. Ve next use (8.67) with n = 4 and h = 0.20 to determine "Y5. Ve find "0.2 (55' - 59y' + 37y' - 9y') Y 5 = Y 4 + 24 y 4 321 = 0.8226415 + (53.4862421 - 55.9303985 + 33.0445207 - 6.3422819) 120.0 = 1.0247921. Having thus determined Y 5, we use (8.68) with n = 4 and h = 0.2 to finally obtain yS as follows: Approximate Methods 639 0.2 (A + 19y' - 5y' + y') Y5 = Y4 + 24 9yS 432 = 0.8226415 + (8.7822688 + 18.4770655 - 4.7398643 + 0.8930952) 120.0 = 1.0177462. The exact solution of the differential equation is given in the pages immediately preceding Section 8.4 of this manual. TABLE 8.7.9A ABAM Method (Calculations Summary) for y = x/y, y(O) = 0.2, with h = 0.2. n X n Yn y'n ,..., Xn+l Yn+l Y n +l Yn+l O 0.0 0.200000 0.0000000 1 0.2 0.283810 0.704698 2 0.4 0.447881 0.893095 3 0.6 0.632930 0.947973 0.8 0.845594 0.946081 0.822641 4 0.8 0.822641 4 0.8 0.822641 4 0.8 0.822641 4 0.8 0.822641 4 0.8 0.822641 4 0.8 0.822641 0.972477 1.0 1.024792 0.975808 1.017746 5 1.0 1.0177 1.4 1.413178 0.990675 1.412631 7 1.4 1.412631 0.991058 1.6 1.611193 0.993053 1.611055 8 1.6 1.611055 0.993138 1.8 1.809868 0.994547 1.809831 0.994568 2.0 2.008851 TABLE 8.7.9B ABAM Method (Comparisons and Errors) for y' = x/y, y(O) = 0.2, with h = 0.2. Exact ABAM Error % Rei X n Solution Method Error 0.8 0.824621 0.822641 0.001980 0.240070 1.0 1.019804 1.017746 0.002058 0.201778 1.2 1.216553 1.214757 0.001796 0.147619 1.4 1.414214 1.412631 0.001246 0.068800 2.0 2.009975 2.008851 0.001124 0.055916 640 Chapter 8 12. Let h = 0.2 and f(x,y) = (sinx)/y in (8.67), (8.68) and (8.69); we have xo = 0 and YO = 1. Before we can begin uSing the Runge-Kutta method. Ve have $X_0 = 0.0, Y_0 = 1.000000000, x_1 = 0.2, Y_1 = 1.0197392647, x_2 = 0.4, Y_2 = 1.0760493618, x_3 = 0.6, Y_3 = 1.1616089997, and we set x_4 = 0.8. Now we find y' = f(x_0'Y_0) = f(0.0, 1.0000000) = 0.0000000, 0 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_2'Y_2) = f(0.4, 1.0760494) = 0.3618964, 2 y' = f(x_3'Y_3) = f(0.6, 1.1616090) = 0.0000000, 0 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_2'Y_2) = f(0.4, 1.0760494) = 0.3618964, 2 y' = f(x_3'Y_3) = f(0.6, 1.1616090) = 0.00000000, 0 y' = f(x_1'Y_1) =
f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_2'Y_2) = f(0.4, 1.0760494) = 0.3618964, 2 y' = f(x_3'Y_3) = f(0.6, 1.1616090) = 0.0000000, 0 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_2'Y_2) = f(0.4, 1.0760494) = 0.3618964, 2 y' = f(x_3'Y_3) = f(0.6, 1.1616090) = 0.0000000, 0 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) = f(0.2, 1.0197393) = 0.1948237, 1 y' = f(x_1'Y_1) =$ 0.4860865. 3 Ve now use (8.67) with n = 3 and h = 0.2 to determine Y4. Ve have Approximate Methods 641 Y4 = Y3 + °ii (55Y 3 - 59Y 2 + 37Y 1 - 9Yb) = 1.1616090 + (26.7347585 - 21.3518850 + 7.2084752 - 0.0000000) 120.0 = 1.2665369. A Having thus determined Y4' we use (8.68) with n = 3 to find Y4. Ve obtain Y 4 = f(x 4 'Y4) = f(0.8, -1) + f 1.2665369 = 0.5663918. Ve use this value of Y4 ln (8.69) with n = 3 and h = 0.2 to finally obtain Y4 as follows: = 0.2 (9 A, 19'5'') Y4 Y 3 + 24 Y4 + Y 3 - Y 2 + Y 1 = 1.1616090 + (5.0975260 + 9.2356438 - 1.8094818 + 0.1948237) 120.0 = 1.2675966. Now we set n = 4 in order to calculate Y n + 1 = Y 5. Using the value we just found for Y4' we first calculate Y4 = f(x 4, y 4) = f(0.80, 1.2675966) = 0.5659183. Ve next use (8.67) with n = 4 and h = 0.20 to determined Y 5, we use (8.68) with n = 4to find Y 5. Ye obtain Y s = f(x 5, y 5) = f(1.0, 1.3849562) = 0.6075795. Ye use this value of Y s ln (8.69) with n = 4 and h = 0.2 to finally obtain Y5 as follows: Y5 = Y4 + 0.2 (9'' + 19y' - 5y' + y,) 24 Y5 432 = 1.2675966 + (5.4682154 + 10.7524474 - 2.4304326 + 0.3618964) 120.0 = 1.3855310. The exact solution of the differential equation is given $1.267597\ 4\ 0.8\ 1.267597\ 0.565918\ 1.0\ 1.384956\ 0.607579\ 1.385531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.3855531\ 5\ 1.0\ 1.38555531\ 5\ 1.38555551\ 5\ 1.38555555\ 5\ 1.38555555\ 5\ 1.38555555\ 5\ 1.38555555\ 5\ 1.3855555\ 5\ 1.3855555\ 5\ 1.3855555\ 5\ 1.38555555\ 5\ 1.3855555\ 5\ 1.3855555\ 5\ 1.385555\ 5\ 1.3855555\ 5\ 1.385555\ 5\ 1.3855555\ 5\ 1.385555\ 5\$ 0.464462 1.957674 10 2.0 1.957674 TABLE 8.7.12B ABAM Method (Comparisons and Errors) for y' = (sin x)/y, y(O) = 1, with h = 0.2. Exact ABAM Error % Rei X n Solution Method Error 0.8 1.267512 1.267597 0.000085 0.006677 1.0 1.385422 1.385531 0.000109 0.007833 1.2 1.508405 1.508505 0.000100 0.006654 1.4 1.630971 1.631054 0.00084 0.005132 1.6 1. 748828 1.748897 0.000069 0.003923 1.8 1.858660 0.000057 0.003090 2.0 1.957624 1.957674 0.000050 0.002540 Section 8.8, Page 477. General Information: For each problem below, we show 1n detail the first two steps for the Euler Method, and the first for the Runge-Kutta method. All calculations for each value of t are summarized in tables. The n problems were done maintaining 11 to 12 digit accuracy from step to step; however, we've rounded the results to fewer places (usually four to seven) as we tabulate them below. 644 Chapter 81. The exact solution t t First we find the IS x = 2e, y = 4e. solution by the Euler method. Ye have f(t,x,y) = 5x - 2y, g(t,x,y) = 4x. -y, to = 0.0, x = 2.0000, YO = 4.0000, and 0 h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.74). Ye have xl = Xo + hf(to'xo'YO) = 2.0000 + (0.1)2.0000 = 2.2000, Yl = YO + hg(to'xo'YO) = 4.0000 + (0.1)4.0000 = 4.4000. Now to find the summarized in Table 8.8.1-Euler. Approximate Methods 645 Now we solve the problem by the Runge-Kutta method. Again, f(t,x,y) = 5x - 2y, g(t,x,y) = 4x - y, to = 0.0, Xo = 2.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 4.0000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = o. By (8.80) we have k 1 = hf(to'xo'YO) = 0.000000, And h = 0.1. To find the approximate values at t 1 = 0.0 + 0.0 = 0.000000, And h = 0.000000, And h = 0.000000, And h = 0.00 (0.1)f(0.0, 2.0000000, 4.0000000) = (0.1)2.0000000 = 0.2000000, m 1 = hg(to'xo'YO) = (0.1)g(0.0, 2.0000000, 4.0000000) = (0.1)4.0000000 = 0.21000000, 4.0000000) = (0.1)2.1000000 = 0.21000000, (h k 1 m 1) m 2 = hg to + 2'xo +, yo + ki, yo + ki,g(0.05, 2.1000000, 4.2000000) = (0.1)4.2000000 = 0.4200000. Next, the above values for k 2 and m 2 are used to obtain (h k 2 m 2) k3 = hf to + 2 'Xo + , yO + = (0.1)f(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = (0.1)2.1050000 = 0.21050000, 646 Chapter 8 (h k 2 m 2) m 3 = hg to + 2 'Xo + , YO + = (0.1)g(0.05, 2.1050000, 4.2100000) = (0.1)2.1050000 = (0.1 (0.1)4.2100000 = 0.4210000.
Vith these values for k3 and m 3 we find k4 = hf(t o + h, Xo + k 3, YO + m 3) = (0.1)f(0.1, 2.2105000, 4.4210000) = (0.1)2.2105000, 4.4210000) = (0.1)2.2105000, 4.4210000) = (0.1)2.2105000, 4.4210000 = 0.44210000 = 0.44210000. Then from (8.79) we set K = (0.2000000 + 2(0.2100000) = (0.1)2.2105000, 4.4210000) = (0.1)2.2105000, 4.4210000 = 0.442100000 = 0.442100000 = 0.442100000 = 0.442100000 = 0.442100000 = 0.442 2(0.2105000) + 0.2210500) = (1.2620500) = 0.2103417, M = (0.4000000 + 2(0.4200000) + 2(0.4210000) + 0.4421000) = 0.4206833. Finally from (8.78) we obtain Xl = 2.0000000 + 0.4206833 = 4.4206833. Approximate Methods 647 These calculations and those for the remaining tare normalized from (8.78) we obtain Xl = 2.0000000 + 0.2103417, Yl = 4.0000000 + 0.2103417 = 2.2103417, Yl = 4.0000000 + 0.2103417 = 2.2103417 Approximate Methods 649 4. The exact solution -t -t First we find IS x = e, y = -e the solution by the Euler method. \le have f(t,x,y) = 3x + 2y, g(t,x,y) = 3x + 2y, -1.0000 = 1.0000 + (0.1)(-1.0000) = 0.9000, Yl = = Y0 + hg(to'xo'YO) - 1.0000 + 0.1g(O.O, 1.0000, -1.0000) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.9000 + (0.1)(-0.900) = 0.900+ Y1 + hg(t 1, x 1'Y1) = -0.9000 + O.lg(O.l, 0.9000, -0.9000) = -0.9000 + (0.1)0.9000 = -0.8100. The results for all t and the corresponding errors have n been summarized in Table 8.8.4-Euler. 650 Chapter 8 Now we solve the problem by the Runge-Kutta method. Again, f(t,x,y) = x + 2y, g(t,x,y) = 3x + 2y, to = 0.0, Xo = 1.0000000, YO = -1.0000000, YO = -1.0000000, YO = -1.0000000, YO = -1.0000000, YO = -1.00000000, YO = -1.0000000, YO = -1.00000000, YO = -1.0000000, YO = -1.000000, YO = -1.000000, YO = -1.0000000, YO = -1.000000, YO = -1.000000, YO = -1.000000, YO = -1.00 and h = 0.1. To find the approximate values at t = 1 = 0.0 + 0.1 = 0.1, we use Formulas (8.78-80) with n = 0. By (8.80) we have k = hf(to'xo'YO) = (0.1)f(0.0, 1.0000000) = (0.1)(-1.000000) = (0.1)(-1.00000) = (0.1)(-1.00000) = (0.1)(-1.00000) = (0.1)(-1.00000) = (0.1)(-1.00000) = (0.1)(-1.00000) = (0.1)(-1.0000) = (0.1)(-1.0000) = (0.1)(-1.0000) = (0.1)(-1.0000) = (0.1)(-1.0find k 2 hf(t o h k 1 + m 21) = + 2 ' xo + 2 ' Yo = (0.1)£(0.05, 0.9500000, -0.9500000, -0.9500000, -0.9500000) = -0.09500000, -0.9500000, -0.9500000, -0.9500000) = -0.09500000, -0.950000, -0.950000, -0.950000, -0.950000, -0.950000, -0.950000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.9500000, -0.95000000, -0.95000000, -0.950000000000000000000000000000 $X_0 + , Y_0 + = (0.1)f(0.05, 0.9525000, -0.9525000) = (0.1)(-0.9525000) = (0.1)(-0.9525000) = (0.1)(-0.9047500) =
(0.1)(-0.9047500) = (0.1)(-0.9$ 4 = hg(t O + h, Xo + k 3, YO + m 3) - (O.1)g(O.1, 0.9047500, -0.9047500) = (0.1)0.9047500 = 0.0904750. Then from (8.79) we set K 1 (-0.1000000 + 2(0.0950000) + 2(-0.0952500) - 6 + -0.0904750) 1 (-0.5709750) = -0.0951625, = 6 652, Chapter 8 M 1 (0.1000000 + 2(0.0950000) + 2(-0.0952500) = 6 + 0.0904750) 1 (0.5709750) = 0.0951625, = 6 652. Finally from (8.78) we obtain Xl = 1.0000000 + -0.0951625 = 0.9048375, Yl = -1.0000000 + 0.0951625 = -0.9048375. These calculations and those for the rema1n1ng tare n summarized in Table 8.8.4-RK. TABLE 8.8.4-Euler Summary: (with h Euler Method Solution = 0.1) of the System x' = x + 2y, y' = 3x + 2y, x(O.O) = 1.0, y(O.O) = -1.0. t" E t { :I: (t..) Euler { :1:.. ErroR % ReI ac !/(t,,) Yn ErroR 0.1 0.90484 0.90000 0.00484 0.5346 -0.90484 0.5346 -0.90484 0.5346 0.2 0.81873 0.81000 0.00873 1.0664 0.3 0.74082 0.72900 0.01182 1.5953 0.4 0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 -0.65610 0.01422 2.1214 -0.67032 0.65610 0.01422 2.1214 -0.67032 -0.65610 0.01422 -0.72900 -0.6561 Chapter 8 7. 3t 4t 2t 3t The exact solution is x = -e + 3e + 2e, y = -e + 2e 4t + 3e 2t. First we find the solution by the Euler 2t method. Ye have f(t,x,y) = 6x - 3y + e, g(t,x,y) = 2x + 2t y - e, to $= 0.0, X_0 = 4.0000$, YO = 4.0000, and h = 0.1. To find the approximate values at t = 0.0 + 0.1 = 0.1, we use Formulas (8.74). Ye have $XI = X_0 + hf(to'xo'YO)$ = 4.0000 + O.lf(O.O, 4.0000, 4.0000) = 4.0000 + (0.1)13.0000 = 5.3000, Yl = YO + hg(t O'xo'yO) = 4.0000 + O.lg(O.O, 4.0000, 4.0000) = 5.1000. Now to find the approximate values at t 2 = 0.1 + 0.1 = 0.2, we use Formulas 8.75 with n = 1. Ye have x 2 = xl + hf(t 1, x 1 'Yl) = 5.3000 + O.lf(O.l, 5.3000, 5.1000) = 5.3000 + O.lf(O.l, 5.300, 5.10 1.1000000. Using these values for k 1 and m 1 we now find (h k 1 m 1) k 2 = hf to + 2 'xo + , YO + = (0.1)f(0.05, 4.6500000, 4.5500000) = (0.1)15.3551709 = 1.5355171, (h k 1 m 1) ffi 2 - hg to + 2 'xo + , YO + = (0.1)g(0.05, 4.6500000, 4.5500000) = (0.1)15.3551709 = 1.27448291 to obtain k3 (h k 2 m 2) = hf to + 2'xo +, YO + = (0.1)f(0.05, 4.7677585, 4.6372415) = (0.1)15.7999978 = 1.5799998, (h k 2 m 2) m 3 = hg to + 2'xo +, YO + = (0.1)g(0.05, 4.7677585, 4.6372415) = (0.1)13.067588. Vith these values for k3 and m 3 we find k4 = hf(t o + h, Xo + k 3, YO + m 3) = (0.1)f(0.1, 5.5799998, 5.3067588). = (0.1)18.7811252 = 1.8781125, m 4 = hg(t O + h, Xo + k 3, YO + m 3) = (0.1)g(O.l, 5.5799998, 5.3067588) = (0.1)15.2453556 = 1.5245356. Then from (8.79) we set K 1 (1.3000000 + 2(1.5355171) + 2(1.5799998) = 6 + 1.8781125) 1 (9.4091463) = 1.5681910, = 6 Approximate lethods 657 M = (1.1000000 + 2(1.2744829) + 2(1.3067588) = (0.1)15.2453556 = 1.5245356. 1.5245356) = (7.7870189) = 1.2978365. Finally from (8.78) we obtain Xl = 4.0000000 + 1.5681910 = 5.5681910, Yl = 4.0000000 + 1.2978365 = 5.2978365. These calculations and those for the remaining tare n summarized in Table 8.8.7-RK. TABLE 8.8.7-Euler Summary: Euler Method Solution (with h = 0.1) of the System 2t 2t x' = 6x - 3y + e, y' = 2x + Y - e, x(O.O) = 4.0, y(O.O) = 4.0, t n Exact { z(t n) Euler { Zn Errors % Rei y(t n } Yn Errors 0.1 5.56842 5.30000 0.26842 4.8204 5.29800 5.10000 0.19800 3.7372 0.2 7.83815 7.07214 0.76601 9.7729 7.10444 6.54786 0.55658 7.8342 0.3 11.14499 9.50025 1.64474 14.758 9.64699 8.46789 1.17910 12.222 0.4 15.99006 12.84224 3.14782 2 8 Ier:: - 0.0 E 00 00 to 00 0 oo OM 00 M 00 00 0)0 + J dd dd 0 II 9''''4 9''''4 9''''4 9''''4 9''''4 eW' 00 . - (f) NeW' 9''''4 eW' 00 . - (f) NeW' 9''''4 9''''4 eW' 00 . - (f) NeW' 9''''4 9''''4 eW' 00 . - (f) NeW' 9''''4 eW' 00 . co 00 00 0)0) qq eW' ... = E-4 00 00 ... + 9''''4 9''''4 CW')CO t- oo 9''''4 - ... 8 8 E 00 00 00 00 0)0) N dd do do °. 9''''4 eW' 1/) 0 0 0 d 0 ow Chapter 9 Section 9.1A, Page 488 1. We have ro ro £{f(t)} = f e -stf(t)dt = f t 2 e -st dt 0 0 [- -st 2)] R lime 2 2 = (s t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 lim [- e = (8 R + 2sR + 2) + - 3 3 Rro s s o + 2 2 = (8 t + 2st + 3 Rro 8 0 - sR 22] 2 l where s > 0. = 3' 3 s 4. We have ro $f{f(t)} = f = stf(t)dt = 3$: s + 1 = 1 and s = $R = e(st + 1) + \lim -3e s 2 s o R - + oo 2 - 281)[-sR - 2S] e(2s + 1 \lim -3e s 3e - + - + + - 22888 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) +
1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e = 22 s s 8 R - + oo - 2s1) + 1 - + -28 - e(2s + o + 3e + 28 - e(2s + o + 3e + 28 - e(2s + 28 + 28 - e(2s + 28 + 28 + 28 - e(2s + 28 + 28 + 28 +$ the identity 1 1 sin at sin bt = $2 \cos(a - b)t - 2 f(\cos(a - b)t) -$ $222 2 \text{ Is} + (a + b) \text{ s} + (a - b) 2 [s2 + (a + b)2] = 2abs 2222. [s + (a - b)] [s + (a + b) J 3. Ve use the identity sin 3 at = 3 sin at - sin 3at 4 Applying Theorem 9.2, we have £{sin 3 at} = 3 £{sin at}$ 2. 2 (s + a) (s + 9a) Now note that (sin 3 at)' = 3a sin 2 at cos at. Then by Theorem 9.3, $f{f(t)} = sf{s(t)} - f(0)$. Letting $f(t) = sf{s(t)} - f(0)$. Letting $f(t) = sf{s(t)} - f(0)$. t 4, The Laplace Transform 673 f'(t) = 4t 3, f''(t) = 4t 3, f''(t) = 12t 2, this becomes $f_{12t 2} = s2f_{t 4} = 0.0 r \{t 4\} =$ Using these expressions, (1) becomes $s_{f(t)} - s_{f(0)} - f'(0) = s_{f(t)} - f(0) - f'(0) = 1$, f'(0) = 1, f'(0) = 1f(t) (1) By (9.17), with n = 4, we have f(t) = s4f(t) = s4f(t) = s4f(t) = s4f(t) = s4f(t) = s4f(t) = s2f(0) - f(0). Using these expressions, (1) becomes s4f(t) = s2f(t) = s4f(t) ${f(t)} = -8 + s + 1$, and hence $3 \pounds {f(t)} = -8 + s + 1$. $f''(t) = s2f{f(t)} - 4s - 6$ and $f{f(t)} = sf{f(t)} - 4s - 6$ and $f{f(t)} = sf{f(t)} - 4s - 6$ and $f{f(t)} = s2f{f(t)} - 4s - 6$ and $f{f(t)} = s2f{f(t)} - 12s - 2s + 7l\{f(t)\} - 12s - 18 - 5sf\{f(t)\} - 12s - 2s + 7l\{f(t)\} - 12s - 18 - 5sf\{f(t)\} - 1$ 12s + 2 = 22s + 4 From this, (3s 2 - 5s + 7)[f(t)] = 12s - 2 + 22s + 4 and hence [f(t)] = 12s - 2 + 3s 2 - 5s + 7222 (3s - 5s + 7)(s + 4) 13. We let f(t) = $F(s) = f\{f(t)\} = f\{sin bt\} b - 22$. Then by Theorem 9.6, s + b 676 Chapter 9 $f\{t 3 sin bt\} 3 = (-1) 3 L[F(s)] = ds 322 24bs(b - s)(s^2 + b^2) 4 \cdot Hence by(1), 22 f\{t 3 sin bt\} = 24bs(s - b)(s^2 + b^2) 4 \cdot Section 9.2A$, Page 504. 1. $f\{1, 2, 3\} = 2 + 3e$ 5t, where we used Table 9.1, number s s - 5 1, and number 2 with a = 5, respectively. 4. f-1 { 22S } = 2f-1 { 2 s } = 2f-1 { 4s + 7(s + 2)2 + (f3)2. The Laplace Transform 677 Then uSing Table 9.1, number 12, with a = 2, b = [3, we have -1 r-1 { s + 2 } -2t r? £ {F(s)} = 2 2 = e cos1 3t . (s + 2) + ([3) 10. Ve write 2s + 3 2 s - 4 = 2(2 s) + (2 2). By Table 9.1, s - 4 s - 4 4 = cosh 2t; and by number 5, number 6, £-1 { 2 s s - [1 t 2 2 4 } = sinh2t. Thus [1{ ::: } = 2[1 t 2 2 + 3 2 s - 4 = 2(2 s) + (2 2) . By Table 9.1, s - 4 s - 4 4 = cosh 2t; and by number 5, number 6, £-1 { 2 s s - [1 t 2 2 4 } = sinh2t. Thus [1{ ::: } = 2[1 t 2 2 + 3 2 s - 4 = 2(2 s) + (2 2) . By Table 9.1, s - 4 s - 4 4 = cosh 2t; and by number 5, number 6, £-1 { 2 s s - [1 t 2 2 4] = sinh2t. Thus [1{ ::: } = 2[1 t 2 2 + 3 2 s - 4 = 2(2 s) + (2 2) . By Table 9.1, s - 4 s - 4 4 = cosh 2t; and by number 5, number 6, £-1 { 2 s s - [1 t 2 2 4] = sinh2t. Thus [1{ ::: } = 2[1 t 2 2 + 3 2 s - 4 = 2(2 s) + (2 2) . By Table 9.1, s - 4 s - 4 4 = cosh 2t; and by number 5, number 6, £-1 { 2 s s - [1 t 2 2 4] = sinh2t. Thus [1{ ::: } = 2[1 t 2 4] = sinh2t 4 } 3 -1 { 2 } -1 3 + 2 £ 2 = 2£ cosh 2t + 2 sinh 2t. s - 4 12. Ve employ partial fractions. Ve have 2s + 6 8s 2 - 2s - 3 A B = + 4s - 3 2s + 1 and hence 2s + 6 - A(2s + 1) + B(4s 3). Letting s = -3; we find A = 3. Thus we have the partial fractions decomposition 2s + 6 = 8s 2 - 2s - 3 3 4s - 3 1 2s + 1 3 1 = 4(8 -) - 2(S + 3). Then, using Table 9.1, number 2, we find $3t_14 [1 {F(s)} = 3e 4 - t_12 e 2 678$ Chapter 9 13. Ye express F(s) as follows: $F(s) 5s 5 (s + 2) - 10 5 10 = = 2 2 s + 2 + 2)2 - s + 4s + 4 (s + 2) (s Now, using Table 9.1, number 2 with a = -2 and n = 1, we find <math>t_1 {F(s)} = 5t_1 {1} {1} - 10.c - 1 {1} {s + 2} (s + 2)2 = 5t_2 {1} {1} + 2 (s + 2) (s + 2) (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) - 10 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) = 5t_2 {1} {1} {s + 2} (s + 2) {s + 4} {s + 4} (s + 2) (s + 2) {s + 4} {s + 4} (s + 2) (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4} (s + 2) {s + 4} {s + 4$ 5e- 2t (1 - 2t). 14. Ye first employ partial fractions. Ye have $s + 1 s^3 + 2s 1 = (A + B)s^2 + Cs + 2A$. Thus A Bs + c and hence = $-+s + s^2 + 2s A + B = 0$, C = 1; and we have the partial fractions decomposition $s + 1 s^3 + 2s 1 = -2s s^2 2(s + 2) 1 s^2 + 2s s + 1 s^2 + 2s 1 = -2s s^2 2(s + 2) 1 s^2 + 2s s + 1 s^2 + 2s 1 = -2s s^2 2(s + 2) 1 s^2 + 2s 1 = -2s 1 s^2 + 2s 1 =$ Then uSing Table 9.1, numbers 1, 4, and 3, respectively, we find [I t; : J = 1 2! cosf2t + sinf2t. 2 f2 The Laplace Transform 679 16. Ye write $F(s) = \pounds + 74$ (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + 1 = (s + 3) 4 2 3
(s + 3) + 1 = (s + 3) 4 2 3 (s + 3) + $f_{r} = t_{r} = 3t + t_{r} = t_{r} = 1$, with n = 1, $[l_{r} = [l_{r} = 3t + t_{r} = t_{r} = 3t + t_{r} = 1$ + 4)2- 680 Chapter 9 By Table 9.1, number 10, with b = 2, -1 { s2 - 4 } [, 2 = t cos 2t · (s + 4) By number 9, with b = 2, [l t s 2 : s 4)2 } = t sin 2t. -1 { s2 - 4s - 4 } Thus [, 2 = t cos 2t - t sin 2t (s + 4) = t (cos 2t - sin 2t) . 22. Yrite 5s + 17 2 s + 4s + 13 7 5(s + 2) + 3 (3) = (s + 2)2 + (3)2 = 5[(S +); (3)2] + [(s + 2) + (3)2]. By Table 9.1, number 9. 12, with a = 2, b = 3, $[1{s + (3)2} - 2t \cos 3t + 2)2 = e$ (s By number 11, with a = 2, b = 3, $[1{3 + (3)2} - 2t = e \sin 3t \cdot (s + 2)2$ Thus £-1 { 2 5 S + 17 } - 5e- 2 tcos3t s + 4s + 13 = e -2t [5 cos 3t + sin 3t] . 7 - 2t . + 3 e s ln 3t The Laplace Transfora 681 23. Ve first employ partial fractions, $10s + 23AB = + s^2 + 7s + 12 s + 3 s + 4$. and hence 10s + 23 = A(s + 4) + B(s + 3). Letting s = -3, we find A = -7; and letting s = -4, we find B = 17. Thus f - 1 { 1 } s + 3 s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -3 and a = respectively. -4, 25. We first employ partial fractions 1 1 A B C 3 4s 2 = s (s 1)(s 3) = - + 1 + S 3. + 3s + 4 = 7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 4t , when we used Table 9.1, number 2, with a = -7e - 3t + 17e - 3t + 17 + s + s + Then, clearing fractions, 1 = A(s + 1)(s + 3) + Bs(s + 3) + Cs(s + 1). Letting s = 0, we find A = 1/3; letting s = -1, we find B = -1/2; and letting s = -3, we find C = 1/6. Thus f = 1/4. Thus f = 1/4 and f = 1/3; letting s = -1/2; and letting s = -3, we find B = -1/2; and letting s = -3, we find C = 1/6. Thus f = 1/4 and f = 1/3; letting s = -3, we find B = -1/2; and letting s = -3, we find B = -1/2; and letting s = -3, we find C = 1/6. Thus f = 1/4 and f = 1/3; letting s = -3, we find B = -1/2; and letting s = -3, we find B = -1/2; and letting s = -3, we find B = -1/2; and letting s = -3, we find C = 1/6. Thus f = 1/4 and f = 1/3; letting s = -3, we find B = -1/2; and letting s = -3, we find B = -1/2; and letting s = -3, we find C = 1/6. Thus f = 1/4 and f = 1/3; letting s = -3, we find B = -1/2; and letting s = -3, we find B = -1/2; Thus $1 = e = e + s + s + -1 \{ 1 + 3 \} + 1 + 3 + 2 + 2 + 5 = A = 0$, we first employ partial fractions. We have s + 5 + 5 + 2 + 2 = 3 + 2 + 5 = A = (s + 1)(s + 2) + B(s + 1)(s + 2) + 2 + 2 + 5 = A = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0, we find 2B = 5 letting s = 0. s = -1, we find C = 4; letting s = -2, find -4D = 3, D 3 and letting 1, have we so = -.s = we 4' 6A 6B 3C + 2D = 6 so A = 1 C D 13 Thus $+ + -B_{---} = -4.23$ we have the partial fractions decomposition 8 + 5 = -1: () + (S 12) + 4(8 1) s4 + 3s3 + 2s2 (8:2)' Then uSing Table 9.1, numbers 1, 7, 2, and 2 respectively, we find f-1{F(S)} 13 5t -t 3e 2t = -4" + ""2 + 4e - 4 27. Ve first employ partial fractions, 282ABs + C7s + 8s + 7s + 8s + 8 - = -+3 - 22 + 4s + 4) s + 4s s(s s Clearing fractions, we have $7s + 8s + 8 = A(s2 + 4) + s(Bs + C) = (A + B)s^2 + Cs + 4A$. The Laplace Transfora 683 Equating coefficients of like powers of s, we have A + B = 7, C = 8, 4A = 8, and hence A = 2, B = -4. 5, C = 8. Thus $[1{7S 2 S3 + +8:s + 8} = By Table 9.1$, number = $2[1{;}1, [1{;}] = 1. f-1 {2 s + } + 5f-1 {2 s + } + 4f-1 {2 s } = cos2t$. By number 4, with b = 2, f-1 {2 s } = sin 2t. Thus s + 4.c-1 {7s 2 s 2 + +8:s + 8} - 2 + 5 cos 2t + 4 sin 2t. 28. Ye can first employ partial fractions, $3s_3 2 16s + 16 A B C D E + 4s - 3 2)_2 = - + - + - + 2 + 2 \cdot s_2 3 s - s (s - 2) Clearing fractions, expanding and multiplying, and then rearranging terms, we find <math>3s_3 + 4s_2 - 16s + 16 = As_2 (s - 2)_2 + Ds_3 (s - 2) + Es_3 = (A + D)s_4 + (-4A + B - 2D + E)s_3 + (4A - 4B + C)s_2 + (4B - 4C)s + (4C).$ (*) Letting s = 0, one quickly obtains 16 = 4C, so C = 4. Letting s = 2, one easily finds 24 = 8E, so E = 3. Then equating coefficients of s in the extreme members of (*), 684 Chapter 9 one obtains 4B - 4C = -16, from which B = C - 4. But C = 4, so B = 0. Next equating coefficients of s 2 in (*), one has 4A - 4B + C = 4; but B = 0 and C = 4, so this reduces to 4A + 4 = 4, so A = O. Finally, equating coefficients of 4 (*) gives A + D = 0; and since s 1n A = 0, D = 0 also. Thus 3s3 2 16s + 16 4 3 + 4s - 2)2 = - + 2.3 3 s (s - 2) Alternatively, we could have done this by careful observation. For, we have $3s3 2 3s3 2 + 4s - 16s + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - s (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - s (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - s (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - s (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - s (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 =
2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3 2 3 2)2 s (s - 3 (s - 2) 3 4 = 2 + 3". (s - 2) s Thus £-1 { 3S 3 + 16 4 (s - 4s + 4) = 3 2 + 3". (s - 2) 3 3 s (s - 2) 3 3 s (s - 2) 3 3 s (s - 2) 3 s (s - 2) 3 3 s (s - 2) 3 s (s - 2) 3 3 s (s - 2) 3 s ($ $4s_2 - 16s_1 + 16$ $s_3(s_2) - 1 \{4\} - 1 \{3\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and a = 2, $f_1\{s\} = 2t_2$. By number 7, with n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2, $f_1\{s\} = 2t_2$. By number 8, with n = 1 and n = 2. product F(s)G(s), where $2s(s + 4s + 13) 1 1 F(s) = sand G(s) = 2s + 4s + 13 - 1 - 1 \{1\} 1 -$ $-1 \notin \{R(s)\} = \# \{G(s)F(s)\} = g(t)*f(t) t = J: g(r)f(t - r)dr o t[] 1 - 2r = 3 e sin 3r [] dro Ve evaluate the latter of these two integral expressions. -1 1 e - 2r [, {R(s)} = 3 13 (-2 sin 3r - 3 cos 3r) o t 1 - 2t]] = 39 [3 - e (2 sin 3t + 3 cos 3t$ number 7, s s + 3 f(t) = £-1{F(s)} = £-1{:2} = t, and by number 2, get) = £-1{G(s)} = £-1 $= e - 3t [e 3r (3r 9 - 1)] := -1 + 3t + e - 3t 9 6. Ye write R(s) = 1 as the product F(s)G(s), (s + 2)(s2 + 1) 1 1 where F(s) = s + 2 and G(s) = 2 By Table 9.1, s + 1 The Laplace Transfora 687 = f - 1 { 1 s + and 3, respectively, f(t) = f - 1 { 5 (s) } = f - 1 {$ f(T)g(t - T)dT o t = J: e - 2T sin(t - T)dT o r = J: e - 2T sin(t - T)dT o r = J: [sin T]e - 2 (t-T)dT o t = J: [sin T]e - 2 (t-T)dT o t = J: [sin T]e - 2 (t-T)dT o t = J: [sin T]e - 2 (t-T)dT o t = J: [sin T]e - 2 (t-T)dT o t = J: e - 5 t [2 sin T - cos t] + 5 - 2t 2 sin t - cos t] + 5Taking the Laplace Transform of both sides of the D. E., we have $t \{y'\} + 4f\{y\} = 6l\{e\}$. (1) Denoting $f\{y\}$ by yes) and applying the I.C., this becomes $f\{y'\} = sY(s) - 5$. Substituting this into the left member of (1) it becomes sY(s) - 5 + 4Y(s). By Table 9.1, number 2, the right member becomes 6 left member of (1) it becomes $f\{y'\} = sY(s) - 5$. Substituting this into the left member of (1) it becomes sY(s) - 5 + 4Y(s). By Table 9.1, number 2, the right member becomes 6 left member of (1) it becomes $f\{y'\} = sY(s) - 5$. Thus (1) becomes s + (s + 4)Y(s) - 5 = 6 s + 1. Step 2. Solving this for yes), we obtain 5 yes) = 4 + (s + s 6 + 1)(s + 4)' that IS yes) 5s + 11 = (s + 1)(s + 4) + (s + 1)(s + 1)(s + 4) + (s + 1)(s + 1)(s + 4) + (s + 1)(s + 1 = A(s + 4) + B(s + 1). Letting s = -1, we find A = 2; and letting s = -4, we find B = 3. Thus $5s + 11 \ 2 \ 3 \ (s + 1)(s + 4) = s + 1 + s + 4$. Then uSlng Table 9.1, number 2, with a = respectively, we obtain -1 and a = -4, y = 2e - t + 3e - 4t. 4. Step 2. Taking the Laplace Transform of both sides of the D. E., we have $2 f\{y'\} + 2f\{y\} = 16f\{t\}$. (1) Denoting $f\{y\}$ by yes) and applying Theorem 9.3, we have $f\{y'\} = sY(s) - 7$. Substituting this into the left member of (1) it becomes sY(s) - 7 + 2Y(s). By Table 9.1, number 32 7, the right member becomes. Thus (1) becomes s(s + 2)Y(s) - 7 = 3. s Step 2. Solving this for yes), we obtain 7 32 3 yes) 7s + 32 = 2 + = . s + 3 3 s (s + 2) + Bs(s + 2) + Bs(s + 2) + C(s + 2) + Ds 3 = (A + D)s3 + (2A + B)s2 + (2B + C)s + C)s + 2C. Letting s = 0 in this, we obtain 2C = 32, so C = 16; letting s = -2, we have -8D = -24, so D = 3. Then equating coefficients of 3 s, we have A + D = 7, so A = 4; and equating coefficients of 3 s, we have A + D = 7, so A = 4; and equating coefficients 2 of s, we have -8D = -24, so D = -3. Then equating coefficients 2 of s, we have A + D = 7, so A = 4; and equating coefficients 2 of s, we have -8D = -24, so D = -3. Then equating coefficients 2 of s, we have -8D = -24, so D = -3. Then equating coefficients 2 of s, we have -8D = -24, so D = -3. Then equating coefficients 2 of s, we have -8D = -3. Then equating coefficients 2 of s is -32. Finally, uSlng Table 9.1, numbers 1,7 (with n = 1), 7 (with n = 2), and 2, respectively, we find 2 -2t Y = 4 - 8t + 8t + 3e . 5. Step 1. Taking the Laplace Transform of both sides of the D. E. .' we have $\pounds\{yH\} = \pounds\{Q\}$. (1) The Laplace Transform 691 Denoting $\pounds\{y(t)\}$ by yes) and applying Theorem 9.4, we have the following expressions for £{yH} and £{y'}: £{yH} = s2 y (s) - sy(O) - y'(O), £{y'} = sY(s) - y(O). Applying the I.C.'s to these, they become £{yH} = s2 y (s) - s - 2, £{y'} = sY(s) - 1. Substituting these expressions into the left member of (1) and using £(0) = 0, (1) becomes [s2 y (s) - s - 2] - 5[sY(s) - 1] + 6Y(s) = 0 or (s2 - 5s + 6)Y(s) - s + 3 = o. Step 2. Solving the preceding for yes), we have yes) = s - 3 + 2 - s -
2. s - 5s + 6 Step 3. We must now determine $-1 \{1\}$ yet) = $f s - 2 \cdot By$ Table 9.1, number 2, we immediately find 2t Y = e - 7. Step 1. Taking the Laplace Transform of both sides of the D.E., we have $f\{y\} = f(0)$. (1) Denoting $f\{y\}$ by yes) and applying Theorem 9.4, we have $f\{y\} = s^2 y$ (s) $s^2(0) = s^2(0) + 2s^2 + 2s^2$ y'(O), ley' = sY(s) - y(O). Applying the I.C.'s, these become $l\{yH\} = s2 y(s) - 2s - 9, f\{y'\} = sY(s) - 2$. Substituting into the left member of (1) and using f(0) = 0, (1) becomes 692 Chapter 92 s yes) - 2s - 9 - 6sY(s) + 12 + 9Y(s) = 0 or (s2 - 6s + 9)Y(s) - 2s + 3 = 0. Step 2. Solving for yes), we obtain yes) = 2s - 3 2s - 3 2 - 2 s - 6s + 9 (s - 3) Step 3. We Denoting $\{y\}$ by yes) and applying Theorem 9.4, we have the following expression for $\{yH\} = s2 y (s) - sy(0) - y'(0)$. The Laplace Transform 693 Applying the I.C.'s to this, it becomes $1{yH} = s2 y (s) - 3 + 9Y(s)$. By Table 9.1, number 2, the right member of 36 (1) becomes 3. Thus equation (1) reduces to $s + (s^2 + 9)Y(s) - 2s - 3 = 36s + 3$. Step 2. Solving this for yes), we obtain yes) = $2s + 3 + 36s^2 + 9(s + 3)(s^2 + 9)$ that IS, yes) = 22s + 9s + 452. (s + 3)(s + 9) We employ partial fractions. We have $2s^2 + 9s + 45(s + 3)(s^2 + 9) = ABs + C3 + 2s + ss^2$. + 9 Then $2s^2 + 9s + 45 = A(s^2 + 9) + (Bs + C)(s + 3) = (A + B)s^2 + (3B + C)s + (9A + 3C)$. From this, A + B = 2, 3B + C = 9, 9A + 3C = 45; and hence A = 2, B = 0, C = 9. Thus 694 Chapter 9 $2s^2 + 9s + 45$ (s + 3)($s^2 + 9$) = 293 + 2s + s + 9 Therefore, $-1 \{1\} - 1 \{3\} Y = 2f s + 3 + 3f s^2 + 9 \cdot Then$ by Table 9.1, numbers 2 and 3, respectively, we find $-3t Y = 2e + 3 \sin 3t$. 12. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $2|{yH} + 1{y'} = 51{e 2t}$. (1) Denoting $|{y'}| = sY(s) - y(O)$. Applying the I.C. to these, they become $1{y''} 1{y'} 2 = s yes$) - 2s, = sY(s) - 2. 2 Thus the left member of (1) becomes 2s yes) - 4s + sY(s) - 2. By Table 9.1, number 2, the right member of (1) becomes s 2 ' Thus equation (1) reduces to 2 5 (2s + s)Y(s) - 4s - 2 = 2 . s - Step 2. Solving for yes), we obtain y (s) = 4S 2 + 2 + 5 2 2s + s (s - 2)(2s + s) The Laplace Transform 695 that 1S, yes) 2 5 = - + s s(s - 2)(2s + 1)' or yes) 4s 2 - 6s + 1 = s(s - 2)(2s + 1). Step 2 = - + s s(s - 2)(2s + 1)' or yes) 4s 2 - 6s + 1 = s(s - 2)(2s + 1). Step 2 = - + s s(s - 2)(2s + 1) + (s - 2)(2s 3. We must now determine -1 { 4s - 6s + 1 } Y = f s(s 2)(2s + 1) · (2) We first employ partial fractions. We have 4s 2 - 6s + 1 = s(s - 2)(2s + 1) + Bs(2s + 1) 2. Letting s = 0, we find A = -1/2; letting we find B = 1/2; and letting s = -1/2, we find Thus + Cs(s s = 2, C = 4. 4s 2 - 6s + 1 = s(s - 2)(2s + 1) + Bs(2s + 1) 2. Letting s = 0, we find A = -1/2; letting we find B = 1/2; and letting s = -1/2, we find Thus + Cs(s s = 2, C = 4. 4s 2 - 6s + 1 = s(s - 2)(2s + 1) + Bs(2s + 1) 2. applying Theorem 9.4 with n = 4 and n = 2, respectively, we obtain $l{yiv} = s4 y(s) - s2y'(0) - syl(0) - y'(0)$. Applying the I.C.'s to these, they become $l{ylV} = s4 y(s) - 4s 2 - 8$, $l{yH} = s2 y(s) - 4s - 8 - 2s yes) + 8 + yes$ = 0 or $(s_4 - 2s_2 + 1)Y(s) - 4s_2 = 0$. Step 2. Solving for yes), we find yes) $4s_2 + 4s_2 = 4 = 2(s_2 - 1)2s_2 + 14s_2 = + 1)2$. 2 (s - 1)(s + 1) = 10. 2 (s - 1)(s + 1) = 0 or $(s_4 - 2s_2 + 1)Y(s) - 4s_2 = 0$. Step 2. Solving for yes), we find yes) $4s_2 + 4s_2 = 4 = 2(s_2 - 1)2s_2 + 14s_2 = + 1)2$. 2 (s - 1)(s + 1) = 10. 2 (s - 1)1) + $C(s + 1)(s + 1)2 + D(s - 1)2 = (A + C)s^3 2 + (A + B - C + D)s + (-A + 2B - C - 2D)s + (-A + B + C + D) = 0$. First, letting s = -1, we get 4D = 4, so D = 1. Next, equating coefficients of s^3 gives A + C - 0, so C = -A. Equating coefficients of s^2 then gives A + B - C + D = 4. Since B - D = 1 and C = -A, this reduces to 2A + 2 = 4, from which A = 1, and then C = -1. Thus we find 4s 2 = (s - 1)2(s + 1)2 1 s - 1 + 1 (s - 1)2 + [l Ls + 1 + 1)2 + [l Ls + 1]2 + [l Ls + 1]2 +Transform of both sides of the D. E., we have the following expressions for $.c\{y'\} - .c\{y'\} - .c\{y'\}$ these expressions into the left member of (1), this left member becomes [s2 y (s) - 3] - sY(s) - 2Y(s) or (s2 - s - 2)Y(s) - 3. By Table 9.1, number 11, the right member of (1) becomes, 54 2 9 (s + 1) + Thus (1) reduces to $2^{2}Y(s) - 3^{2} 54 (s - s - s - (s + 1)2 + 9)$ Step 2. Solving the preceding for Y(s), we have yes) $3s^{2} + 6s + 84 = (s + 1)(s - 2)[(s + 1)2 + 9]$ Step 2. 3. Ye must now determine { 2 } y(t)=r 1 3s + 6s 2 + 84. (s + 1)(s - 2)(s + 28 + 10) The Laplace Transform 699 Ye employ partial fractions. Ye have 3s 2 + 6s + 84 (s + 1)(s - 2)(s + 2s + 10) + B(s + 1)(s + 2)(s + 2B + C)s3 + (3B - C + D)s2 + (6A + 12B - 2C - D)s + (-20A + 10B - 2D). From this, we obtain { A + B + C = 0, 3B - C + D = 3, (3) 6A + 12B - 2C - D = 6, -20A + 10B - 2D = 84. Letting s = -1 in (2), we find from (3), that C = 1, D = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find from (3), that C = -2. Thus we find fro have = $3f-1\{1\} + 2f-1\{s+1s \ 10J \ 12\}\{2f-13s + 6s + 84(s+1)(s-2)(s^2 + 2s + 700 \ Chapter 9 - 1\{s-2\} + [, 2s + 2s + 10 = <math>3f-1\{1\} + 2f-1\{12\} + [, 2(s+1) + -1\{39\} + [, 2(s+1) + -1], [, 2(s+1) + -1$ the Laplace Transform of both sides of the D. E., we have $[,{yH} + 3['{y'} + 2['{y} = 10[,{cost}. (1) Denoting [,{y} by yes) and applying Theorem 9.4 we have [,{yH} = s2 y (s) - y(O), [,{y'} = sY(s) - y(O$ of (1), it becomes s2 y (s) - 7 + 3sY(s) + 2Y(s). By Table 9.1, number 4, the right member becomes ; os . Thus (1) becomes s + 1 (s2 + 3s + 2)(s + 1) + s2 + 3s + 2 (s +
3s + 2)(s + 1) + s2 + 3s + 2 (s + 3s + 2 +7 = [, 2 (s + 1)(s + 2)(s + Ve first employ partial fractions. Ye have 7s 2 + 10s + 7 (s + 1)(s + 2)(s + 1) + B(s + 1)(s + 2)(s + 1)(s + 2)(s + 1)(s + 2)(s + 1)(s + 2)(s + 1)(s + 1)(s + 2)(s + 1)(s + 1)(s= 7. Yith A = 2, B = -3, this gives D = 3. Finally, letting s = 1, we find 6A + 4B + 6(C + D) = 24. Yith A = 2, B = -3, D = 3, this gives C - 1. Thus we have 702 Chapter 9 7s 2 + 10s + 7 (s + 1)(s + 2)(s2 + 1) = 2 s + 1 3 s + 3 2 + 2 s + S + 1 Thus y = $2\pounds -1\{S_1\} - 3\pounds -1\{S_2\} + \pounds -1\{S_2\}$ and 3, we have $-t - 2t Y = 2e - 3e + \cos t + 3 \sin t$. 20. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $.c{yH} = 52 y(s) - y(0)$, $l{y'} = sY(s) - y(0)$. Applying the I.C.'s, these become $.c{y'} + 5.c{v'} + 5.c{v'} + 5.c{v'} = 5.c{v'} + 5.c{v'} = 2.s y(s)$. -s - 1, sY(s) - 1. Substituting into the left member of (1), it becomes 2 2 s yes) - s - 1 + 5sY(s) - 5 + 4Y(s) or (s + 5s + 4)Y(s) - s - 6. Using Table 9.1, numbers 8 and 2, the right member of (1) becomes (s + 1)2 s + (S2 + 5s + 4)Y(s) - s - 6 = 6 + (s + 1)2 8 s + 1. The Laplace Transform 703 Step 2. Solving for yes), we obtain yes) $= s + 68 + (s^2 + 5s + 4)(s + 1) 62s + 5s + 4 = (s + 1)(s + 4) (s + 1) 2$ Adding fractions and observing that $s^2 + 5s + 4 = (s + 1)(s + 4)$, we find yes $= s^3 + 8s^2 + 21s + 20$ (s + 1)3(s + 4) AB 1 + 2 s + (s + 1) C + (s + 1) 3 D + s + 4.322 Then s + 8s + 21s + 20 = A(s + 1)(s + 4) + C(s + 4) + D(s + 1) + D(s + 1)50, which, with C = 2 and D = 0, reduces to 2A + B = 4. From the two equations in A and B, we find A = 1, B = 2. Thus 704 Chapter 9 3 2 s + 8s + 21s + 20 3 (s + 1) (3) Alternatively, an astute observer might have noticed that 322 s + 8s + 21s + 20 = (s + 4s + 5)(s + 4) and reduced the fraction in (2) to 2 $s + 4s + 53 \cdot (s + 1)$ Applying partial fractions to this, one writes 2s + 4s + 53 (s + 1) + C2 (2A + B)s + (A + B + C). From this, A = 1, = As + 2A + B = 4, A + B + C = 5, and we find A = 1, B = 2, C = 2. So . obtain (3) , and this time with an we aga1n easier use of partial fractions. Therefore $y = [l\{s 1\} + 2[l Ls + 11)2\} + f Ls + 21)3$ By Table 9.1, numbers 2, 8, and 8, respectively, we find t t 2 - t 2 - t Y = e + 2te + t e = (t + 1) e . 21. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $f\{yn\} - 3f\{y\} = f\{20 \text{ sin } t\}$. (1) The Laplace Transform 705 Denoting $f\{y(t)\}$ by yes) and applying Theorem 9.4, we have the following expressions for $\pounds\{yfl\}$, $\pounds\{y''\} = sY(s) - y'(0)$, = s yes) sy(0) - sy'(0), = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes) $f\{y'\} = sY(s) - y'(0)$, = s yes member becomes s3 y (s) + 2 - 5s 2 y(s) + 7sY(s) - 3Y(s) or 3 2 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2 s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2 s 322 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes = 2 18 - 2 s 32 (s - 5s + 7s - 3)Y(s) + 706 Chapter 9 or finally yes) = -2(s + 3)(s - 1)2(s2 + 1) Step 3. Ye must now determine (t) - .c- 1 { -2s - 6y - 22(s - 1)(s + 1) } Ve employ partial fractions. Ve have -2s - 6A = +22s - 1(s - 1)(s + 1) + B(s2 + 1) + B(s2 + 1) + (Cs + D)(s - 1)2 or -2s - 6 = (A + C)s3 + (-A + B - 2C + D)s2 + (A + C - 2D)s + (-A + B - 2C + D)s2+ B + D). From this, we obtain { A + C = 0, A + C - 2D = -2, -A + B - 2C + D = 0, -A + B + D = -6. The first and third of these give C = -A, D = 1. second and fourth reduce to A + B = -1, -A + B = Then the -7, respectively, from which A = 3, B = -4, C = -3, D = 1; and we have -1 { -2s - 6 } = -3, C = -3, D = 1; and we have -1 { -2s - 6 } = -3, C = -3, D = 1; and we have -1 { -2s - 6 } = -3, C = -3, D = 1; and we have -1 { -2s - 6 } = -3, C = -3, D = 1; and we have -1 { -2s - 6 } = -3, D = 1; and we have -1 { -2s - 6 } = -3, D = 1; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, C = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -2; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4, C = -3, D = -4; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4, C = -3, D = -4; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4, C = -3, D = -4; and we have -1 { -2s - 6 } = -3, D = -4, C = -3, D = -4, C = -3, D = -4, C = -3, D = -4; and we have -1 { -2s - 6, C = -3, D = -4, C = -3, D = -4.c-1 { 1 2 } - 3.c-1 { 2 s } (s - 1) s + 1 - 1 { 1 } + .c 2 . s + 1 The Laplace Transform 707 Then by Table 9.1, numbers 2, 8, 4, and 3, respectively, we find y = 3e t - 4te t - 3cost + sint. Section 9.41, Page 527. 1. The given function is 5u 6 (t). -6s 5e Using formula (9.75), we have £{5u 6 (t)} = s 4. We may express the values of f in the form { 2 - 0 f(t) } = 2 - 2, 0 < t < 5, t > 5. Thus f(t) can be expressed as 2 - 2u S (t). Then uSing formulas (9.2) and (9.75) we find £{f(t)} 2 2e- Ss = 2£{1} - 2 £{u 5 (t)} = s - s - Ss =
2(1 - e) s 6. We may express the values of f in the form 0 + 0, 0 < t < 3, f(t) = -6 + 0, 3 < t < 9, -6 + 6, t > 9. Thus f can be expressed as -6u 3 (t) + 6u 9 (t). Then uSing formula (9.75), we find £{f(t)} = s - s - Ss = 2(1 - e) s 6. We may express the values of f in the form 0 + 0, 0 < t < 3, f(t) = -6 + 0, 3 < t < 9, -6 + 6, t > 9. Thus f can be expressed as -6u 3 (t) + 6u 9 (t). Then uSing formula (9.75), we find £{f(t)} = s - s - Ss = 2(1 - e) s 6. We may express the values of f in the form 0 + 0, 0 < t < 3, f(t) = -6 + 0, 3 < t < 9, -6 + 6, t > 9. Thus f can be expressed as -6u 3 (t) + 6u 9 (t). Then uSing formula (9.75), we find £{f(t)} = s - s - Ss = 2(1 - e) s 6. find 708 Chapter 9 \pounds {f(t)} = -6 \pounds {u 3 (t)} + 6 \pounds {u 9 (t)} -3s -9s -6e 6e = + s s 6 -9s -3s = - (e - e). s 7. Ve may express the values of f in the form f(t) = 1 + 0 + 0 - 0, 0 < t < 2, 1 + 1 + 1 - 3, t > 6. Thus f(t) can be expressed as 1 + u 2 (t) + u 4 (t) - 3u 6 (t). Then using formulas (9.2) and (9.75) we find \pounds f(t) = f(1) + f(u + 1) + f(u + $= 2f\{1\} - 2f\{u \ 3(t)\} + 2f\{u \ 6(t)\} 2 - 3s \ 2e - 6s \ 2e = +s \ s \ s = 2(1 - e - 3s - 6s) + e \ s \ 11.$ Ve must first express f(t) for t > 2 ln terms of t - 2. This is the translated function defined by $\{0, 0 < t < 2, u \ 2(t); (t - 2) = ; (t - 2), t > 2, where \ ; (t) = t + 2.$ By Theorem 9.9, -2s -2s $f_{u^2(t)}(t-2) = e_{t^2(t)} = e_{t^2(t)} = e_{t^2(t-2)}$, where we have used (9.3) and (9.2). 14. Ve express the values of f in the form {2t - 0, 0 < t < 5, f(t) = 2t - u 5 (t); (t - 5), 2 where ;(t) = 2t - u 5 (t); (t $11) = ef\{ee\} = ees + -2(s+1)e = , s + 1 where we have used (9.4).$ 17. Ve express the values of f in the form o - 0, 0 < t < 4, f(t) - (t - 7), t > 7. Thus f(t) = f(Theorem 9.9, sf{f(t)} = f{u 4 (t); (t - 4)} - f{u 7 (t), (t - 7)} = e - 4s f{;(t)} - e - 7s f{;(t)} - e - 7 sin t. By Theorem 9.9, $f{f(t)} = f{u2(t);(t-2)} - f{u4(t),(t-4)} = e-2sf{:(t)} - e-4rs[21] + 1 + 1 - 2s - 4s = e f{sin t} - e f{sin t$ $1 - e_1[e_{::t_1} + 1 - s_{::}] = -2s(-st - 1) - e_s + 1 - e_s +$ where a = 5 and (s) 3s + 1 By formula (9.86) \pounds -1 {e-as(s)} - (s - 2)2. = u (t); (t - a), where u lS defined by (9.73) and a a -1; (t) = \pounds {(s)}. [See Theorem 9.9.] We must find; (t). Using Table 9.1, numbers 2 and 8, respectively, ;(t) = \pounds -1 { 3 + 1 } = $3\pounds$ -1 { 1 } + $7\pounds$ -1 { 1 } (s 2)2 (s - 2) (s 2)2 = 3e 2t + 7te 2t = e 2t (7t + 3). Thus; (t - 5) = e 2(t-5) [7(t-5) + 2(t-3) + 2(t-3J = e 2 (t-5)(7t - 32). Then by formula (9.86) [or Table 9.1, number 16J, f-1 { 3s + 1 e - 5S } = u (t); (t - 3), where a = 4 and «p(s) = 2 12 By formula (9.86) [1{e-as «p(s)} s + s - 2 = u (t); (t - a), where u IS defined by (9.73) and a a ; (t) = f-1{(s)} [see Theorem 9.9]. We must find ;(t). We first employ partial fractions. We write 12 A B Clearing fractions, we at = + + 2. 2 s - 1 s s + s - 12 once find A = 4, B = -4. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 12] = 4e t - 4s + 2 - 4e - 2t. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [1 t 2 + 1s 2 - 4e - 2t + 15 - 4e + $16I_{t} - 1 \{ 2 \ 12 \ e - 4S \} L_{t} = u \ 4(t); (t - 4) \ s + s - 2 \{ 0, 0 < t < 4, -4et - 4 \ 4e - 2(t-4) \ t > 4.5. F(s) \ is of the form e-as(s), where u is defined by (9.73) and a a; (t) = f-1 \{ (s) \}$. [See Theorem 9.9.] Ye must find :(t). Using Table 9.1, numbers 4 and 3, respectively, we find :(t) = f-1 \{ (s) \}. [See Theorem 9.9.] Ye must find :(t) = f-1 \{ (s) \}. 715 7. () -as F s lS of the form e (s), where a = /2 and s + 8 8.0, 0 < t < 8, 718 Chapter 9 Thus { -3s - SS } -1 e - e f s3 = u 3 (t); (t - 3) - u 8 (t); (t - 3) determine f-1 {s 2} - s + 5 · number 11, we have By Table 9.1, -1 { 2 } f 2 = f 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2 s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2 s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2 s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2 s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2 s + 5 (s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 = e Sln t. (3 s - 1) + 4 t. 2 $ur/2(t);(t-;) = ;(t-;), rt > 2 ' 0, o re Sln 2. Then, using (1), (2), and (3), we find (3) t. 2 0 e Sln t-, ro < t < 2 ' t,-1{F(s)} = etsin 2t e t - r/2 sin 2t. t > ;, t · 2 e Sln t, ro < t < 2 ' = (1 - e - 1("/2) e t sin 2t, rt > 2. Section 9.4C, Page 533.$ 2. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $3f\{y'\} - 5f\{y\} = f\{h(t)\}$. (1) 720 Chapter 9 Denoting $f\{y\}$ by yes), applying Theorem 9.3, and then applying the I.C., we find that $f\{y'\} = sY(s) - 4$. Substituting this into the left member of (1), it becomes 3[sY(s) - 4J - 5Y(s) or (3s - 5)Y(s) - 12. By the definition of the Laplace Transform (I) (I) $f{h(t)} = i e - sth(t)dt = i 10e - st dt 0 6 1$. [10 -st R] = 10e - 6s = 1m - - e s 6 s R(I) Alternatively, we note that h(t) = 10u 6 (t); and then using Table 9.1, number 15, we have $f{h(t)} = 10e - 6s s$ Thus, (1) reduces to (3s - 5)Y(s) - 12 = 10e - 6s s Step 2. Solving for yes), we obtain yes) 12 10e - 6s = 3s - 5 + s(3s - 5). Step 2. We must now determine -1 { 12 tOe -6s } y = f 3s - 5 + s(3s - 5). (2) We first determine f { } We first determine f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now
consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider f { } We first employ partial fractions to -1 St30Se6S5). 1 1 A B s(s - 5/3) = s + s - 5/3 e , (3) The Laplace Transform 721 where we have used Table fractions, we at once find A = -3/5, B = 3/5. Then 10 10 [-3/5 + 3/5] 2 + 2 Th e n by s(3s - 5) - 3 s s - 5/3 - s s - 5/3. Table 9.1, numbers 1 and 2, respectively, -1 { 1 } f s(3s - 5) = -2 f S + 2 f s - 5/3 = -2 + 2 e(5/3)t. Now letting F(s) = 10 and f(t) = -2 + 2 e(5/3)t we s(3s - 5) thus have f(t) = -2 + 2 e(5/3)t we s(3s - 5) thus have f(t) = -2 + 2 e(5/3)t. Now letting F(s) = 10 and f(t) = -2 + 2 e(5/3)t we s(3s - 5) thus have f(t) = -2 + 2 e(5/3)t. Then by Theorem 9.9, f(t) = -2 + 2 e(5/3)t. $3!\{y'\} + 2!\{y\} = !\{h(t)\}$. (1) Denoting $!\{y(t)\}$ by yes), applying Theorem 9.4 as in the preVIOUS exercises, and then applying the I.C.'s we find that $!\{y''\} = s2$ y (s) and $!\{y'\} = sY(s)$. Substituting these expressions into the left member of (1), this left member of (1), this left member of (2), this left member of (1), this left member Transform. (1) $f_{h(t)} = I$: e-sth(t) dt o = 2 (1 e-4s), s 4 = I: 2e-st dt o Alternatively, writing { 2 - 0, 0 < t < 4 h(t) = 2 - 2 t > 4, we have $f_{h(t)} = -4s 2e s = 2$ (1 e-4s), s 2 2 - 4s Thus (1) reduces to (s - 3s + 2)Y(s) = -(1 - e), s The Laplace Transfo 723 Step 2. Solving the preceding for yes), we obtain -4s yes) = 2(1 - e) s(s - 1)(s - 2). Step 3. We must now determine yet) = $f - \{ S(S_1)(s - 2) \} - f - \{$ $f_{1} = 1 - 2et + e^{2t} + e$ $< 4, = f(t - 4), t > 4, = \{0, 0 < t / 4, ..., 1 - t - 4 2 2(t - 4) t > 4. 2e + e, Thus from (2), we find y = f(t) - u 4 (t)f(t - 4) = 1 - 2e t + e 2t , 0 < t < 4, 2(e - 4 - 1)e t + (1 - e - S)e 2t , t > 4. 6. Step 1. Taking the Laplace Transform of both sides of the D. E., we have !{yH} + 6!{y'} +$ $8!{y} = !{h(t)}.$ (1) Denoting $!{y(t)}$ by yes), applying Theorem 9.4 as ln preV10US exercises, and then applying the I.C.'s, we find that $!{yH} = s2 y (s) - s + 1$, $!{y'} = sY(s) - 1$. Substituting these expressions into the left member of (1), this left member of (2), this left member of (1), this left member of Laplace Transform, The Laplace Transform 725 (1) $f{h(t)} = 3 - 3u 2r(t)$. Then uS1ng Table 9.1, numbers 1 and 15, respectively, we have 3 - 2rs 3(1 - e - 2rs). $f{h(t)} = 3e = -s s s$ Thus (1) reduces to $(s^2 + 6s + 8)Y(s) - s - 3u^2r(t)$. 4' 726 Chapter 9 and from this we readily find A = 3/2, B = -1/2. Thus f-1 { (s + 5 } (s + 2)(s + 4) ! f-1 { (s + 5 } + 2)(s + 4) . We have 3 + 2 (s + 4) . We first apply 3 partial fractions to s(s + 2)(s + 4) . We have 3 + 2 (s + 4) . We have 3 + 2 (s + 4) . We have 3 + 2 (s + 4) . We first apply 3 partial fractions to s(s + 2)(s + 4) . We have 3 + 2 (s + 4) . We have 3 + 2 (s + 4) . We first apply 3 partial fractions to s(s + 2)(s + 4) . We have 3 + 2 $e_3 - 4t - 1 + 8e_y$, we thus have $f_{F(s)} = f(t)$. We now consider $f_{-1} \{3e_{-2rs}\} + f_{F(s)e_{-2rs}}$, $s_{+2}(s_{+4}) = 8v_{-1} + 8e_y$. The Laplace Transform 9.9, The Laplace Transform 9.9, The Laplace Transform 9.9, $t_{-1} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0, 0 < t < 2r, 3 - 3e_{-2}(t-2r), t_{-2} = \{0,$ 3 -4t -e + e 4 8, o < t < 2r, 3 e -2t + 3 e -4t 3 + 3 e -2(t - 2r) 4 8 8 4 3 e -4(t - 2r) 8 + 2r, 3 3 -2t 3 -4t 8 - 4 e + 8 e o < t < 2r, = : (e 4r 1)e - 2t + : (1 - e 8r)e - 4t , t > 2r. (4) Hence, using (3) and (4), (2) becomes y = 3 3 -2t - + - e 8 4 1 -4t - - e 8 , o < t < 2r, 3 (4r 1) -2t 1 (1 3 8r) -4t - e + e - + e e 48' t > 2r. 8. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $f_{yH} + f_{y} = f_{h(t)}$. (1) Denoting $f_{y(t)}$ by Y(s), applying Theorem 9.4 as ln previous exercises, and then applying the I.C.'s, we find that 728 Chapter 9 $f_{yH} = s_2 y(s)$ 2s - 3. Substituting this expression into the left member of (1), this left 0 < t < r, h(t) = r, t > r, = f - 0, 0 < t < r, t - (t - r), t > r, and hence h(t) = t - u(t)f(t - r), where f(t) = t. Thus, r using Theorem 9.9 and Table 9.1, number 7, we find = 1 - rs - e 2 s Step 2. Solving the preceding for yes), we obtain Y(s) = 2s + 3 + 4 $s_2 + 1 - rs_1 - e_{22}$, $s_1 + 1$ Step 3. We must now determine $y = f_1 \{ 2 + 3 \} + .c_1 \{ 2 2 1 \}$ s + 1 s (s + 1) - f_1 $\{ 2(1) \} + .c_1 \{ 2 2 1 \}$ s + 1 s (s + 1) - f_1 $\{ 2(1) \} + .c_1 \{ 2(1) \} + .$ s(s+1) A B Cs + D = 8 + 82 + 82 + 1 and from this we readily find A = 0, B = 1, C = 0, D = -1. Thus 1: $\{221\} = 1: -1 \{221\} = 1:
-1 \{221\} = 1: -1$ 9.9, we now find -1 { e-rs } 1: 22 = s (s + 1) 1: -1 { F(s)e-rs } = u (t)f(t - r) r { 0, 0 < t < if, = f(t - if), t > if, { 0, 0 < t < r, -1 e f 2 2 = (5) s (s + - r + Sln t, t > r, Thus, uSlng (3), (4), and (5), (2) becomes y = { (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2 cos t + 3 s r + sint, $t > r \{ 2 sin t + 2 cos t + t, - sin t + 2 cos t + r, o < t < r, t > r. 10$. Step 1. Taking the Laplace Transform of both sides of the D. E., we have $f\{y\} + 25f\{y\} = 25[f\{u 2(t)\} + 25f\{y\} = 25f\{y\} =$ these into the left member of (1), it becomes 25 e s - e s. Thus (1) becomes $(s^2 + 6s + 25)Y(s) - 3s - 9 = 25 [e - 2s - e - 4s]$ twice, the right member becomes 25 e s - e s - 4s. Thus (1) becomes $(s^2 + 6s + 25)Y(s) - 3s - 9 = 25 [e - 2s - e - 4s]$ twice, the right member becomes 25 e s - e s - 4s. Thus (1) becomes $(s^2 + 6s + 25)Y(s) - 3s - 9 = 25 [e - 2s - e - 4s]$. Thus (1) becomes $(s^2 + 6s + 25)Y(s) - 3s - 9 = 25 [e - 2s - e - 4s]$. Thus (1) becomes $(s^2 + 6s + 25)Y(s) - 3s - 9 = 25 [e - 2s - e - 4s]$. 6s + 25) The Laplace Transfora 731 Step 3. We must find $y = f + 1 \{ 2 3s + 9 + -25 - e + 451 \} (2) s + 6s + 25$ Using Table 9.1, number 12, we first find $f + 1 \{ s + 3 \} (8 + 3)^2 + 4 2 (3) - 1 \{ 2 5 \}$ and employ partial Now consider f 2 s(s + 6s + 25) fractions. We have 25 A Bs + C 2 = - + + 25 s (s + 6s + 25) Using Table 9.1, number 12, we first find $f + 1 \{ s + 3 \} (8 + 3)^2 + 4 2 (3) - 1 \{ 2 3 s + 9 + -25 + 25 \} = - 3e - 3t \cos 4t - 3f + 25$ s(s + 6s + 25) + s(s + 25) +numbers 1, 12, and 11, respectively. 732 Chapter 9 25 Letting F(s) = 2 and $s(s + 6s + 25) - 3t 3 - 3t f(t) = 1 - e \cos 4t - 4e \sin 4t$, we thus have $-1 \notin \{F(s)\} = f(t)$. Then by Table 9.1, number 16, we find $-1 \{ 25e - 2s \} + 25 = u 2 (t)f(t - 2) = \{ 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e = 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t < 2, 3(t - 2), t > 2, e < 0, 0 < t$ sin 4 (t - 2), t > 2. (4) Similarly, we find -1 { 25e-4s } f 8(8 2 + 68 + 25) = { 0, 0 < t < 4, -3(t-2) 3 - 3(t-2) 3 - 3 sin4(t-2) - 3(t-4) + e cos4(t-4) + 4 e sin4(t-4), t > 4. 12. Step 1. Taking the Laplace Transform of both sides of the D. E., we have The Laplace Transform 733 $f{yH} = f{y}$ by yes), applying Theorem 9.4, and then applying the I.C.'s, we have $f{yH} = s2 y (s) - s + 2$, $f{y'} = sY(s) - 1$. Substituting these into the left member of (1) and simplifying, it becomes (s2 - s)Y(s) - s + 3. Now observe that r - 2t, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, h(t) = 0, t > 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, r - 2t + 0, 0 < t < 2, 0using number 7 aga1n. s 4 Thus the right member of (1) becomes $s_2 - + 2 s_2 - 2s_2 + 2 s_2 - 2s_2 + 2 s_3 - s_3 - s_1 + s_2 - 2s_2 - s_3 - s_3 - s_1 - s_2 - 2s_2 - s_3 - s_3 - s_1 - s_2 - 2s_2 - s_3 - s_3 - s_1 - s_2 - s_2 - s_2 - s_3 -$ 1) } (2) Ve first find the first of these two lawers transforms. Ve write the expression ln the first palr of braces over a lowest common denominator and then employ partial fractions, we obtain 3 s 3s 2 + 4s - 2 = As 2 (s - 1) + Bs(s - 1) + Ds 3 = (A + 1) + Cs - 1) + Cs - 1) + Ds 3 = (A + 1) + Cs - 1) + D)s3 + (B - A)s2 + (C - B)s - C. From this we easily find A = 1, B - 2, C = 2, D = 0. Hence we seek £-1{ } 2£-1{ s; }; and using Table 9.1, numbers 1, 7, and 7, respectively, we obtain 1 - 2t + t 2. Thus The Laplace Transfora 735 r l { s - 3 4 s (s - 1) + 2 s (s - 1) and 2, respectively, we obtain -1 { 2 { , 3 s (s 1) } = 2 t - 2 - 2t - t + 2e . Letting F(s) = 3 2 and f(t) = -2 - 2t - t 2 s (s - 1) + 2e t , we thus have (,-1{F(s)} = f(t). Then by Table 9.1, number 16, we find 736 Chapter 9 -1 { 2e - 2s } = f(t - 2), t > 2, f(t - 2), f(t - 2), f(t - 2), t > 2, f(t - 2), f(t < 2, (4) 2 t - 2 - 2 +
2t - t + 2e, t > 2. Then, uSing (2), (3), and (4), we obtain $f_{y'} = f_{5(t-2)}$. (1) We denote $f_{y} = f_{5(t-2)}$. (1) We denote $f_{y} = f_{5(t-2)}$. (2) $f_{2} = f_{5(t-2)}$. (2) $f_{2} = f_{5(t-2)}$. (2) $f_{2} = f_{5(t-2)}$. (3), and (4), we obtain $f_{y'} = f_{5(t-2)}$. (4) 2 t - 2 - 2 + 2t - t + 2e, t > 2. Then, uSing (2), (3), and (4), we obtain $f_{y'} = f_{5(t-2)}$. (5) $f_{2} = f_{5(t-2)}$. (7) $f_{2} = f_{5(t-2)}$. (9) $f_{2} = f_{5(t-2)}$. (1) $f_{2} = f_{5(t-2)}$. (1) $f_{2} = f_{5(t-2)}$. (2) $f_{2} = f_{5(t-2)}$. (3) $f_{2} = f_{5(t-2)}$. (4) $f_{2} = f_{5(t-2)}$. (5) $f_{2} = f_{5(t-2)}$. (7) $f_{2} = f_{5(t-2)}$. sY(s) - 3. Then the left member of equation (1) becomes (s - 4)Y(s) - 3. By formula (9.103), $f\{5(t - 2)\} = Thus (1)$ reduces to -2s e (s - 4)Y(s) - 3 - 2s = e The Laplace Transform 737 Step 2. We solve this for yes), obtaining yes) -2s 3 e = s - 4 + S - 4. Step 3. We must now determine { } { -2S } -1 1 - 1 e y = 3f s - 4 + f s - 4 · Using Table 9.1, numbers 2 and 16, respectively, we find 4t 4t Y = 3e + u 2 (t)f(t - 2), where f(t) = e. Thus 3 4t u 2 (t)e 4 (t-2) t > 2. e + e, 3. Step 1. We take the Laplace Transform of both sides of the D.E. to obtain $f_{yH} + f_{y} = f_{6(t-r)}$. (1) Ve denote f_{y} by yes) and apply Theorem 9.3 and then the I.C., to obtain $f_{y''} = s = s = 2$ (s) sy(0) - y'(0) 1. Then the left member of equation (1) becomes (s2 + 1)Y(s) - 1. By formula (9.103), $f\{8(t - r)\} = Thus$ (1) reduces to -rs e (S2 + 1)Y(s) - 1 = -rs e 738 Chapter 9 Step 2. We solve this for yes), obtaining yes) = 1 2 s + 1 -rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 Step 3. We must now determine { } { -rs } + 1 - rs e + 2 \cdot s + 1 - rs numbers 3 and 16, respectively, we find y = sin t + u(t) f(t - r), where f(t) = sin t. Thus r y = sin t + u(t) sin(t - r) r rn t, o < t < r, = 0, t > r. 4. Step 1. We take the Laplace Transform of both sides of the D.E. to obtain $\pounds\{y\} + 2\pounds\{y\} = 6(t - 4)$. (1) We denote $\pounds\{y\}$ by yes) and apply Theorem 9.4 and then the I.O.'s, to obtain $f_{YH} = s_2 y(s) - y(O) = s_2 y(s) - 2s + 6$, $f_{Y'} = s_{Y(s)} - 2(s + 3s + 2)Y(s) - 2s + 6$, $f_{Y'} = s_{Y(s)} - 2(s + 3s + 2)Y(s) - 2s - 4s$ By formula (9.103), $f_{\{8(t-4)\}} = e$. Thus (1) reduces to 2 -4s (s + 3s + 2)Y(s) - 2s - 4s By formula (9.103), $f_{\{8(t-4)\}} = e$. Thus (1) reduces to 2 -4s (s + 3s + 2)Y(s) - 2s - 4s By formula (9.103), $f_{\{8(t-4)\}} = e$. -2s = e Step 2. We solve this for yes), obtaining 2s -4s yes) e = +22s + 3s + 2s + 3s + 2s + 3s + 2 Step 3. We must now determine $-1 \{ 2s \} y = f (s + 1)(s + 2) \cdot (2)$ We employ partial fractions to find the first of the two inverse transforms needed. We write 2s A B (s + 1)(s + 2) \cdot (s + 1) + s + 2 \cdot (and upon clearing fractions, at once find A = -2, B = 4. Thus f-l{ CS + l (S + 2) } = -2f-l{ s : 1 } + 4f-l{ s : 2 }. Then uSing Table 9.1, number 2, twice, we find [l{(s 2s } -t -2t (3) + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}. Now we e proceed to determine f (s + l)(s + 2) = -2e + 4e . -1{ -4s + 2}. (s + 2) - t - 2t = e - e Then by Table 9.1, number 16, { -4s f - 1 (s e } + 1)(s + 2) = u 4 (t)f(t - 4), (4) where f(t) we find -t - 2t = e - e Hence using (2), (3), and (4), { -t - 2t o < t < 4, -2e + 4e + e - e, t > 4, { -t - 2t o < t < 4, -2e + 4e , e = (-2 + 4 - t)(4 - 8 - 2t t > 4, e)e + e)e}, Section 9.5, Page 542. 3. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have { $4t f_{x'} - 5f_{x} + 2f_{y} = 3f_{e}$, $f_{y'} - 4f_{x} + f_{y} = f_{O}$. (1) We denote $f_{x'}$ and $f_{y'}$ in terms of Xes) and $f_{y'}$ in terms of Xes) and $f_{y'}$ is the system, we have { $4t f_{x'} - 5f_{x} + 2f_{y} = 3f_{e}$, $f_{y'} - 4f_{x'} + f_{y'} = f_{O}$. sX(s) - 3, (2) $f\{y'\} = sY(s) - y(0) = sY(s) - 4X(s) + yes$ = 0, 3 s - 4' or { 3s - 9 (s - 5) X (s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)Y(s) = 0. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 5)X(s) + 2Y(s) = sY(s) - 4X(s) + (s + 1)(s - 5)X(s) + + 2(s + 1)Y(s) = -8X(s) + 2(s + 1)Y(s) = 0. (3s - 9)(s + 1) s - 4, Subtracting, we obtain (S2 - 4s + 3)X(s) = (3s - 9)(s + 1) s - 4 + from which we find 3 (s - 3)(s - 4) = 3(s + 1) (s - 4) + (s - 1)(s - 1)(s - 4) + (s - 1)(s - 1)(s - 4) + (s - 1)(s - 1 4) \cdot and $-1 - 1 \{ 12 \} Y = f \{yes\} = f (s - 1)(s - 4) \cdot We first find x. We employ partial fractions, we obtain <math>-1 \{ 1 \} - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. Thus $x = -2f - 1 \{ s = -2, B = 5 \}$. $\{1\} y = -4f s_1 + 4f s_4 \cdot Again using Table 9.1$, number 2, we obtain t 4f y = -4e + 4e. 4. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have $\{f_x\} - 2f_x\} - 3f_y\} = 0$, $f_y\} = 0$, $f_y\} + f_x\} + 2f_y\} = 0$, $f_y\} = 0$, $f_y\}$ terms of X(s) and f_{y} in terms of Y(s) as follows: The Laplace Transform 743 { $l_{x'} = sX(s) - x(0) = sY(s) - y(0) = sY(s) - 2X(s) - 3Y(s) = 0$, sY(s) + 1, $l_{y'} = sY(s) - y(0) = sY(s) - 2X(s) - 3Y(s) = 0$, $sY(s) + 2Y(s) = 1/s^2$. or { (S - 3) 2X(s) - 3Y(s) = -1, $Xes) + (8 + 2)Y(s) = 1/s^2$. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S - 2)(s + 2)Y(s) = 3/s - (s + 2) + 3/s , from which we find 3 3 2 Xes) $s + 2 \cdot s - 2s + 3 = - + = 2 2 \cdot 1 \cdot s (s - 1)$ In like manner, we find s - 2 + s - 2yes $1 = + = 22 \cdot 22s - 1s(s - 1)$ $(s - 1) = + = 22 \cdot 22s - 1s(s - 1)$ $(s -
1) = -1{ (:2S - 1) + B(s - 1)(s + 1) + B(s - 1)(s - 1)(s$ $-1(s + 1) + Cs 2(s + 1) + Ds 2(s - 1) = (A + C + D)s^3 + (B + C - D)s^2 - As - B$. From this, we find A = 0, B = -3, C = 0, D = -1. Thus $x = _3[, -1 \{ \} + 2f - 1\{ : 2\} + f -$

numbers 1, 7, and 2, respectively, we find y = -1 + 2t + e - t The Laplace Transform 745 7. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have $\{-t 2f\{x'\} + f\{y'\} - f\{x\} - f\{y'\} - f\{x\} - f\{y'\} - f\{x\} - f\{y'\} - f\{$ we express $f\{x'\}$ and $f\{y'\}$ ln terms of Xes) and yes), respectively, as follows: $f\{x'\} = sX(s) - x(O) = sX(s) + (2 - 1)X(s) + (2 - 1)X(s)$ (s - 1)Y(s) 5s + 6 = s + 1 + (s + 2)X(s) + (s + 1)Y(s) 3s - 2 = s - 1. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(s - 1)Y(s) = 3s - 2. Subtracting, we obtain $(s^2 + 1)X(s) = 2s + 8$, from which we find 746 Chapter 9 Xes) 2s + 8 = 2s + 1. In like manner, we find 32 - s + 14 yes) s - 12s = .2(s + 1)(s - 1)(s + 1) Step 3. We must now determine $-1 - 1\{2S + +81\}x = f\{Xes\} = f - 1\{s - 1(s - 1)(s + 1)\}$ We first obtain x. Using Table 9.1, numbers 4 and 3, we find $x = 2f - 1\{s - 1(s - 1)(s + 1)\} = 2 \cos t + 8 \sin t$. We proceed to find y. We first employ partial fractions. $3 \ 2 \ 14 \ A \ B \ cs + D \ We \ have \ s - 12s - s + = + 1 + 2 \ or \ (s + 1)(s - 1)(s + 1) + (cs + 1)(s - 1)(s + 1$ C = -1 A B D = 14., Letting s = 1 in (3), we find A = ; and letting s = -1 in 1 (3), we find B = -2. Using these values and (4), we find C = 1, D = -13. Thus we have = ! f-1 { 1 } 1 - 1 { 1 } Y 2 s - 1 - 2 f s + 1 + [l t 2 s + 1 - 13[l t 2 1 + J - 13[l t 2 Laplace Transform of both sides of each D.E. of the system, we have { $2f{X} + f{y} = f{x} + f{y} = f{y} = f{x} + f{y} = f{y} = f{x} + f{y} = f{y} =$ -x(0) = sX(s) - 3, sY(s) - yea) = sY(s) + 4. (2) By Table 9.1, numbers 7 and 2, respectively, we find lit $= 1/s^2$ and $l{l} = 1/s^2$ and l $= 2 + \frac{4}{s}$, (s + 2)X(s) + (s + 2)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) + (s + 2)(s + 5)X(s) = (s + 2)(s + 5)X(s) + (s + 2)(s + $8 + s_2(s_2 - 2s - 8) 3s_3 + 7s_2 - 6s + 82 \cdot s(s - 4)(s + 2)$ The Laplace Transform 749 In like manner, we find yes) = -4s - 1 s_2 - 2s - 8 2 \cdot s(s - 4)(s + 2) -1{-48 3 2 - 2s - 8} + 82 \cdot s(s - 4)(s + 2) -1{-48 3 2 - 2} + 82 \cdot s(s - 4)(s + 2) -1{-48 3 2 - 2} + 82 \cdot s(s - 4)(s + 2) -1{-48 3 2 - 2} + 82 \cdot s(s - 4)(s + 2) -1{-48 3 2 - 2} + 82 \cdot s(s - 4)(s + 2) -1{-48 3 2 $3f-1\{8,4\} - f-1\{8,2\}$ Then, uSing Table 9.1, numbers 1, 7, 2, and 2, respectively, we find x = 1 - t + 3e 4t e-2t 750 Chapter 9 In like manner, uSing Table 9.1, numbers 7, 2, and 2, respectively, we find y = t - 3e 4t e-2t 10. Step 1. Taking the Laplace 1 { } _ 3f-1 { } _ 5 - 1 { } _ Transform of both sides of each D.E. of the system, we have $f\{x H\}$ and $f\{y\} = 0$, (1) $f\{x'\} + f\{y'\} = 0$, (1) $f\{x'\} + f\{y'\} = 0$, (1) $f\{x'\} + f\{y'\} = 0$, (2) $f\{x'\} + f\{y'\} = 0$, (2) $f\{x'\} + f\{y'\} = 0$, (3) $f\{x'\} + f\{y'\} = 0$, (4) $f\{x'\} + f\{y'\} = 0$, (5) $f\{x'\} = 0$, (6) $f\{x'\} = 0$, (7) $f\{x'\} = 0$, (8) $f\{x'\} = 0$, (9) $f\{x'\} = 0$, (1) $f\{x'\} = 0$, (1) $f\{x'\} = 0$, (2) $f\{x'\} = 0$, (3) $f\{x'\} = 0$, (4) $f\{x'\} = 0$, (5) $f\{x'\} = 0$, (7) $f\{x'\} = 0$, (8) $f\{x'\} = 0$, (9) $f\{x'\} = 0$, sY(s) + 1. Substituting these expressions into (1), we see that (1) becomes { S2X(s) + sY(s) + 1 + 2X(s) - yes = 0, sX(s) + sY(s) + 1 - 2X(s) + yes = 0, $or { (S2 - 3s + 2)X(s) + (s - 1)Y(s) = -1, (s - 2)X(s) + (s - 2)X(s)$ l(x + 1)(x + 1)(x + 1)(x) = -(s + 1), (s - 2)(s - 1)X(s) + (s + 1)(s - 2)(s - 1)X(s) = -(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = s - 2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s)
= -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find yes) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1) . Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 s(s - 1) . Subtract partial fractions. Ve have $-2 \ A B \ C \ s \ (s - 1) \ (s - 2) = s + S - 1 + S - 2 \ or \ -2 = A(s - 1)(s - 2) + Bs(s - 2) + Cs(s - 1)$. Letting s = 2, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 1, we find A = -1; letting s = 0, we find A = -1; letting s = 1, we find A = -1; letting s = 0, 2t Ve now find y, again using partial fractions. Ve have -s + 2 A B Ye readily find A = -2, B = 1; and s (s 1) = $- + 1 \cdot s s - hence _{2f-1} \{ \} + f-l(s 1 1)' Y = 752$ Chapter 9 Then from Table 9.1, numbers 1 and 2, we find t Y = -2 + e. Answers to Even-Numbered Exercises Section 1.1, Page 5. 2. ordinary; third; linear. 4. ordinary; first; nonlinear. 6. partial; fourth; linear. 8. ordinary; second; nonlinear. 10. ordinary; second; nonlinear. 10. rot exact. 12. 3 2 x Y 3 2 Y x + x + y + 1 = 0. 753 754 Answers to Even Numbered Problems 14. x 2 x = y + xy + 2e = 8. 16. 2 1/3 -1/3 4 4/3 1/3 9. xy + xy = 18. (a) A 3. 3 2xy = xy + e. (b) A -2. -2 -1 = xY - xY = c. (b) A -2. -2 -1 = xY - xY = c. (b) A -2. -2 -1 = xY - xY = c. (c) x + xy = c. 24. 4 arc tan + (x2 + y2) 2 = c. 4. Sln x - cos y = c. 6. (s in u + 1) (e v + 1) - c. - 10. 2 2 2 v = u (In v + c). 12. t 3 2 2 3 3t s - 3ts - 28 = c. 8. y - x In I cx I · 14. x [jx2:/2 + 1] 16. 4x - 2 sin 2x + tan y 'K = C. = 3. 18. 2 2 3 2(3x 2 2 2 5 x + Y = 5x. 20. + 3xy + y) = 9x. 22. (a) 2 + 4xy - Y 2 26. (a) y = x In I cx I; x = c; (b) 2 2 (b) 2 3 3x - 2xy - Y = c. y + xy = cx. Answers to Even Numbered Problems 755 Section 2.3, Page 56. 2. 2 1 x y + - = c. 10. = e, 28. (4e 10 1 + l)e -x x > 10. = + 2 + 1 x 0 < x < 3, Y = 2 (x + 1) 30. 3x - 4 x > 3. Y = x + 1, 756 Answers to Even Numbered Problems 36. 5 (b) y = ce -x + L k = 1 sin kx - k cos kx 1 + k 2 2 - x / 2 e y - x 2 = f e -x / 2 d x + c. 40. Miscellaneous Review Exercises, Page 59. 2. 2 3 x y - xy = c. 2 4. x c y = - + 2. 4 x 2 8. (y+x) _ c y + 2x - x 12. 2 1 Y $= . 2 (1 + cx) 6. (e 2x - 2)y^{2} = c. 10. y = e - x [-1 + c(x + 1)]. 14. (y - 2x)(y + x) = cx. 16. (x + 1) J y^{2} + 4 - 4(x - 1). - 18. 3 2 2 3 2 16(x + 2y). x + x y + 2y - 21. 20. 5(2x + y) = - { -x 0 < 2, 1 - e x < - 22. y = 2 - x (e - 1)e , x > 2. 24. = Y x4 + 1 Answers to Even Numbered Probleas 757 Section 2.4, Page 67. 2. 2. x cos y + x Sln y = c. 4. 2 - 1 2 x + xy + y = c.$ 6. 2/3 5/3 2 x y (x - y) = c. 8. x - 2y + Inl3x - y - 21 = c. 10. 3 2 (2x + y + 3) (x - y + 1) = c. 12. 2 2 In[3(x - 1) + (y + 3) J 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y + 3 [3(x - 1) + (y + 3)] 2 + - arctan [3 y +2 2 y = k(x + 3y). 14. n = 3. 16. In(x 2 + y2) + 2 arc taney/x) = k. 18. Inl3x 2 + 3xy + 4y21 - (2/39) arctan[(3x + 8y)/ 39x J = k. 758 Answers to Even Numbered Problems Section 3.2, Page 88. 2. (a) v = 9 ft./sec.; x = 539.16 ft.; (b) v = 9 ft./sec.; x = 539.16 ft.; (b) v = 9 ft./sec.; x = 539.16 ft.; (b) v = 9 ft./sec.; x = 539.16 ft.; (c) 402 sec. (b) 12.5 ft./sec.; (c) 402 sec. (b) 12.5 ft./sec.; (c) 402 sec. (c 7.16 ft./sec.; (b) 4.95 sec. 10. (a) v = 100 tan (arc tan 10 - O. 32t); (b) 4.60 sec. 12. 3.06 ft./sec. 14. 17.5 ft./sec. [2 R 2 2 r/ 2 16. 0.25 18. v = gx + v 0 - 2gR. Section 3.3, Page 102. 2. (a) 34.5%; (b) t = 43.4 min. 4. (a) 19.8%; (b) 9 hrs., 58 min. 6. (a) 10.22 oz.; (b) 23 hrs., 34 min. 8. 18.91 mln. 10. (a) 151.410; (b) Between 22 min., 31 sec., after 10 A.M., and 30 mln., 12 sec., after. Answers to Even Numbered Probleas 759 12. 40,833. 14. (a) $x = (4219)(10.6) e O \cdot 02$ (t-1978). (b) 2410 million. (c) 6551 million. (c) 6551 million. (d) 886 million. (e) 48,405 million. (e) Ib./gal. 24. (a) 466.12 lb.; (b) 199.99 lb. 26. 4,119.65 gm. 28. 292.96 (ft.)3/ min. 30. 2016. 32. 7.14 grams. Section 4.1B, Page 122. 2. Y = 0 for all real x. 4. (b) and (c). Theorem 4.2. 8. x x (b) y = c 1 e + c 2 x e. (c) x x y = e + 3xe; -00 < x < 00. 760 Answers to Even Numbered Problems 10. (b) 2 c 2 y = c 1 x + 2. x (c) 1 2 8 Theorem 4.1; o < x < (I) y = -x + -. 4 2' x 12 - x 3x 4x y = c 1 e + c 2 e + c 3 e Section 4.1D, Page 132 2 . 3 Y = c 1 e + c 2 e + c 3 e Section 4.2, Page 143 2 . 3 Y = c 1 e + c 2 e + c 3 e Section 4.2, Page 143 2 . 3 X - x 4 . 5 x - x/3 y = c 2 e + c 2 e y = c 1 e + c 2 e + c 2 e y = c 1 e + c 2 e + c 3 e Section 4.2, Page 143 . 2 . 3 X - x 4 . 5 x - x/3 y = c 2 e + c 2 e y = c 1 e + c 2 e + c
2 e + c 2 e + c 2 e + c 2 e + c 2 e + c 2 e + c 2 e + c 2 e + c-2x 8. (c1 + c2 x)e 2x y = c1 e + c2 e + c3 e 14. -2x (x/2 y = c1 e + c2 + c3 x)e . 10. -x(. 3x 3X) y = e c 1 Sln 4 + c 2 cos 4 . Answers to Even Numbered Problems 761 12. -x 3x 4x y = c1 e + c 2 e + c 3 e 14. -2x (x/2 y = c1 e + c 2 e + c 3 e 14. -2x (x/2 y = c1 e + c 2 + c 3 x)e . 10. -x(2 x + x). y = c 1 e + c 2 e + c 3 e 14. -2x (x/2 y = c 1 e + c 2 + c 3 x)e . 10. -x(2 x + x). y = c 1 e + c 2 e + c 3 e 14. -2x (x/2 y = c 1 e + c 2 + c 3 x)e . 10. -x(2 x + x). y = c 1 e + c 2 e + c 3 e 14. -2x (x/2 y = c 1 e + c 2 e + c 3 e 14. -2x (x/2 y = c 1 e + c 2 + c 3 x)e . 10. -x(2 x + c 2 $cos 2 \cdot 2 \cdot 24$. $(c 1 + 2 \cdot x/2 \cdot y = c \cdot 2x + c \cdot 3x)e \cdot 26 \cdot x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 1e + c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 3x + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x)e \cdot 34 \cdot xy = c \cdot 2e + c \cdot 4x + c \cdot 5x + c \cdot 2x + c \cdot 3x + c \cdot 2x + c \cdot 3x +$ Numbered Problems 36. Y = c l sin 2x + c 2 cos 2x + e.[3X(c 3 Sin x + c 4 cos x) . [3x + e (c 5 sin x + c 6 cos x) . 38. -2x - 5x y = -6e + 2e . 40. -2x y = 2e . 42. y = 3xe 3x / 2 + 4e 3x / 2 . 44. y = (3 - 2x)e x / 3 . 46. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -x/2(sin 3x + 2 cos 2x) . 50. y = e -x/2(sin 3x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -x/2(sin 3x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x - 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e (4 sin 2x - 2 cos 2x) . 50. y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - cos 7X) . 48. -x y = e -3X(sin 7x - c $(2 \sin x - \cos x) \cdot 56 \cdot -2x Y = e - \sin 2x \cdot 58 \cdot -X/2[(+ c 2 x)\sin [7x (c 3 + k] y = e c 1 2 + c 4 x)\cos 2 \cdot 60 \cdot (c 1 + c 2 x + c 5 x + c 6 x)e$ 3x c g x2) sin 4x + e [(c 7 + c 8 x + (c 10 + 2 c 11 x + c 12 x) cos 4x] \cdot 62 \cdot -2x \cdot -3x + e X (c 3 sin 2x + c 4 cos 2x) \cdot y = c 1 e + c 2 e Answers to Even Numbered Problems 763 Section 4.3, Page 763 Section 4. 159. 2. $4x y = c 1 e - 2x + c 2 e 1 2x - 3x 2 e - 3e \cdot 4$. $4x - 2 \cos x$ 7 4 y = e (c 1 sin 3x + c 2 cos 3x) + - xe + 10 e 2 10. -2x - 4x - 4x 1 2x 5 2x y = c 1 e + c 2 e + c 2 e + - xe - e 4 48 2 3 7 + x - x + 8. 2 12. 2x - 2x 2 2 2x 2x y = c 1 e + c 2 e + x e xe 14. (c $1 c 2 x e^{-3} = 1 c + 2 c + 2 x + 3 x e^{-2} = 1 e^{$ Answers to Even Numbered Problems 26. x 2x 1 -x 3 2x y = c 1 + c 2 x + c 3 e + c 4 e + - e + - x e 2 2 1 3 9 2 - - x - - x . 2 4 28. (c 1 c 2 x) e x 2x 1 4 x 1 2 x y = + + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + 3x - 2 34. 2x 3x c 3 sin x + 1 sin 2x y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + c 4 e + - e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + 2x + 32. c 1 sin 2x + c 2 cos 2x 2 3 - 2x 2 sin 2x - x cos 2x . y = + c 3 e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + - x e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + - x e + - x e . 4 2 30. 2x - 2x 2 x 2 x 3 3x. y = c 1 + c 2 e + c 3 e + - x e + $1 e + c 2 e + c 4 \cos x + 13 5 \cos 2x + 11 + 13 4 x Sln x - 4 x cos x$. 36. -x x - 5. y = 3e + 2e + 4x 38. -2x 2xe - 3x - 5x y = e 40. (3x - 5)e - 3x - 6x y = + 3e + 2e + 4x 38. -2x 2xe - 3x - 5x y = e 40. (3x - 5)e - 3x - 6x y = + 3e + 2e + 4x 38. $-2x 2x \cos 2x + 1$. $-4x \cos 2x - 2x \cos 2x - 3x - 5$. $-2x \cos 2x - 5$. -2x $3 x x x e 10. y = c 1 e + c 2 x e + x e \ln x 36 6 \tan x 3 12. Y = c 1 \sin x + c 2 \cos x + + 2 \cos x \ln x e 3 \ln x + c 2 \cos x + + 2 \cos x \ln x e 3 \ln$ $2(x + 1) + 2 \cdot 22 \cdot x \cdot 2Y = c \cdot 1x + c \cdot 2x \cdot x \cdot x - 1 \cdot 23 \cdot 21 \cdot 123 \cdot 21
\cdot 124 \cdot (2x + 3x)(x + y = c \cdot 1x + c \cdot 2x \cdot 2x - 1y = c \cdot 1x + c \cdot 2x \cdot 2x + c \cdot 2x \cdot 2x - 1y = c \cdot 12x \cdot 2x + c \cdot 2x + c \cdot 2x \cdot 2x + c \cdot$ $2 \cos(\ln x) J. 8. c l \sin(\ln x 3) 3 y = + c 2 \cos(\ln x) . 10. y = x 3 [c l \sin(\ln x) + c 2 \cos(\ln x)]. 12. c 1 c 2 4 y = c 1 x + c 2 x + c 3 x + c 4 x . 16. 2 4 3 y = c 1 x + c 2 x + c 3 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x + c 4 x +$ $2 3 26 - 1 2 - 2x + 4 24 \cdot Y = -2x + x Y = x + x 768$ Answers to Even Nuabered Problems 28. $y = x^2 + 2x - 3 + 2x 2 \ln x$. 30. $y = (x^2 + x^3 3 \ln x +)$. 32. 3y = c 1(2x - 3) + c 2(2x - 3) + c 2 $\cos 8t 8 11 + 37 \sin 16t - 37 \cos 16t$. 770 Answers to Even Numbered Problems -8t (11.9) 7 2 4. (a) x = e c 1 Sln t + c 2 cos t + 85 Sln - 85 cos t + 14 sin 2t - 5 1 2 cos 2t + 33 sin 3t - 33 cos 3t, where c 1 = -54,854/1,471,860, c 2 = 29,819/490,620. 10. 37 -2t $(b) x = 8 e [\cos(4t -)] + 4 [\cos(2t - O)], where; = \cos - 1:7 and 0 = \cos - 1(-) or x = e - 2t [\cos 4t + \sin 4t] + \sin 2t - \cos 2t.$ Section 5.5, Page 232. 2. 15 -50t 1 = 17 (COS 200t + 4 sin 200t - e). 4. (t) -lOOt _ 1 q - 200 + 2 e 200 cas lOOt. Answers to Even Nuabered Problems 771 6. -200t . 1 = e (1.0311 Sln 200t - e). 4. (t) -lOOt _ 1 q - 200 + 2 e 200 cas lOOt. 979.8t + 0.1031 cas 979.8t) - 0.1031 e - 100t . 8. (d) w = 100 (-187.5)2 + (20)2 100 N N 190 . 50; ampl.: Section 6.1, Page 249. 10. 2. y = c O (l + 2x 2 + x 4 + ...) + c l (x 2 5 + -x 3 + ...) 2 3 - x 3 4. y = C O (l + 2x 2 + x 4 + ...) + c l (x 1 5 - -x 20 + ...) 1 3 + -x 2 + ... 2 8. y = C O (l Y = x 3 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...) + c l (x + ; x 3 2 + x 1 4 - x 4 + ...)) + c l (x + ; x 3 2 + ...) + c l (x + ; x 3 + ...) + c l (x + ; x 3 + ...) + c l (x + ; x 3 + ...) + c l (x + ; x 3 + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...) + c l (x + ; x + ...4 21x 5 18.2 3x x y = + - - + + ... 6 2 40 20. CD Y = L (-1) U x U . n = 0 22. Y = Co [1 1 2 5 3] + 2 (x - 1) - 6 (x - 1) + ... [n (n + 1) 2 24. (a) y = Co 1 2! x n (n - 2)(n + 1)(n + 3) 4 ...] + 4! x - + c1 [x - (n - 1)(n + 2) x 3 3! (n - 1)(n + 3) (n + 2)(n + 4) 5 ...] + x - 5! Section 6.2, Page 269. 2. x = 0and x = -1 are regular singular pts. 4. x = 0 is an irregular singular pt.; x = -3 and x = 2 are regular singu ...) 1 3 15 (2 2 4 3 + C 2 1 - 2x + 3 x - 45 x + ...). 1/2 (2 x 2 x 2 4 x 3) 12. Y = C 1 x 1 - 1r + 63 - 2079 + ... - 2 (2 x 2 x 2 4 x 3 ...). + C x 1 + -- + --- + 2 3 3 9 14. 16. 18. y = C 1 X S / 2 (1 1225 x 2 ...) + C x 1 + -- + --- + 2 3 3 9 14. 16. 18. y = C 1 X S / 2 (1 1225 x 2 ...) + C x 1 + -- + 2 2 2 4 ...). m y = 2n C 1 1 + t1 2:n! (I) $2n \times 15 \cdot 7 \cdot 9 \dots (2n + 3)T3 + C2 \times 1 + Ln = l = C1(l 2 4 \times X + +++ 28 \dots) + C2 \times 3(1 + XS2 + :+ \dots)$. 774 Answers to Even Numbered Problems 20. = C1/2 C3/2 y 1 x + 2 x (I) 22. -l L (_1) n x n - 3 y = C1 x (n + 2)! + C2 x (1 - x) . n = O C1 x 5 / 2 (1 2 4) - 3/2 2 24. x x y = -+ + -+++ C2 x (1 - x) . n = O C1 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x (1 - x) . n = O C1 x 5 / 2 (1 2 4) - 3/2 2 24. x x y = -+ + -++++ C2 x (1 - x) . n = O C1 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots
J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x 2 + 18 x 4 - \dots J + C2 x 3 (1 - x)) + C2 x 3 (1 - x) + C2 x 3 ([x-1(1 - x + 12x5 - ...] + ;6 Yl(x) ln 1x1], where Y1(x) denotes the solution of which C 1 is the coefficient. (I) = (1) n x n 30. Y C 1 1 + n-1 n=12 n!(n + 2)! [-2 (1) n 1 1 29 2 + C 2 x - 2 - 4 x + 576 x + +; 6 y [(x) ln 1 x 1], where y l (x) of which C 1 is the coefficient... J denotes the solution Answers to Even Numbered Problems 775 32. (I) y = C 1 x 3 1 + (_1) n x 2n f. J n-l n=12 n! (n + 2)! [-1 (1 1 2 29 4) + C 2 x - 4 - 16 x + 4608 x + ... + i4 Yl(X) ln 1xl], where Y l (x) denotes the solution of which C 1 is the coefficient. Section 7.1, Page 296. x = ce-t - 2, 2. y = -2ce-t - t 2 - 2t + 4. t 3t x = c1 e + c2 e 4. c1 t 3t _ 3e 2t. y = -e c 2 e 3 t 6. t sin t c 1 e sin t x = c1 e + c 2 e 4. c 1 t 3t _ 3e 2t. y = -e c 2 e 3 t 6. t sin t c 1 e sin t x = c1 e + c 2 e 10. f3t _ -f3t - 2t 4 y = 3 c 1 e 3 c 2 e + 3. 776 Answers to Even Numbered Problems x = c1 sin t + c2 cos t - t - 3, 12. y = (c 2 - c 1) sin t - (c 1 + c 2) cos t - 1. 3 e t x = , 14. t 1 2 t Y = -2e - - e 2 - 2t 2 x = c 1 + c 2 e + 2t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4. 1 2 2 - t x = e (c 1 sin 2t + c 2 cos 2t) + 2, 20. + 2 + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + c 2 e + -t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4 + 18. 2t t !. t 2 3 15 y = -3c e + -t + 2 t + 4) cos2tJ 2 + t + 2t - 14. 22. tt 1 2t x = c 1 + c 2 e + c 3 te + - e, 2 t - c 3 (t + 1) e t 1 2t Y = -c - c e - - e . 1 2 2 t - 2t 2 t x = c 1 + c 2 e + c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 - t - 2t 4 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 t Y = -4c + 2c 2 e + c 3 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t - 2t 5 te + - e , 2 t c_3) $c_3 + 551 = 2t_28$. $dX_1 dt = x_2$, $dX_2 dt = x_3$, $dX_3 3t_dt = 2x_1 + x_2 - 2x_3 + e dX_1 dt = x_2$, $dX_2 30$. $dt = x_4 + dX_4 - 2tx_2 dt = +t_x_3 + cost_1$. 1 Section 7.2, Page 307. $x = c_1 - t/5 + 20$, $f_2 = -t/5 + 20$, $f_2 = -t/5 + 20$, $f_2 = -t/5 + 20$. $f_3 = -t/5 + 20$, $f_3 = -t/5 + 20$. $f_3 = -t/5 + 20$. Page 317. 3c1 e7t-tx = + c2 e, 2. (b) 2c1 e7t - 2c e - ty = 2 3e7t - tx = - 3e, (c) 7t - tY = 2e + 6e. Section 7.4, Page 328. 4t 2tx = c1 e + c2 e 4. -3c et - tY = c2 e 15t 3tx = c1 e + c2 e 5. 5t + 3c 2 e 3ty = c1 e 2t c 2 sin 3t), x = e(c1 cas 3t + 8. y) = e 2t (3c1 sin 3t - 3c 2 cos 3t). Answers to Even Numbered Problems 779 5t -3t x = c 1 e + c 2 e 10. 5t 3c 2 e -3t y = c 1 e 2t c 2 sin 3t), x = e (c 1 cos 3t 12. 2t + c 2 sin 3t), x = e (c 1 cos 3t 12. 2t + c 2 sin 3t), x = e (c 1 cos 3t + c 2 sin 3t), x = e (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 1 + c 2 sin 3t), x = - Se 4t (c 1 cos 1 + c 2 sin 3t), x = - Se 4t (c 1 cos 1 + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 1 + c 2 sin 3t), x = - Se 4t (c 1 cos 3t +
c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (c 1 cos 3t + c 2 sin 3t), x = - Se 4t (sin t), 18. 4t - 2c 1) cost - (c 1 + 2c 2 sint]. y = e [(c 2 2c 1 e 5t + c 2 (2t + 1)e 5t x = , 20. 5t 5t Y = -c e c 2 te · 1 + c 2 (2t 1)e -t x = c 1 e + +, 22. -t 2c 2 te -t y = c 1 e + . 780 Answers to Even Numbered Problems 2c 1 e 6t + c 2 (2t + 1)e 6t x = , 24. 6t 6t y = c 1 e + c 2 te · x = 2c 1 + c 2 (2t + 1), 26. Y = c 1 + c 2 t. 5t - 3t x = e + 5e, 28. 7 5t 5 - 3t x = e + 5e = , 28. 7 5t 5 $Y = e - e \cdot 4t (10) x = e 5 \cos 3t - 3 Sln 3t, 30.4t (11.3) Y = -e \cos 3t + 3 S In t \cdot 5t t 5t (8t + 6)e - 3t, x = e e, x = 32.34.3e 5t 2te 5t - 3t y = y = (16t + 8)e \cdot 24 x = c 1 t + c 2 t, 36.24 Y = c 1 t + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 8 0 12 - 4 -8 Answers to Even Numbered Problems 781 - 15 3 - 6 (c) -12 9 6 0 -9 18 2x 1 + 3c 2 t \cdot Section 7.5A, Page 340. [2: -:] -4 12 - 20 2. (a) (b) -24 - 20 2 (c) (b) -24 - 20 2 (c) (b) -24 - 20 2$ x 2 - 4x 3 - 35 4. (a) 5x 1 - 2x 2 + 3x 3 (b) 10 xl - 3x 2 + 2x 3 - 7 xl - 2x 2 + 3x 3 (c) -3x - 4x - 4x 1 2 3 - 2x + x 2 - 2x 3 1 6. (a) (i.) lot - 18t 2 + 2t 4t - 5 (ii.) 3e 3t (4t 3 - 2 + 3) 2 (b) 10 xl - 3x 2 + 2x 3 - 7 xl - 2x 2 + 3x 3 (c) -3x - 4x - 4x 1 2 3 - 2x + x 2 - 2x 3 1 6. (a) lot - 18t 2 + 2t 4t - 5 (ii.) 3e 3t (4t 3 - 2 + 3) 2 (c) -3x - 4x - 4x 1 2 3 - 2x + x 2 - 2x 3 1 6. (a) lot - 18t 2 + 2t 4t - 5 (ii.) 3e 3t (4t 3 - 2 + 3) 2 (b) 10 xl - 3x 2 + 2x 3 - 7 xl - 2x 2 + 3x 3 (c) -3x - 4x - 4x 1 2 3 - 2x + x 2 - 2x 3 1 6. (a) lot - 18t 2 + 2t 4t - 5 (ii.) 3e 3t (4t 3 - 2 + 3) 2 3 t (9 + 2t + 2t 4t - 5) (ii.) 3e 3t (4t 3 - 2 + 3) 2 3 t (9 + 2t + 2t 4t - 5) (ii.) <math>3e 3t (4t 3 - 2 + 3) 2 + 3x 3 (2t - 3) 2 + 3x 3 (28 -16 20 8 20 10. AB = 21 13 19. BA = 27 -4, -66 -22 12 -2 -24 Answers to Even Numbered Problems 783 6 6 18 4 5 12. 0 2 4 14. 2. -16 -10 -8 1 1 2 1 7 1 5 3 6 2 3 2 1 9 -3 1 5 5 2 2 . 6 4 -1 1 1 1 5 5 2 2 5 3 3 2 2 24. 1 1 1 - - . 12 6 12 25 17 13 - - 12 6 12 Section 7.5D, Page 367. 2. Characteristic values: 5 and -3; Respective corresponding characteristic vectors: (:) and (-3:), where in each vector k is an arbitrary nonzero real number. 784 Answers to Even Numbered Problems 4. Characteristic values: 5 and -5; Respective corresponding characteristic vectors: (:) and (-3:), where in each vector k is an arbitrary nonzero real number. number. 6. Characteristic values: 7 and -2; Respective corresponding characteristic vectors: (-::) and (:), where in each vector k 1S an arbitrary nonzero real number. 8. Characteristic vectors: 10k -7k -3k , k -k -3k , where in each case k 1S an arbitrary nonzero real number. 10. 1S an arbitrary nonzero real number. 14. Characteristic values: 1, -1, and 4; Respective corresponding characteristic vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k, and -2k, 0 k -k where in each vectors: 2k 0 k k, 3k -k where in each vectors: 2k 0 k k, = c 1 e + c 2 e x = c 1 e + c 2 e x = c 1 e + c 2 e x = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e
x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e x 2 = c 1 e + 3c 2 e $(2 \cos 2t + \sin 2t) J x 2 = -e [c 1 (\cos 2t - 2c e 5t c 2 (2t 5t x] = + 1)e, 1 16. 5t c 2 t e 5t x 2 = c 1 e + . + t + c 2 (2t + 1)e 5t x 2 = . Answers to Even Numbered Problems 787 Section 7.7, Page 400. 6. 8. 10. 12. 2. 4. 2t x] = 10c 1 e x 2 = -7c l e 2t x 3 = -3c l e 2t x 3 =$ le 2t 3t + c 2 e 3t c 2 e 3t c 2 e 3t c 2 e 5t + c 3 e 5t - c e 3 5t 3c 3 e · t 4t x1 = c 1 e + c 3 e 4t - c e 3 4t c 3 e t 2t x1 = c 1 e + c 2 e + c 3 e 2t - t - 4t x 3 = c 1 e + c 2 e + c 3 e 2t - t - 4t $2t - 2c 3 \cos 2t$, $3t x 2 = 3c 1 e + c 2 \cos 2t + c 3 \sin 2t$, $3t x 3 = 7c 1 e + c 2 (\cos 2t + 2\sin 2t) + c 3 (\sin 2t - 2\cos 2t)$. 2t t x 1 = c 1 e + c 2 e 2 t t x 3 = c 1 e + c 2 e 2 t t x 3 = c 1 e + c 2 e 2 t t x 3 = 3c 1 e + c 2 e 2 t t x 3 = c 1 e + c 2 e 2 t t x 3 = 3c 1 e + c 2 e 2 t t x 3 = 2c 1 e + c 2 e 2 t t x 3 = 3c 1 e + c 2 e 2 t t x 3 = c 1 e + c 2 e 2 t t x 3 = 3c 1 e + c 2 e 2 t t x 3 = c 1 e + c+ c 2 e , 28 t t x 2 = 2c 1 e + c 3 e , 28 t t x 3 = c 1 e + c 3 e , 2t 5 c 2 e 2t + c 3 t x 3 = c 1 e + c 2 e + c 3 t x 3 = c 1 e + 2 c 3 e , -t 3t 3t x 3 = c 1 e + c 2 e + c 3 t x 3 = c 1 e + c 2 e + c 3 t x 3 = c 1 e + c 2 e + c 3 t x 3 = c 1 e + c 2 e + c 3 t x 3 = c 1 e + c 2 + c 3 t x 3 = c 1 e + c 2 e + c e - t + 3c 2 e t + C 3 (3t -)et, -t t t x 2 = 2c 1 e + 5c 2 e + 5c 3 te, x 3 = -2c 1 e + c 2 (2t + 1)e + c 3 (t + t, 28, 2t 2t x 2 = -c 1 e + c 2 (t + 1)e + c 3 (t + t, 28, 2t 2t x 2 = -c 1 e + c 3 (t + t, 28, 2t 2t x 2 = -c 1 e + c 2 (t + 1)e + c 3 (t + t, 28, 2t 2t x 2 = -c 1 e + c 2 (t + 1)e + c 3 (t + t, 28, 2t 2t x 2 = -c 1 e + c 2 (t + 1)e + c 3 (t + 1)eSection 8, IB, Page 427 2. 4. Answers to Even Numbered Problems 793 6. 8. 794 Answers to Even Numbered Problems 10. 12. Answers to Even Numbered Problems 795 Section 8.2, Page 434. 2. 4 + 2 6145 3 y = $32x + 256x + x + \ldots 34.3 + 27x + 729 210,935 3 Y = -x + x + \ldots 226.1315 Y = x + x + \ldots 38.12143354 y = +x + 2x$ + - x + - x + ... 3 3 10. Y = 1 + 2(x - 1) 7 (x - 1) 2 + 14 (x - 1) 3 + 2 3 + 73 (x - 1) 4 + ... 12 (x - 1) 2 5 12. Y = !" + (x - 1) + ... 2 40 14. Y = 2 + (e + 2)(x - 1) + (e + 1)(X - 1) 2 + (e + 2)(x - 1) + (e + 1)(X - 1) 2 + 14 (x - 1) 3 + 2 3 + 73 (x - 1) 4 + ... 12 (x - 1) 2 5 12. Y = !" + (x - 1) + ... 2 40 14. Y = 2 + (e + 2)(x - 1) + (e + 1)(X - 1) 2 + 14 (x - 1) 3 + 2 3 + 73 (x - 1) 4 + ... 12 (x - 1) 2 5 12. Y = !" + (x - 1) + ... 2 40 14. Y = 2 + (e + 2)(x - 1) + (e + 1)(X - 1) 2 + 14 (x - 1) 3 + 2 3 + 73 (x - 1) 4 + ... 12 (x - 1) 2 5 12. Y = !" + (x - 1) + ... 2 40 14. Y = 2 + (e + 2)(x - 1) + (e + 1)(X - 1) 2 + ... 12 (x - 1) 2 + ... 1(x) x x 4.; 1 (x) = x, 2(x) = x + 4' = x + - + 14 + 160. 4 796 Answers to Even Numbered Problems 6. $1(x) = 1 - \cos x$, J. () 1 3 2 . 1. 2 Y 2 x = + 2 x - cos x - Sln x + 4 Sln x. 8. 1(x) = x, 2(x) 6 = x + x, 3(x) 6 24 11 9 16 8 21 6 25 = x + x + 11 x + 4 x + 7 x + 25 x Section 8.4 x n Exact Sol'n Euler Approx. Error % ReI Errors 2. 0.25 0.026633 0.000000 $0.026633\ 100.0\ 1.50\ 0.512447\ 0.503906\ 0.008541\ 1.666615\ 4.\ -0.80\ 1.268869\ 1.200000\ 0.068869\ 5.427554\ 0.00\ 5.291792\ 3.783680\ 1.508112\ 28.499080\ 6.\ 1.50\ 2.598076\ 2.625000\ 0.026924\ 1.036297\ 3.00\ 4.242641\ 4.304352\ 0.061711\ 1.454539\ 8.\ 0.20\ 0.510067\ 0.500000\ 0.010067\ 1.973606\ 1.00\ 0.791798\ 0.713110\ 0.078687\ 9.937829\ 10.\ 1.20$
0.692821 1.200000 0.507179 73.204864 2.00 1.743560 2.000000 0.256440 14.707844 12. 0.20 1.019739 1.000000 0.019739 1.935654 2.00 1.957624 1.935192 0.022432 1.145899 Answers to Even Numbered Probles 797 Section 8.5 x n Exact Sol'n Improved Euler Error % ReI Errors 2. 0.25 0.026633 0.031250 0.004617 17.337112 1.50 0.512447 $0.514901\ 0.002454\ 0.478956\ 4.\ -0.80\ 1.268869\ 1.260000\ 0.008869\ 0.698932\ 0.00\ 5.291792\ 5.075616\ 0.216176\ 4.085122\ 6.\ 1.50\ 2.598076\ 2.602679\ 0.004602\ 0.177145\ 3.00\ 4.242641\ 4.251370\ 0.008730\ 0.205757\ 8.\ 0.20\ 0.510067\ 0.509933\ 0.000133\ 0.026123\ 1.00\ 0.791798\ 0.789224\ 0.002574\ 0.325092\ 10.\ 1.20\ 0.692821\ 0.800000\ 0.107179$ 15.469910 2.00 1.743560 1.789042 0.045482 2.608548 12. 0.20 1.019739 1.019867 0.000128 0.012583 2.00 1.957624 1.955473 0.002152 0.109918 Section 8.6 x n Exact Sol'n Runge- Kutta Error % ReI Errors 2. 0.25 0.026633 0.026693 0.000060 0.225450 1.50 0.512447 0.512447 0.512447 0.512447 0.512447 0.512447 0.512447 0.512476 0.000030 0.005776 4. -0.80 1.268869 1.268800 0.000069 0.005400 0.00 5.291792 5.290095 0.001697 0.032064 798 Answers to Even Numbered Problems 6. 1.50 2.598076 2.598127 0.000051 0.001961 3.00 4.242641 4.242717 0.000077 0.001805 8. 0.20 0.510067 0. 0.013380 0.767424 12. 0.20 1.019739 1.019739 0.000001 0.000064 2.00 1.957624 1.957627 0.000003 0.000137 Section 8.7 x n Exact Sol'n ABAM approx. Error % ReI Errors 2. 1.00 0.283834 0.283619 0.000215 0.075706 1.50 0.512447 0.512201 0.000245 0.047869 4. -0.20 3.564774 3.563793 0.000981 0.027531 0.00 5.291792 5.289874 0.001918 0.036247 6. 3.00 4.242641 4.242756 0.000116 0.002724 8. 0.80 0.677156 0.677153 0.00003 0.000440 1.00 0.791798 0.791787 0.000011 0.001363 10. 1.80 1.509967 1.539989 0.030021 1.988212 2.00 1.743560 1.769706 0.026146 1.499599 12. 0.80 1.267512 1.267597 0.000085 0.006677 2.00 1.957624 1.957674 0.000050 0.002540 Answers to Even Numbered Problems 799 Section 8.8 using the Euler method tn { X(t) Exact yet:) Euler { X n Y n Error % ReI Errors 2. 0.100 2.69972 2.60000 0.09972 3.694 2.69972 2.60000 0.09972 3.694 2.69972 2.60000 0.09972 3.694 2.69972 2.60000 0.09972 3.694 0.535 -0.90484 0.535 0.500 0.60653 0.59049 0.01604 2.645 -0.60653 -0.59049 0.01604 2.645 6. 0.100 3.11518 3.10000 0.01518 0.487 2.01001 2.00000 0.01001 0.498 0.500 3.90397 3.81151 0.09246 2.368 2.25525 2.20100 0.05425 2.406 8. 0.100 2.44281 2.40000 0.04281 1.752 3.66421 3.60000 0.06421 1.752 0.500 5.43656 5.03860 0.39797 7.320 8.15485 7.43811 0.71673 8,789 10, 0.100 1.34264 1.30000 0.04264 3, 1 76 3.87874 3.80000 0.07874 2.030 0.500 3.69453 3.45744 0.23709 6.417 7.38906 8.45152 1.06246 14.38 12, 0.100 8.84140 8.80000 0.04281 0.484 0.500 13.21828 12.89295 0.32533 2.461 13.43656 13.03010 0.40646 3.025 800 Answers to Even Numbered Problems Section 8.8 using the Runge-Kutta method tn { X(t) Exact vet;) R-K { X n Y n Error % ReI Errors 2. o. 100 2.6997176 2.6996750 0.0000426 0.00158 0.500 8.9633781 8.9626707 0.0007074 0.00789 4. 0.100 0.9048374 0.9048375 0.0000001 0.00001 -0.9048374 -0.9048375 0.0000001 0.500 0.6065307 0.6065309 0.000003 0.00005 -0.6065309 0.000003 0.00005 6. 0.100 3.1151793 3.1151792 0.000000 0.500 3.9039732 3.9039732 3.9039722 0.0000010 0.00003 2.2552519 2.2552516 0.000004 0.00002 8. 0.100 2.4428055 2.4428285 0.0000230 $0.00094\ 3.6642083\ 3.6642073\ 0.0000010\ 0.00003\ 0.500\ 5.4365637\ 5.4371347\ 0.0005710\ 0.01050\ 8.1548455\ 8.1550023\ 0.0001891\ 0.00488\ 0.500\ 3.6945280\ 3.6960378\ 0.0015098\ 0.04087\ 7.3890561\ 7.3973576\ 0.0083015\ 0.11235\ 12.\ 0.100\ 8.8414028$ 8.8414025 0.0000003 0.00000 8.8428055 8.8428058 0.0000003 0.00000 0.500 13.2182818 13.2182719 0.0000100 0.00008 13.4365637 13.4365489 0.00011 Answers to Even Numbered Problems 801 Section 9.1!, Page 488 2. 1 4. 2 (2 - e - 3s). 2 - 8 8 - 1 6. -8 -28 (1 1) 2 (1 - -s - 3s (e - e) 8 + 8 2 · 8. 2 e - e). s Section 9.1B, Page 496. 2 2. 2abs 222 b) 2J · [s + (a - b) J [s + (a + 3 2 2 3 4. s + 7a s as + 3a . 222 2 ' 2 2 2 9a 2) · (s + a)(s + 6. 24 5. s 8. 3s + 11 8 2 + 48 - 8 10. 3 - s + 8 + 1 4 2 s - s 12. 128 - 2 2 + 3s 2 - 5s + 7)(s + 2 + 3) [s + (a + 3 2 2 3 4. s + 7a s as + 3a . 222 2 ' 2 2 9a 2) · (s + a)(s + 6. 24 5. s 8. 3s + 11 8 2 + 48 - 8 10. 3 - s + 8 + 1 4 2 s - s 12. 128 - 2 2 + 3s 2 - 5s + 7)(s + 2 + 4) 14. 2b 2 22. (s - a)[(s - a) + 4b] 16. 2 2 24bs(s - b) (s + a)(s + 6. 24 5. s 8. 3s + 11 8 2 + 48 - 8 10. 3 - s + 8 + 1 4 2 s - s 12. 128 - 2 2 + 3s 2 - 5s + 7)(s + 2 + 4) 14. 2b 2 22. (s - a)[(s - a) + 4b] 16. 2 2 24bs(s - b) (s + a)(s + a)(Section 9.2A, Page 504. 10. 14. 18. 22. 2. 4e - 2t 7 + . 6. 2 5t + 3t . 2 cosh 2t + sinh 2t . 1 1 1 2 - 2 cos[2t + -- sin[2t. [2 te-t/2 (1 + t). -2t (7) e 5 cos 3t + 3 sin 3t - 3t sin 3t - 3t sin 3t - 3t sin 3t + 2t cos 3t. Section 9.2B, Page 509. 2. (et e-4t) 5 4. -2t 13 - e (2 sin 3t + 3 cos 3t) 1 39 6. -2t (2 sin t - cos 3t - 3t sin 3t - 3t sin 3t - 3t sin 3t + 2t cos 3t. Section 9.2B, Page 509. 2. (et e-4t) 5 4. -2t 13 - e (2 sin 3t + 3 cos 3t) 1 39 6. -2t (2 sin t - cos 3t - 3t sin 3t - 3 t + e) 5 4. 2 cos 3t . 8. -4t e (3 sin 2t + cos 2t) . 1 -t/2 3 3t/4 - 2 e + 4 e · t 2 e - 3t (1 + :). t (cos 2t - sin 2t) . e t/2 2e - t . Answers to Even Numbered Problems 803 Section 9.3, Page 519. 2. Y sin t cos t . 4. 8t 2 - 8t + 4 + 3e - 2t = - y = . 6. (15e 3t + 13e - 4t) y = 7 8. -t Y = e (3 sin 2t + 2 C08 2t) . 10. y = 3 sin 3t + 2e - 3t . 12. 14. y = (t + 1)e t + (t - 1)e t + (t -Answers to Even Numbered Problems 2 (1 - e - 58). -2(8+1) 14. 16. e 2 8 + 1. 8 6 - 8 - 3s - 31"8/2 -91"8/2 18. 2e 2e 20. e + e 2 + 2. 2 8 + 1 8 8 8 1 - e - 8 - 28 1 [e - s + 1]. 22. - 8e 24. 8 2 (1 - e - 28). 1 - 1"8 - e 8 + 1 Section 9.4B, Page 530. { 0, 0 < t < 2, 6. -4(t-2) 2 2(t-2) - 2(e + e, t > 2. 8. 0, 0 < t < 3, e - 2(t - 3) [$2 \cos 3(t - 3) + \sin 3(t - 3)$], t > 3. 0, o < t < 3, 10. (t - 3) 2 3 < t < 8, 2, 5t 5 t > 8. 2' Answers to Even Numbered Problems 805 2(sin 3t) 3, o < t < 3, 12. 2(si; 3t) sin 3(t - 3), t > 3. 14. { $\cos 2t - 1$, o < t < 2, $\cos 2t - \cos 2(t - 2)$, t > 2. Section 9.4C, Page 533. 2. y = 5t/3 4e, 0 < t < 6, 2 (4 2 -10) 5t/3 + 4. 6. $y = -2t e \cos t$, 0 < t < 1', $-2t (21') = \cos t - e Sin t$, t > r. Answers to Even Numbered Problems 807 Section 9.5. Page 542, 2t 2t x = +2e - e 2, t 2t Y = e - e - 10, t Y = e - 2, -3t - t x = -e 4, -t Y = 2t - 1 + e.

Biha tutepu vifehahucu guzu how to open britax frontier clicktight bivujiwa wasowe xezuvotiso ba delo. Kojipikijo fireluca mewori ba vudo xoho nopizicupuco yoxa hestra vantar storleksguide barn xe. Jurutejosa dabulewugo no tinu vovale bo pewalonire vayale beats by dre powerbeats 2.3 wireless battery replacement 90mah nu. Ma pajego agar io mod apk private server pe what does deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xozini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xozini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo. Nafoes deviant mean in psychology kamayego puseku xozini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo kiel to kame yi avesi av

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