Introduction to ordinary differential equations (4th ed.) by shepley l. ross

Differential Equations Class Notes Introduction to Ordinary Differential Equations, 4th Edition by Shepley L. Ross, John Wiley and Sons (1989). Copies of the classnotes are on the internet in PDF format as given below. These notes and supplements have not been classroom tested (and so may have some typographical errors). They are based on a sophomore differential equations class I taught at Louisiana State University in Shreveport (MATH 355) in spring 1992. A copy of the book can be browsed online here (accessed March 4, 2019). Announcement for the Class. PDF. Syllabus for the LSUS Class. PDF. Chapter 1. Differential Equations and Their Solutions. Section 1.1. Classification of Differential Equations; Their Origin and Application. PDF. Section 1.2. Solutions. PDF. Section 1.3. Initial-Value Problems, Boundary-Value Problems and Existence of Solutions. PDF. Chapter 2. First-Order Equations for Which Exact Solutions Are Obtainable. Section 2.1. Exact Differential Equations and Integrating Factors. PDF. Section 2.2. Separable Equations and Equations Reducible to This Form. PDF. Section 2.3. Linear Equations and Bernoulli Equations. PDF. Section 2.4. Special Integrating Factors and Transformations. Test 1 (1.1-1.3, 2.1-2.4). PDF. Chapter 3. Applications of First-Order Equations. Section 3.1. Orthogonal and Oblique Trajectories. Section 3.2. Problems in Mechanics. PDF. …

[illegible]

substitutions to obtain v + x - = dx 2 1 + 2v 2 v - 4v or dv 1 2 3 + 6v - 4v 2 v dv dx obtain (v - 4v) We now separate variables to x- = dx 2 v - 4v 2 dv We integrate to obtain - 2 3 - x 1 + 6v - v 1 2 v 3 1n Ix I 3 In\I + 6v = + In Co or In 11 + 6v 2 - v 3 \ = In Co Ixl. We multiply by -3 First-Order Equations 35 and simplify to obtain In 11 + 6v 2 - v 3 1 = -3 In Co I x I or Inl1 + 6v 2 - v 3 1 = co-3Ixl-3 or 1 1 + 6v 2 _v 3 1 = I 1 -3 -3 c x , where c = Co We now resubstitute v = yjx to obtain 11 + 6(yjx2) - (yjx)3 1 = clxl- 3 . We multiply by Ixl3 to obtain Ix 3 + 6xy2 - y31 = c. For suitable values of x and y, this may be expressed 323 somewhat more simply as x + 6xy - y = c, or 332 x - y + 6xy = c. Now let us solve the glven D.E. as an exact D.E. 222 Here M(x,y) = x + 2y , N(x,y) = 4xy - y. From these we find M (x,y) = 4y = N (x,y), so the D.E. lS exact. y x 2 2 We seek F(x,y) such that F (x,y) = M(x,y) = x + 2y x y2. From the first of and F (x,y) = N(x,y) = 4xy - y these, F(x,y) = M(x,Y)8x + ;(y) = (x2 + y2)8x + x 3 2 ;(y) = + 2xy + ;(y). From this, Fy(x,y) = 4xy + ;/(y). But we must have F (x,y) = N(x,y) 4xy - 2 - Y - . Y 4xy - Y 2 or ;'(y) 2 So = -y - 3 2 3 F(x,y) x 2xy Y - - + - - + c - 3 3 0 Therefore 4xy + ;'(y) = 3 ;(y) = - Y3 + cO. Thus the one-parameter family of solutions F(x,y) = c 1 lS 3 2 3 x 2xy Y - c where We multiply - + - 2' c 2 = c 1 - cO. 3 by 3 to obtain x 3 + 6xy 2 3 where 3c 2 . So y = c, c -= the one-parameter family of solutions of the glven D E . 3 3 6 2 . . lS X - Y + xy = c. 36 Chapter 2 25. Since the D.E. is homogeneous, it can be expressed in the form = g( ). Let x = r cos B, y = r sin B. Then dy = dx r cos -r Sln () d () + s ln e dr e de + cos e dr = r cos e + sin e dr/d() -r sin e + cos e dr/de Substituting into = g( ). The D.E. reduces t O r cos e + sin e d r / de - (t an successively -r sin e + cos e dr/de - g e), dr . dr r co s () + sin () d () = g (t an ()) [- r s 1 n e + co s e d e J , dr [sin () - cos () g(tan ())J d() = -[g(tan e) sin e + cos OJr, dr Sln e g(tan e) + cos e = e de, which lS separable ln r r co s e g (tan e) - s 1 n and e. 26. (a) The D.E. of Exercise 8 is (x + y) dx - xdy = O. This can be written = 1 + y/x, and so 1S homogeneous. Using the method of Exercise 25, we let x = r cos e , . e Th dy r cos e de + sin e dr We y = r s 1 n. en dx = _r sin e de + co s e dr. substitute this into = 1 + y/x. The D.E. reduces successively to r cos e de + sin e dr -r sin e de + cos e dr = 1 + tan e, r cos e de + sin edr = (1 + tan e) (-r sin e de + cos edr), [sin e - (1 + tan e) cos eJ dr = -r[cos e + (1 + tan e) sin e] de, First-Order Equations 37 dr cos () + 1 + tan s ln () [cos () + sin () + tan () sin ()] d() = ()) cos () - r 1 + tan s ln () [cos () + sin () + tan () sin ()] d() = cos () (1 + tan () 2 = + tan ()) d () , 2 = (s ec () + tan ()) d () , or finally dr 2 = (s e c () + tan ()) d () . r This lS separable. Integrating, assumlng r > 0, cos () > 0, we o bt a in 1 n r = tan () - 1 n co s () + 1 n c , where c > o. Now resubstitute, ac cording to x = r cos (), y = rsin(); that is, let r = j x 2 + y2,tan() = y/x. We obtain successively ln j x 2 2 y/x - In(x/ j x 2 + y2) In c , + y = ln j x 2 2 + In x/ j x 2 + y2 + In c = c/x, + y In(cx) - y/x, or finally y = x In(cx). - Section 2.3, Page 56 The equations of Exercises 1 through 14 are linear. In solving, first express the equation in the standard form of equation (2.26) and then follow the procedure of Example 2.14. 38 Chapter 2 1. The D.E. is already ln the standard form (2.26), with P(x) = 3/x, Q(x) = 6x 2 . An I.F. is eJP(x)dx = e J (3/x)dx = 3 e3lnlxl = elnlxl = Ixl 3 = *x 3 (+ if x 0, - if x < 0). Ve multiply the D.E. through by this I.F. to obtain x3 3 2 6 5 d (3) 5 I . b . 3 + x = x or dx x y = 6x. ntegratlng, we 0 taln x y 6 3-3 = x + e or y = x + ex 3. The D.E. is already in the standard form (2.26), with P(x) = 3, Q(x) = 3x 2 e -3x. An I.F. is e J P(x) dx = eJ 3 dx = e 3x Ye multiply the D.E. through by the I.F. to obtain 3x dy 3 3x 3x 2 e dx + e y = d (3X) 2 Integrating we obtain 3x 3 or - e y = 3xe . e y = x + e dx y = (x 3 +) -3x or e e . 6. 2 This equation is linear in v. Ye divide through by u + 1 dv 4u 3u to put it in the standard form du + 2 v = 2 u + 1 u + 1 with P(u) = 4u and Q(u) = u 2 + 1 3u u 2 + 1 An I.F. lS eJP(u)du = f 4u exp 2 u + du 1 = e 2ln(u 2 +1) = 2 2 (u + 1) . Ye multiply the standard form equation through by this to . (2 2 dv 2 2 obtaln u + 1) du + 4u(u +l)v = 3u(u + 1) or d 2 2 3 du [(u + 1) v] = 3u + 3u. Integrating, we obtain 2 2 3u 4 3u 2 (u + 1) v = + 2 + e. First-Order Equations 39 7. dy 2x + 1 Ye first divide through by x to obtain dx + 2 y = x + x x - 1 x h P( ) 2x 2 + 1 and Q(x) - _ x X - 1 were x = x + x An I.F. lS eJp(x)dx = exp[: : : dX) = elnlx2+xl = Ix 2 + xl = * (x 2 + x) (+ if x < -lor x > 0, - if -1 < x < 0). In any case, upon multiplying the standard form equation through by the I. F., we obtain (x 2 + x) + (2x + 1) y = (x + 1)(x - 1) 2 dy (2x + 1) 2 - 1 that or (x + x) dx + y = x , lS, d 2 2 Integrating, we find 2 2 2 x)y dx [(x + x) Y] = x - 1. (x + = 323 x /3 - x + c/3 or 3(x + x)y = 3x - x + c. 8. 2 Ye first divide through by x + x - 2 to put the equation th t d d f dy 3(x + 1) 1 U here ln e s an ar orm dx + (x + 2) (x - 1) Y = x + 2' " 3 (x + 1) d 1 P(x) = (x + 2)(x _ 1) an Q(x) = x + 2 . An I.F. lS J P (x) dx _r [3 (x + 1)] d = e - exp J (x + 2) (x _ 1) x elnlx+21+21nlx-11 = Ix + 21(x - 1)2 = *(x + 2)(x - 1)2, (+ if x -2, - if x -2), where partial fractions have been used to perform the integration. In either case (x > -2 or x -2), upon multiplying the standard form equation through by the 1. F ., we obtain (x + 2)(x - 1) 2 + 3(x + l)(x - l)y = (x - 1)2, that is, d: [(x + 2)(x -1)2yJ = (x - 1)2. Integrating, we find (x + 2)(x - 1)2y = 3 {x -3 1) + 0 r 3 (x + 2) Y = x - 1 + c (x - 1) - 2 . 40 Chapter 2 9. Ye first put the equation in the form dy xy + y - 1 = dx + x 0, and then ln the standard form + (1 + ;)y = ; , where P(x) = 1 + l/x and Q(x) = l/x. An I.F. 18 eJP(x)dx = e(1+1/x) dx = eX+lnlxl = Ixle x = * xe x , (+ if x 0, - if x < 0). In either case, we multiply the standard form equation through by the I.F. and obtain x dy x xe dy + (x + 1) e y = e d x x x e or dx [x e yJ = e. Integrating, we find x e y = e + c -1 -x or y = x (1 + c e). 10. This D.E. is linear ln x (like Example 2.16). We divide through by y and dy to put it in the standard form 2 dx xy dy + + x y - y = 0, dx or - + dy I. F. is (y +) x = 1, where P(y) 1 - y + y' Q(y) = 1. An 2 2 eJP(y)dy = e J (y+1/y)dy = eY 2/2+1nlyl = lyle Y /2 = 2/2 :I: ye y (+ if y 0; - if y < 0). Mul t iplying the standard form equation through by this, we obtain First-Order Equations 41 y2/2 dx 2 y2/2 ye dy + (y + l)e x 2 / 2 = ye Y d 2/2 or dy (ye Y x) = 2 / 2 e Y + e or xy = 2 / 2 ye Y 2/2 Integrating, we find ye Y x- 2 / 2 1 + ee- y . 12. This D.E. lS linear in r. Ye first divide through by dO b 0 0 dr 0 fj 4 0 0 0 dr to 0 taln eos dO + r Sln u - eos = or eos dO + r sin 0 = eos 4 0 . Next we di vide through by eos 0 to put dr 3 the equation ln the standard form dO + (tan 0) r = eos 0, where P(O) = tan 0 and Q(O) = eos 3 0. An I.F. is e J P (0) dO = e J t an 0 d 0 = e In I see 0 I = I see 0 I = * see 0 (+ O f 4k - 1 0 4k + 1 f . k d 1 . f 1 2 1r < < 2 1r 0 r some 1 n t eger , an - 4k + 1 4k + 3 0 2 1r < () < 2 1r for some lnteger k). In any case, upon multiplying the standard form equation through by the I.F., we obtain seeO + (seeOtanO)r = eos 2 0, that is, d 2 dO [1" see OJ = eos O. Upon integrat ing, we find r see 0 = 0 + sin 20 + cO' where the right-side was integrated as follows: f eos 2 0 d 0 = f 1 + e 2 0s 2 0 d 0 = 0 + sin 20. Substituting 2 sin () eos 0 for sin 2 (), the solutions may be expressed as r see 0 = 0 + sin 0 eos 0 + cO' Finally, we mul t iply through by 2 eos () to obtain 2r = (0 + sin Oeos ()+ e)eos 0, where e = 2e O . 42 Chapter 2 13. Ye first put the D.E. in the form (1 + sin x) + (cos x)y = 2 cos x, and then ln the standard form dy cos x dx + 1 + sin x 2 cos x Y = 1 + sin x ' where P(x) cox x = 1 + sin x ' Q(x) = 1 2 cos x + Sln x An I.F. lS f cos x dx /P(x)dx 1 + sin x Inl1+sinx I 11 + sin x I = 1 e = e = e + sin x , since 1 + Sln x > o. Multiply the standard form equation through by this, to obtain (1 + s in x) + (cos x) y = 2 cos x, which is in fact the form preceding the standard form. Ye observe that this is d: [(1 + sin x)y] = cos 2 x. x sin 2x Integrating we obtain (1 + sin x)y = 2 + 4 + c 1 or 2(1 + sinx)y = x + sinxcosx + c, where c = 2c1 and we have employed double-angle formulas. 14. This D.E. lS linear ln y. We first divide through by dy to obtain y sin 2x - cos x + (1 + sin 2 x) = 0 or (1 . 2) dy (. 2) N d . . d + Sln x dx + Sln y y = cos x . ext we lVl e through by 1 + sin 2 x to put the equation in the standard First-Order Equations 43 form + 1 sin 2x o 2 + Sln x y = cos x where P (x) = o 2 ' 1 + Sln x sin 2x 1 . 2 + Sln x and Q(x) = COS x 1 02 + Sln x An I. F. i S eJP(x)dx = exp [f sin 2.X 2 dX] = exp 1 + Sln x [f 2 s in x .co 2 s x dx] 1 + Sln x e ln (1+sin 2 x) _ _ 1 2 = + sin x. Upon multiplying the standard form equation through by the I.F., we obtain (1 + sin 2 x) + (sin 2x)y = cos x, that lS, d: [(1 + sin 2 x)y] = cos x. Integrating, we find (1 + sin 2 x)y = sin x + c. 15. This lS a Bernoulli D.E., where n = 2. We multiply -2 -2 dv 1 -1 1 through by y to obtain y - x y - - x Let l-n -1 dv -2 v = y = y ; then dx = -y dx. The preceeding D. E. d O l f 0 h 1 0 . dv 1 1 rea 1 y trans orms lnto t e lnear equatlon - d + - v = x x x An I.F. lS e Jdx / x = elnlxl = Ixl = * x. Multiplying through by this, we find x : + v = 1 or d: (xv) = 1. -1 Integrating, we find xv = x + c, from which v = 1 + cx But v = l/y. Thus we obtain the solution in the form l/y = 1 + cx- 1 . 18. This is a Bernoulli D.E. in the dependent variable x and independent variable t, where n = -1. We multiply tprough o dx t + 1 2 t + 1 1- n by x to obtaln x dt + 2t x = t Let v = x 2 dv dx x ; then dt = 2x dt . The preceeding D. E. readily = 44 Chapter 2 t f . h 1 . . dv rans orms lnto t e lnear equatlon dt + t + 1 t v = t + 1 dt 2(t; 1). An I.F. is ef 1 = et+lnltl = It let = * t e t. Multiplying through by this, we find t e t + t t d t t (t + 1) e v = 2 (t + 1) e or dt [t e v J = 2 (t + 1) e . I . b . t 2 t 2 ntegrat lng, we 0 taln t e v = t e + c or v = + c t -1 e -t. But v = x 2 . Thus we obtain the solution ln 2 -1 -t the form x = 2 + c t e . The equations of Exercises 19 through 24 are linear. 21. x Ye divide thru by (e + 1) and dx to put ln the form x x -1 x dy e [y(e + 1) - 3(e + l)J + dx = 0 and thence in the standard form dy - + dx x e = 3e x (e x + 1), y eX + 1 where P(x) = x e , Q(x) - 3e x (e x + 1). An I.F. lS JP(x)dx e x e + 1 x = exp f x e e + x dx = e 1 n (e + 1) = eX + 1. 1 Ye multiply obtaining d x dx [(e + (ex + l)y the standard form equation through by this, x dy x x x 2 (e + 1) dx + e y = 3e (e + 1) or l)yJ = 3e x (e x + 1)2. Integrating, we obtain x 3 = (e + 1) + c or x 2 x -1 Y = (e + 1) + c(e + 1). (*) First-Order Equations 45 Ye apply the I.C. y(O) = 4: Let x = O,y = 4 in (*) to obtain 4 = 4 + c/2. Thus c = 0, and the particular x 2 solution of the stated I.V.P. lS Y = (e + 1) . 24. This equation is linear in the dependent variable x with t as the independent variable, and is already in the standard form, with pet) = -1, Q (t) = sin 2t. An I.F. lS eJP(t)dt = e J (-l)dt = -t e Ye multiply the equation -t dx -t -t. through by this to obtain e dt -e x = e Sln 2t or d -t -t dt (e x) = e sin2t. Ye next integrate, using integration by parts twice, or an integral table, on the right. We obtain -t -t -Sln 2t - 2 cos 2t e x = e + c 5 or - (sin 2t + 2 cos 2t) t (*) x = e + ce 5 . Ye apply the I.C. x(O) = 0: Let t = x = 0 in (*). Ye obtain 0 = - 1/5(0 + 2) + c from which c = 2/5. Thus the general solution of the stated I.V.P. is x = 2 t . 2 2 2 e - Sln t - cos t 5 25. This is a Bernoulli D.E. with n - 3. Ye first multiply 3 3 d y 4 1-n throu g h b y Y to obtain Y + y - x Now let v = Y dx 2x - . = y 4 . Then : = 4 y3 and the preceeding D. E. 46 Chapter 2 transforms into : + 2 v x = x, which is linear In v. In dv 2 the standard form this linear equation is dx + x v = 4x, with P(x) = 2/x, Q(x) = 4x. An I.F. is eJP(x)dx = e J (2/x)dx = e 21n / x / = 2 x. Ye multiply the standard form through by this to obtain 2 dv 3 d 2 3 . x dx + 2 x v = 4x or dx (x) = 4x. Ye lntegrate to obtain x 2 v = x 4 + c. But v = y4. Hence we obtain the one-parameter family of solutions of the given Bernoulli Equation in the form 244 x y = x + c. (*) We apply the I.C. y(l) - 2. Let x = 1, Y = 2 ln (*) - to obtain 16 = 1 + c, so c = 15. Thus the particular solution of the stated I.V.P. 2 4 4 + 15. lS x y = x 26. Ye first wri te the D. E. In the form x + y = x 3 / 2 y3/2 and multiply through by l/x to obtain dy + !. y = x 1 / 2 y3/2. dx Ye recognlze this D.E. as a Bernoulli D.E. with n = 3/2. Ve now multiply through by y -3/2 to obtain y -3/2 + 1 -1 / 2 1/2 1-n -1 / 2 x y = x · Now let v = y = y . Then : = : ; Y -3/2 and the preceeding D.E. transforms into -2 dv dx 1 + -v x 1/2 h . h . 1 . . = x , W lC IS lnear In v. In the standard First-Order Equations 47 dv (- 2x 1) V = x1/2 form this linear D.E. lS dx + 2 ' with P(x) = x 1 / 2 An I F . JP(x)dx _J(- 1/2x)dx = 2 . . . lS e - e 1 1 I 1 -1/2 -1 / 21n x In x 1 1 -1 / 2 -1 / 2 e = e = x = x (assuming x > 0). We multiply the standard form through by this I.F. to b . -1 /2 d v 1 _1 d (-1 /2) 1 T.T o ta1n x dx - 2x 3 / 2 v - - 2 or dx x v = - 2 ' "e -1/2 1 -1/2 integrate to obtain x v = - 2 x + c. But v = Y . 1 - 2x ' Q(x) = Hence we obtain the one-parameter family of solutions of -1/2 -1/2 the glven Bernoulli equation in the form x y = 1 2 x + c or 1 fXY 1 = - 2x + c (*) We apply the I.C. y(l) = 4. b . 1 1 1 to 0 taln 2 = - 2 + c, so c = · Let x = 1, Y = 4 in (*) Thus the particular solution of the stated I.V.P. lS 1 fXY 1 = - 2x + 1. 27. Here we actually have two I.V. problems: (I) For 0 < x < 1, (II) For x > 1 , dy y = 2, dy y = 0, - + - + dx dx y(O) = 0, Y (1) = a, where a is the value of lim (x) and denotes the x-+1- solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 1. 48 Chapter 2 We first solve (I). The D.E. of this problem 1S linear in standard form with P(x) = 1, Q(x) = 2. An I.F. is eJP(x)dx = e J (l)dx = eX. We multiply the D.E. of (I) h h b h . b . x dy x x d [X J t roug y t lS to 0 taln e dx + e y = 2e or dx e y = 2e x . We integrate to obtain eXy = 2e x + c or y = 2 + -x ce We apply the I.C. of (I), y(O) = 0, to this, obtaining 0 = 2 + c or c = -2. Thus the particular -x solution of problem (I) is y = 2 - 2e . This is valid for 0 < x < 1. Letting denote the solution just obtained, we note that lim (x) = lim (2 - 2e- x) = 2 - 2e- 1 . This is the x-+1 x-+1 x of Problem (II); that lS, the I.C. of (II) is y(l) = 2 - 2e- 1 . Now solve (II). The D.E. of this problem is linear lS standard form with P(x) = 1, Q(x) = o. An I.F., as ln x problem (I), is e. We multiply through by this to obtain x dy x d x e dx + e y = 0 or dx (e y) = o. Ye integrate to obtain eXy = c or y = c e -x Now apply the I. C. of (II), y (1) = 2 - 2e- 1 , to this. Let x = 1, Y = 2 - 2e- 1. We obtain 2 - 2e- 1 = c e- 1 , from which c = 2e - 2. Thus the particular solution of problem (II) is y = (2e - 2)e- x . This is valid for x > 1. We write the solution of the entire problem, showing intervals where each part is valid, as 2(1 - e- x), 0 < x < 1, y = -x 2(e - l)e , x 1 First-Order Equations 49 29. As ln Exercise 27, we actually have two I.V. problems: (I) For 0 < x < 2, (II) For x 2, dy -x dy -2 - + y = e , - + y = e dx dx y(O) = 1, y(2) = a, where a is the value of lim (x) and denotes the x2 solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 2. We first solve (I). The D.E. of this problem is linear in standard form with P(x) = 1, Q(x) = e- x An . fP(x)dx f(l)dx x . I.F. lS e = e = e. We multiply the D.E. of (I) through by this to obtain eX + eXy = 1 or d: (eXy) x -x = 1. Integrating we obtain e y = x + c or y = e (x + c). We apply the I.C. of (I), y(O) = 1 to this, obtaining 1 = o + c, or c = 1. Thus the particular solution of problem (I) is y = e-x(x + 1). This is valid for 0 < x < 2. lim x2 Letting denote the solution just obtained, we find -x -2 ;(x) = lim e (x + 1) = 3e . This is the a of x-+2 Problem (II); that lS, -2 the I.C. of (II) is y(2) = 3e . Now solve (II). The D.E. of this problem is linear in -2 standard form with P(x) = 1, Q(x) = e . An I.F., as in Problem (I), is eX. We multiply through by this to obtain x dy x x-2 d x x-2 e dx + e y = e or dx [e yJ = e · We integrate to 50 Chapter 2 obtain x x-2 -x x-2 c) -x e y = e + c or y = e (e + or y = ce + -2 Now apply the I.C. of (II) , y(2) 3e -2 to this. e = , Let x 2, -2 We obtain 3e -2 -2 from = y = 3e . = ce + e , which c = 2. Thus the particular solution of problem (II) -x -2 is y = 2e + e This is valid for x > 2. We write the solution of the entire problem, showing intervals where each part lS valid, as -x e (x + 1), o < x < 2 , y = -x -2 2e + e , x > 2 30. As in Exercises 27 and 29, we actually have two I.V. problems: (I) For o < x < 3, (II) For x 3, (x + 1) dy y (x + 1) dy y 3, - + = x, - + = dx dx y(O) = 1/2. y(3) = a, where a is the value of lim (x) and denotes the x-+3 solution of (I). This is prescribed so that the solution of the entire problem will be continuous at x = 3. We first solve (I). Ye first express the D.E. ln the standard form d d x Y + 1 Y = x l ' where P(x) x + 1 x + = l/(x + 1), Q(x) = x/ex + 1). An I.F. is eJP(x)dx = e Jdx/x+1 - - e ln l x+1 1 _ - Ix + 11 . Since x + 1 1 > 0 ln (I), we have Ix + 11 = x + 1. Ye multiply the standard First-Order Equations 51 form equation through by this to obtain (x + 1) + Y = x, which turns out to be exactly the equation that we started with. Ve note that this is d [(x + l)yJ = x. 2 Integrating we obtain (x + l)y = x /2 + c. Applying the I.C. y(O) = 1/2, we obtain c = 1/2. Thus the particular 1 2 solution of Problem (I) is (x + l)y = 2 (x + 1) or y - 2 x + 2(x + 1 1) . This is valid for 0 < x < 3. Letting denote the solution just obtained, we find 2)] 5 lim (x) = 1. [x + This the a of Problem 1m _ 2(x + = 4 . lS x3 x3 (II) ; that is, the I.C. of (II) lS y(3) = 5/4. Now solve (II). We put the D.E. ln standard form dy + 1 3 and find, as ln Problem (I), that an dx x + 1 y = x + 1 I.F. lS (x + 1). We multiply the standard form equation through by this to obtain (x + 1) + y = 3, which 18 exactly what we started with here ln Problem (II). We d note that this is dx [(x + l)yJ = 3. Integrating we obtain (x + l)y = 3x + c. we obtain 5 = 9 + c or c = Applying the I.C. y(3) = 5/4, -4. Thus the particular solution of Problem (II) is (x + l)y = 3x - 4 or 3x - 4 Y This is valid for x > 3 = x + 1 . . The solution of the entire problem lS 52 Chapter 2 x 2 + 1 2(x + 1)' o < x < 3, y = 3x - 4 x + 1 ' x > 3 31. (a) The D.E. is linear. In the standard form it is dy - + dx b k -x with P(x) b Q(x) k -x An I.F. -y = - e = a ' = - e a a a a . fP(x)dx f(b/a)dx (b/a)x Multiplying the lS e = e = e . standard form equation through by this, we obtain e (b/a)x dy + b e (b/a)x y = k e (b/a-)x dx a a d [e (b/a)x yJ or dx = k e(b/a-)x. a (1) Ve now consider two cases: (i) f ; and (ii) = In case (i), where f b , integrating (1) we obtain a e (b/a)x y = : (_ r 1 e (b/a-)x + c or y = -x ke b - a -bx/a + ce . (2) b d (b/a)x In case (ii), where = a '(1) becomes dx [e yJ - k Integrating, we obtain e(b/a)x y = kx + c or a a k -bx/a x e y = -bx/a + ce . (3) a First-Order Equations 53 (b) Suppose = o. Then since b > 0 we can have onl y a ' case (i), * b , here. Then with = 0, (2) becomes y a = + ce -bx/a. As x !D, e -bx/a 0 and y . Suppose > o. Then either case (i) or case (ii) can occur. In case (i), y is given by (2); and since e-x 0 and e- bx / a 0 as x CD, we have y 0 as x CD. In case (ii), y is given by (3); and since -bx/a 0 -bx/a xe and e 0 as x CD , we again have y 0 as x m . 33. (a) The function f such that f(x) = 0 for all x E I has derivative f'ex) = 0 for all x E I. We substitute this f(x) for you and f'(x) for in the D.E. Ve obtain f/(x) + P(x)f(x) = 0 + P(x)O = 0 for all x E I, so f(x) is a solution. (b) By the Theorem 1.1, there is a unlque solution f of the D.E. satisfying the I.C. f(x O) = 0, where Xo E I. By (a), ;(x) = 0 for all x E I is a solution of the D.E. and obviously it satisfies the I.C. Thus by the uniqueness of Theorem 1.1, we must have f(x) = ;(x), that is, f(x) = 0 for all x E I. (c) Since f and g are solutions of the D.E., so is their difference h = f - g (by Exercise 32(a)). Also h(x O) = f(x O) - g(x O) = o. So h is a solution such that h(x O) = 0 for some Xo E I. By part (b), hex) = 0 for all x E I. But this means f(x) - g(x) = 0 and hence f(x) = g(x) for all x E I. 54 Chapter 2 34. (a) Since f lS a solution of (A), f'(x) + P(x)f(x) = Q(x). (1) Since g lS a solution of (A), g'(x) + P(x)g(x) = Q(x). (2) Then, subtracting (2) from (1), we have f/(X) + P(x)f(x) - g'(x) - P(x)g(x) = Q(x) - Q(x) or f'(x) - g'(x) + P(x) [f(x) - g(x)J = 0 or, finally, [f(x) - g(X)J' + P(x) [f(x) - g(x)J = o. This shows that f - g is a solution of the D.E. dy dx + P(x)y = O. (b) Since f lS a solution of Equation (A), we have f'(x) + P(x)f(x) = Q(x). (3) By part (a), since f and g are solutions of (A), their difference f - g is a solution of dy/dx + P(x)y = O. Thus we have [f(x) - g(x)J' + P(x) [f(x) - g(x)J = o. First-Order Equations 55 Multiplying this by c, where C lS an arbitrary constant, we have c[f(x) - g(x)J' + cP(x) [f(x) - g(x)J = o. (4) Adding (3) and (4), we have {f'(x) + P(x)f(x)} + {c[f(x) - g(x)J' + cP(x) [f(x) - g(x)J} = Q(x). Rearranging terms, this takes the form {c[f(x) - g(x)J' + f'(x)} + P(x){c[f(x) - g(x)J + f(x)} = Q (x) . or {c[f(x) - g(x)J + f(x)}' + P(x){c[f(x) - g(x)J + f(x)} = P(x). We see from this that c(f - g) + f satisfies equation (A). That is, since c is an arbitrary constant, c(f - g) + f is a one-parameter family of solutions of (A) . 37. (a) This is of the form (2.41) with fey) = Sln y, P(x) = 1 - , an d Q (x) = 1 . Ye 1 et v = f (y) = sin y , from w hi ch x dv _ dy dx - (cos Y) dx . Substituting this into the stated D E o b dv 1 1 h 0 h 0 1 0 0 · ., lt ecomes - d + -v = , W lC lS lnear ln v. x x An I.F. is eJP(x)dx = e J (l/x)dx = elnlxl = Ixl. 56 Chapter 2 Multiplying the linear equation through by this, we have x : + v = x or d: (xv) = x. Integrating, we 2 2 find xv = x 2 + cO' or 2xv = x + c, where c = 2c O . Now replacing v by Sln y, we obtain the solution in 2 the form 2x sin y - x = c. 40. This 18 a Riccati Equation of the form (A) of Exercise 38, with A(x) = -1, B(x) = x, C(x) = 1. The solution f(x) = x is given. By Exercise 38(b), we make the transformation y = f(x) 1 1 from which dy = 1 () dV + - = x + - - v v' dx 2 dx. v Substituting ln the given D.E., it takes the form 1 dv 1 - 2" dx = v (x +)2 + x(x +) + 1. 1 dv This reduces to - -- -- = 2 dx v 1 - -- - 2 v x dv or -- - xv = 1, which v dx is linear in v, with P(x) = -x, Q(x) = 1. An I.F. is fP(x)dx _ f(-x)dx _x2/2 e - e - e . Multiplying the linear D.E. through by this, we obtain _x2/2 dv -x2/2 e dx - e x v = 2 -x /2 e or 2 d: (e- X /2 v) = 2 -x /2 e . 2 2 Integrating, we obtain e -x /2v = f e -x /2 dx + c. Since the integral on the right cannot be expressed as a finite sum of known elementary functions, we leave it as First-Order Equations 57 indicated.. 1 But from the transformation y = x + v ' we see that v = 1 (y - x). We replace v accordingly and thus obtain the solution, 2 -x /2 e y - x 2 = f e -x /2 dx + c. Section 2.3, Miscellaneous Review Exercises, Page 59 The solution of each of these review problems 1S given, but most of these solutions are presented 1n abbreviated form with many details omitted. 1. The D.E. is both separable and linear. Upon separating variables and integrating, we obtain 2 In (x 3 + 1) + In I c I = Inlyl, from which we find y = c(x 3 + 1)2. The solution as a linear equation lS almost as easy. 3 -2 The I.F. is (x + 1) . 2. The D.E. lS exact. We seek F(x,y) such that F (x,y) = x M(x,y) = 2xy3 - y and F (x,y) = N(x,y) = 3x 2 y2 - x. y 2 3 the first of these, we find F(x,y) = x y - From this F (x,y) = 3x 2 y2 - x + '(y). But y = N(x,y) = 3x 2 y2 2 3 F(x,y) = x Y 2 3 x y - xy = c. From xy + (y). since F (x,y) y find '(y) = 0, (y) = cO. Thus x, we xy + cO' and the family of solutions lS 58 Chapter 2 3. The D.E. is both separable and linear. First consider it as a separable equation. Upon separating variables and using partial fractions (preparatory to integrating), we + dy - 0 Integrat ion gives In I x 1 - y - 1 - 1)1 = 11 = In c 1 . Upon simplifying and find Ix(y - 1)1 = IC 1 (x + 1)1. find [- 1 1]dX x + lnlx + 11 + lnly - taking antilogs, we Assuming x > 0, y > 1 and simplifying, we can write this as xy = c x + (c - 1), or xy + 1 = c (x + 1), where c = 1 + c 1 . Alternately, consider the D.E. as a linear equation. The I.F. [x (x d: 1)] = % x Multiplying the lS exp 1 . x + I' standard form" of the D.E. through by this we have [xy]' = (x + 1)-2. Integrating, we find xy = x + 1 x + 1 -1 this we find xy + 1 = c(x + 1) , -(x + 1) + c. From as before. 4. The D.E. is linear. The I.F. is x. Multiplying the "standard form" of the equation through by this, we find 2 3 2 3 x y' + 2xy = x and hence (x y)' = x. Integration and 2 simplification at once give y = x 4 + . x 5. Writin g the D.E. ln the form dy dx 5 [1 - 3 = , 1 + y x we recognlze that it lS homogeneous. We let v = vx. Then the D.E. dv 5v- 3 becomes v + x - d = 1 . Simplification yields x + v First-Order Equations 59 (v + 1)dv v 2 - 4v + 3 to the left member, we find -dx x Then upon applying partial fractions = [- v 1 + v - 2 3]dV = - : Integration glves -lnlv - 11 + 2ln Iv - 31 = -lnlxl + lnlc/. Simplification and taking of antilogarthims 2 i v - 3) _ c y results ln l v _ 11 - x But v - x Upon resubstituting accordingly and simplifying, we find (3x - y)2 = Ic(y - x)l. 6. The D.E. lS exact, separable, and linear. Considered as an exact equation, we seek F(x,y) such that F (x,y) = x M(x,y) 2x 2 and F (x,y) = N(x,y) 2x Y - 2y. From the = e y = e y 2 first of these, we find F(x,y) 2x y (y) . From = e - + 2 this, F (x,y) = e 2x y2 + '(y). But since F (x,y) = N(x,y) y y = e 2x y _ 2y, we find '(y) = -2y, (y) = _y2 + cO' Thus 2x 2 2 F(x,y) e y and the family of solutions = y + cO' lS 2 2x 2 2y2 [where we have set F(x,y) e y = c = c , , 1 multiplied through by 2, and let c = 2(c 1 - cO)]. Assuming e 2x - 2 0, this can also be written as y = c 2 (e 2x - 2)-1/2 where c 2 = JC. We can also consider the D.E. as a linear equation. We .. (2x 2) ' 2x 0 d th t . rewrlte It as e - y + e y = an en pu It ln 2x 2x the "standard form" y' + e yj(e - 2) = 0. From this we 60 Chapter 2 find the I.F. is (e 2x - 2)1/2, where we assume e 2x - 2 > O. Multiplying the standard form through by the I.F., we f o d (2x 2) 1/2 I 2x (2x 2) -1/2 ln e - y + eye - = 0 or [(e 2x _ 2)1/2yJI = O. Integration and simplification readily yield y = c(e 2x _ 2)-1/2. 7. The D.E. is both separable and linear. We first solve this as a separable equation. Separating variables, we h 4x3dx dy 4 ave 4 + 3 2y = O. Integration yields In(x 4 + 1) + x + 1 y - 1 2 1n 12y - 31 = Inl c 1 1. Simplify, assumlng 2y - 3 0, to obtain In(x 4 + 1)(2y - 3)1/2 = Inl c 1 1. From this we obtain (x 4 + 1)(2y - 3) = c 1 . Then y = 3/2 + 2 4 c/2 c 1 c(x + 1) , where c = 2 · Consider the D.E. as a linear equation. In standard 3 3 8x y _ 12x form the D.E. is y' + 4 - 4 . The I.F. is x + 1 x + 1 4 2 (x + 1) . Multiplying the standard form through by this we find [(x 4 + 1)2y]' = (x 4 + 1)(12x 3). I t t o d d o . ° b (4 1) 2 h ° 3 negra lon an lV1Slon y x + t en glve y = 2 + 4 -2 c (x + 1) . 8. Vriting the D.E. in the form = I 2 + + [P L we 2 ' see that the D.E. is homogeneous. Let y = vx. Then = dv dv v v 2 v + x dx ' and the D.E. becomes v + x dx = -1 - 2 - 2. First-Order Equations 61 2 S . l . f . t . . ld dv - (v + 3v + 2) and hence lmp 1 lca lon Yle s x dx - - 2 ' 2 dv dx = - -- Use of partial fractions glves v 2 + 3v + x 2 [1 v + 1 v 2]dV = dx x Integration and immediate simplification 2 In v + 1 In c Further glve = - v + 2 x 1 2 siJ]plification v + c But = y glves = - v . v + 2 x x Resubstituting accordingly and again simplifying, we 2 obtain [: ++ 2:] = 9. 322 Rewriting the D.E. 1n the form (4x y - 3x y)dx + (2x 4 y - x 3)dy = 0, we find that it is exact. Ve 322 F(x,y) such that F (x,y) = M(x,y) = 4x y - 3x y x F (x,y) = N(x,y) = 2x 4 y - x 3 y F(x,y) = x 4 y2 _ x 3 y + (y). x 3 + ;'(y). But since F (x,y) = N(x,y) = y seek and From the first of these, 4 From this, F (x,y) = 2x Y - Y 4 3 2x y - x , we must have '(y) = 0, (y) = cO. Thus F(x,y) + cO' and the solution is x 4 y2 - x 3 y = c. 423 = x Y - x Y 10. The D.E. is linear. The standard form is y' + xy x + 1 = e -x(x + 1), and the I. F. is % eX (x + 1). Mul tiplying the standard form through by this, we obtain the equivalent of x ' [x e + Y l] = (x + 1)-2. Integration and multiplication by (x + l)e- x result in y = e- x [-l + c(x + l)J. 62 Chapter 2 , . 11 . Vritin g the D.E. ln the form dy = dx 2 - 7 Y x 3 Y x ... we recognlze , - 8 Then dy = v + dx that it is homogeneous. Ve let y = vx. dv x dx · dv The D.E. becomes v + x dx = dx 2 - 7v 3v - 8. Simplification gives (3v - 8)dv = 3v 2 - v - 2 dx x Partial fractions decomposition puts this in the form [3V: 2 - v = 1] dv = - . Integration gives 2ln (3v + 2) - In Iv - 11 = - In Ix I + Inlcl. Simplification and taking of antilogarithms . c y results 1n nnrv = X But v = x Resubstituting 2 accordingly and simplifying gIves (3y +22X) . = x c x 2 and hence (2x + 3y) = Ic(y - x)l. 12. The D.E. is a Bernoulli Equation with n = 3. Ve multiply -3 2 -3 dy -2 through by y to obtain x y dx + xy = x. Let v = l-n -2 . dv _-3 Y = Y , then dx - -2y dx. The preceeding D. E. readily transforms into the linear equation dv - 2 v dx x 2 An I.F. - J 2 / x dx lS e = 1 2. x Multiplying through by x this, we obtain d: [:2] = - :3 ' Integrating, we find X = 1 -2 2 + c. Replacing v by y and simplifying, we find y x = 1 2. 1 + cx First-Order Equations 63 13. This is a linear equation. The standard form is 2 6x2 y' + x y = 3 An I.F. is (x 3 + 1)2. Multiplying x + 1 x + 1 through by this we obtain [(x 3 + l)yJ' = (x 3 + 1)(6x 2). Integration and division by (x 3 + 1)2 then give the 3 -2 solutions y = 1 + c(x + 1) . 14. dy Writing the D.E. ln the form dx = ,. .. 2 2 + y x 2 Y - 1 x we recognize that it is homogeneous. Ve let y = vx. Then = v + dv x dx · The D.E. becomes v + x : 2 + v 2 = 2v - 1. Simplification gives (2v - l)dv = v 2 - v - 2 dx x Integration gives Inlv 2 - v - 21 c = In -. Taking antilogarithms and simplifying slightly, x we obtain 2 - 2 = c/x. But v = y/x. Resubstituting v - v accordingly and simplifying, we find 2 2 Y - xy - 2x = cx or (y - 2x)(y + x) = cx. 15. The D.E. lS both homogeneous and Bernoulli. Ve first consider it as a homogeneous equation, which form we ,. .. 2 1 + I. x recognize by writing it as = 2 Y x Ve let y = vx. Then = v + x : . dv The D.E. becomes v + x dx = dx = -x' 2 1 + v 2v 2vdv Simplification quickly leads to 2 v - 1 integration glves Inlv 2 - 11 = In c . x and then From this, we 2 readily find v c 1 = -. x simplifying, we find y2 Resubstituting v 2 - x = cx. = I. and x 64 Chapter 2 Now apply the initial condition y(l) = 2. Letting x = 1, y = 2, we get 3 = c, and he nce obta in the solution in the form y2 = x 2 + 3x or y = j x 2 + 3x. Alternatively, we recognize that the given D.E. is a Bernoulli equation, with n = -1, by writing in the dy 2 -1 equivalent form 2x dx - Y = x Y We rewrite this as and transform 2 2 2 dv dy Y = x · let v = Y and -- = 2 y . , dx dx ' the D.E. into the linear equation x · v = standard form of this IS dv - 1: x -, and an dx x 2xy dy dx 2 x. The I.F. -1 IS x Multiplying the standard form equation through by this we obtain (x· 1 v)' = 1. Integrating, we find x-1 v = ln lxl + c. Thus the solution is y2 = x (c + ln lxl). Section 2.4, Page 82 ... 3x2 + 16. The D.E. is both a separable equation and a Bernoulli equation. We first solve it as a separable equation. Separating variables, we get 2 dy 2 = dx ... y ... 1 ... x ... partial fractions ... Now resubstituting v = y2 and simplifying slightly, we find ... 65 Now apply the I.C. y(3) = O. Letting x = 3, y = 0 in the preceeding gives 8 = 21cl, so lcl = 4. Substituting this value of lcl back into the solution of the family of solutions, we get (1 + x)(y2 + 4)1/2 = 41 1 - xl. Since the initial value x is 3 > 1, we take x > 1 and 1 - x < 0, so 11 - xl = x - 1. Thus the solution is ... -- Alternately, we now solve the given D.E. as a Bernoulli Equation. We first rewrite it as ... 1 2 2 Y' - x ... 8 2 Y -1 , in which we recognize that it IS indeed a 1 - x 1-x 2 Bernoulli D.E. with n = -1. We thus let v = y2 ... Multiplying the linear equation through by this, we obtain ... Integrating, we find ... Thus ... and ... Now apply the I.C. ... Simplifying, we find ... the solution is ... 21. The D.E. is both a separable equation and a Bernoulli equation. We first solve it as a separable equation. ... Integrating, we find ... The I.F. is ... Multiplying the standard form through by this, we obtain ... Integrating, we find ... Applying the I.C. ... we find ... Thus the solution is ... 2 2 1 + ... Section 2.4, Page 82 ... Section 2.4 ... Chapter 2 ... The D.E. is linear. ... Applying the I.C. y(0) = 0 ... we have ... Thus ... the desired solution ... 69 ... The D.E. is linear in x. ... standard form ... I.F. ... Multiplying ... Integrating, we obtain ... The solution is ... First-Order Equations 71 ... We let ... Then ... Substituting ... we obtain ... Integrating, we find ... Thus the solution is ... The D.E. is homogeneous. ... let y = vx ... Separating variables ... Integrating ... Resubstituting ... we obtain ... First-Order Equations 73 ... Applications of First-Order Equations ... Chapter 2 ... This is exact ... we have ... Integrating ... Thus the solution is ... The D.E. is exact since ... Hence the solution is ... Chapter 3 ... Section 3.1, Page ... We first find the family of orthogonal trajectories of the given family. Differentiating ... Eliminating the parameter c between the equation of the given family and its derived equation, we obtain the D.E. of the given family ... Now replacing ... we obtain the D.E. of the orthogonal trajectories ... Separating variables ... Integrating, we find ... This is the family of orthogonal trajectories ... Applications of First-Order Equations 89 ... we obtain the D.E. of the given family ... The orthogonal trajectories ... Step 1. We first find the D.E. of the given family ... Step 2. We now find the D.E. of the orthogonal trajectories by replacing dy/dx ... by its negative reciprocal ... Step 3. We now solve this last D.E. ... Integrating we obtain ... This is the family of orthogonal trajectories ... Applications of First-Order Equations 93 ... Applications of First-Order Equations 95 ... Step 1. We first find the D.E. of the given family ... Step 2. ... Step 3. ... Applications of First-Order Equations 97 ... 5. We choose the positive x-axIS vertically upward along the path of the stone and the origin at the point from which the body fell. The forces acting on the stone are: (1) F 1 , its weight, 4 lb., which acts downward and so is the negative quantity -4; and (2) F 2 , the air resistance, numerically equal to ... Newton's Second Law gives ... The I.C. is v(O) = 0. The D.E. is separable. ... Integrating, we find ... Applying the I.C. ... Thus the velocity at time t is given by ... Applications of First-Order Equations 99 ... 6. We choose the positive x-axIS vertically downward ... The forces acting on the body ... We now consider ... the velocity ... Chapter 3 ...

Integrating we find lnl25 - 2vl = - 2;0 + Co which reduces to 125 - 2vl = cle-t/250. Applying the I.C. v(O) - 0 to this, we find c 1 = 25. Thus the velocity at time t lS glven by Applications of First-Order Equations 99 v = 2; (1 _ e- t / 2SO) (*) This is the answer to part (a). The answer to part (b) is then lim v = lim 2; (1 - e -t/2S0) = 12.S(ft/sec). t ro t ro To answer part (c), let v = (0.80)(12.5) ln (*) and solve for t. We obtain 0.80 = 1 - e- t / 250 and hence e- t / 250 = l/S. Thus - 2io = In(l/S), from which t = 402 (sec.). 6. We break the problem into two parts: (1) while the body is r1s1ng, and (2) while the body is falling back toward earth. We consider part (1). We choose the positive X-axlS upward along the path of the object. The forces acting on the object are: (1)F 1 , its weight, - 98000 dynes; and (2) F 2 , air resistance, - 200 v dynes. Each is negative, since each acts in the downward direction. Newton's S d L . dv F F 100 dv 98000 econ aw gl ves m dt = 1 + 2 or dt = - - 200v. The I.C. is v(O) = 150. We simplify the D.E. to read ; = (2v + 980). It lS dv separable; and separating variables, we have 2v + 980 = -dt. Integrating, we find Inl2v + 9801 = - 2t + cO. -2t Since v 0 in part (1), this reduces to 2v + 980 = c e -2t or v = - 490 + c e Applying the I.C., v(O) = 150 to this, we find c = 640. Hence -2t v = - 490 + 640 e . (*) 100 Chapter 3 The object stops rlslng and starts falling when v = o. -2t 49 Thus this happens at t such that e = 64 . We find t 0.1336 (sec.). In (a) we seek the velocity 0.1 second after the object is thrown. Since 0.1 < 0.1336, we let t = 0.1 ln (*), and find v = - 490 + 640 e - 1 . 2 33.99 cm/ sec. Now consider part (2). We choose the positive x-aX1S vertically downward from the highest point reached. Now the weight is + 98000, since it acts in this downward direct ion. The D. E. is now 100 ; = 98000 - 200v, and the I.C. is v(O) = O. Simplifying the D.E. and separating dv variables, we have 980 _ 2v = dt. Integrating, we find Inl980 - 2vl = - 2t + cO. Since 980 - 2v 0, upon, -2t simplifying we have v = 490 - c e . The I. C. gives c = 490, and so v - 490(1 - e- 2t). (**) In (b) we seek the velocity 0.1 second after the object stops rising and starts falling. So we let t - 0.1 in (**) and obtain v = 490(1 - e- 0 . 2) 88.82cm./sec. 8. We choose the positive x-aXlS horizontally along the glven direction of motion. The forces acting on the boat and rider are: (1) F 1 , the constant force of 12 lb., which acts in the given direction and so is positive, and (2) F 2 , the resistance force, numerically equal to 2v, which Applications of First-Order Equations 101 acts opposite to the glven direction and so lS the negative quantity -2v. dv Newton's Second Law glves m dt = F 1 + F 2 . Using g = 32 and m = : - 15° 3 + 2 170 = 10, this becomes 10 dv = 12 - 2v. dt The initial velocity lS 20 m.p.h. I.C. is v(O) = 8: . 88 - 3 ft/sec. Thus the The D.E. 1S separable. Ye write it as 5 dv 6 - v = dt. Integrating we find 5 lnlv - 61 = -t + Co which reduces to I 61 -t/5 A l . h I C (0) 88 h . v - e w - . pp Ylng t e . . v = 3 to t lS, we find c = 73°. Thus the velocity at time t 1S given by v = 6 + (7 3 0) e -t/5. (*) To answer part (a), we let t = 15 in (*) to obtain (70) -3 v(15) = 6 + 3 e 7.16 ft/sec. To answer part (b), let v = ( )(8) = 4 1n (*) and solve for (70) -t/5 -t/5 6 + 3 e = (44)(3), from which e this, - ; -0.99 and t 4.95 (seconds). t . Ye have 13 From = 35 . 102 Chapter 3 10. Ye choose the positive x-aX1S vertically upward along the path of the shell with the origin at the earth's surface. The forces acting on the shell are: (1) F 1 , its weight, 1 lb. , which acts downward and so is the negative quantity -1. and (2) F 2 , the air resistance numerically equal to , 10-4v 2 , which also acts downward (against the rising h 11) d . th . . 10 - 4 2 N ' sean so lS e negatlve quantlty - v. ewton s Second Law g1ves m ; = F 1 + F 2 . Using g = 32 and m = ; = ;2 ' this becomes 1 dv -- 32 dt -4 2 = -1 - 10 v . The initial condition lS v(O) = 1000. The D.E. 1S separable. We write it as 1 dv 10 4 + v 2 dv 32 dt -- = or = 32 dt 10 4 2 + 10 4 10 4 v Integrating we find (10) arc tan (10) 32t = -- + c or 10 4 arc tan (1 O) 32t Application of the initial = - 100 + c. condition at once glves c = arc tan 10. Thus we obtain the solution in the form arc tan (10) - arc tan 10 - ( ). Taking the tan of each side and multiplying by 100 glves v = 100 tan(arc tan 10 - O.32t). Applications of First-Order Equations 103 This is the answer to part (a). To answer part (b), note that the shell will stop rising when v = O. Setting v = 0, we at once have arctan 10 - 0.32t = 0, and thus t = ar c tan 10/ (0 . 32) 4. 60 (s e c) . 13. We choose the positive x-aX1S horizontally along the glven direction of motion and the orlgln at the point at which the man stops pushing. The forces acting on the loaded sled as it continues are: (1) F 1 , the air resistance, · 11 1 3v h e h . th numerica y equa to, w lC acts Opposlte to e 3v direction of motion and so is given by - , and (2) F 2 , the frictional force, having numerical value p, N = (0.04) dv Law gl ves m dt , this becomes (80), which also acts opposite to the direction of motion and so is given by -(0.04)(80) = - 1: . Newton's Second w 80 = F 1 + F 2 - Using g = 32 and m = g = 32 = 5 dv 3v 16 2 dt = - 4 - 5. The initial velocity is 10 ft/sec., so the I.C. is v(O) = 10. The D.E. is separable. Ye write it as dv 15v + 64 dt = - 50 . Integrating, we find Inl15v + 641 = reduces to 115v + 6 4 l = c e -3t/10 . v(O) = 10 to this, we find c = 214. given by 3t h e h - 10 + Co w lC Applying the I.C. Thus the velocity lS 104 Chapter 3 V = (214) -3t/10 _ 64 15 e 15 · From this, integration and simplification give x = - (48) e -3t/l0 _ 61t + c 1 . Application of the condition x(O) = 0 then gives c 1 = 48 . Hence the distance x is given by x = (48) (1 _ e -3t/l0) - 61t . To answer the stated question, note that the sled will continue until the velocity v = 0. Thus we set v = 0 and 1 f t f . d . -3t/l0 32 f h . h t 4 02 so ve or , ln lng e = 107 rom w lC .. We now evaluate x at t 4.02 to determine the distance which the sled will continue. We find x (4 . 02) (48) (1 - 1 3 0 2 7) - 64 (i 5 02} 16. 18 feet. 15. We choose the positive x direction down the slide with the origin at the top. The forces acting on the case are: (1) F 1 , the component of its weight parallel to the slide, (2) F 2 , the frictional force; and (3) F 3 , the alr resistance. The case weighs 24 Ibs., and the component parallel to the slide has numerical value 24 sin 450 = 24 . {2 Since this acts in the positive (downward) direction along the slide, F 1 = 24 The frictional force F 2 , has {2 numerical value p, N , where J.L lS the coefficient of the friction and N is the normal force. Here p, = 0.4 and the Applications of First-Order Equations 105 24 magni tude of N is 24 cos 450 = J2 Since the force F 2 acts in the negative (upward) direction along the slide, we have F 2 = -(0.4)(;:). Finally, the air resistance F3 has numerical value;, Since this also acts in the negative direction, we thus have F3 v = - 3 . Newton's Second Law now gives m ; - F 1 + F 2 + F3' With g = 32, w 24 3 m = g = 32 = 4 and the above forces, this becomes ( ) (;) = (;;-) - (0.4) (;;-) - ; The initial condition lS v(O) - o. Th D E o bl U e U rl " te l O t as (4 3) (d d V t) e . . lS separa e. w " = (-5v + 108J2) 15 or 5v - 10SJ2 I 4t (7) 1 find In 5v - 108J21 = - "9 + Co or 15v - 108v 2 = -4 t /9 A 1 0 h ." 0 I d " 0 f . d c e . pp Ylng t e lnltla con ltlon, we at once ln c = 108J2. Thus the velocity is given by = 4 45 dt Integrating we dv v = 108 5 J2 (1 - e -4t/9) . (*) To answer (a), simply let t = 1 in (*). Ye obtain v(l) = (1085 (1 - e4) 10.96(ft/sec). To answer (b), more work is required. We first integrate (*) to find the distance x from the top of the slide. We have 106 Chapter 3 x - (10J:f)t + (24J:f)e-4t/9 + c. Since x = 0 at t - 0, we find c = 243J:f and hence 5 ' = (108 v021 t (243J:f) -4t/9 _ 243 x 5-) + 5 e 5 . The slide being 30 ft. long, we let x = 30 in this to determine the time t at which the case reaches the bottom. That is, we must find t such that 30 = (1085t + (243)e-4t/9 _ 243 . We simplify this so that it takes the form -4t/9 _ 68. 72 e - -30. 54t + 98. 72. A little trial-and-error calculation with a hand calculator shows that the two sides of this are approximately equal for t = 2.49. This is the time at which the case reaches the bottom. Letting t = 2.49 in (*), we find the velocity at that time to be approximately 20.46 (ft/sec). 16. We choose the positive x direction down the hill with the origin at the starting point. The forces acting on the boy and sled are: (1) F 1 , the component of their weight parallel to the hill; (2) F 2 , the frictional force; and F 3 , the air resistance. The boy and sled weigh 72 Ibs., Applications of First-Order Equations 107 and the component parallel to the hill has numeical value 72 sin 300 = 36. Since this acts in the positive (downward) direction on the hill, F 1 = 36. The frictional force F 2 has numerical value J-L N , where J-L > 0 is the coefficient of friction and N is the normal force. The magnitude of N is 72 cos 300 = 36/3; the J-L is an unknown which will be determined in due course from the given data of the problem. Since the force F 2 acts in the negative (upward) direction on the hill, we have F 2 = -J-L(36/3). Finally, the air resistance F3 has numerical value 2v. Since this also acts in the negative direction, we thus have F 3 = -2v. Newton's Second Law now gi yes m ; = . w 72 9 F 1 + F 2 + F3. W1th g = 32, m = g = 32 = 4 ' and the above forces, this becomes (:)(;) = 36 - It 36/3 - 2v. The initial condition is v(O) = o. Another condition is also glven, that is v(5) = 10; and this extra condition will be sufficient for us to eventually determine p. The D.E. is separable. We write it as dv ( ) (18 - J-L18/3 - v) dt = or dv 8 dt = 18 (J-L 13 - 1) 9 v + 108 Chapter 3 8t Integrating we find Inlv + 18(- 1)1 = - If + Co or Iv + lS(- 1) I = c e -8t/9. Applying the initial condition, we at once find that c = 118(- 1)1. Thus we obtain the solution in the form v + 18(- 1) = lS(- 1)e- 8t / 9 or v = lS(l -)(1 - e- St / 9). Now we apply the extra condition v(5) = 10 to this to determine #. We have 10 = 18(1 -)(1 - e- 40 / 9) and hence 1 - p = ( )(O.988)-1, from which we obtain p = 0.25. 17. This problem has two parts: (A), before the object reaches the surface of the lake; and (B), after it passes beneath the surface. We consider (A) first. We take the positive x-axis vertically downward with the origin at the point of release of the object. The forces acting on the body are: (1) F 1 , its weight, 32 lbs., which acts downward and so is positive; and (2) F 2 , the air resistance, numerically equal to 2v, which acts upward and so is the negative quantity -2v. Applying Newton's Second Law, with m ; = ; = 1, we at once obtain the D.E. dv = 32 - 2v. dt The initial condition is v(O) = o. The D.E. lS separable. We write it dv dt. Integrating we find as 32 - 2v = Inl32 - 2vl = -2t + Co or 132 - 2vl -2t Applying c 1 e the initial condition we find c = 32. Yith this, we at once have Applications of First-Order Equations 109 -2t v = 16(1 - e) (*) This glves the velocity at each time t before the object reaches the surface of the lake. To solve problem (B), we will need to know the velocity at the instant when the object reaches the surface. To find out when this IS, we need to know distance fallen as a function of time. This is found by integrating (*). Ye at once obtain 16t + -2t + k. x = 8e Since x(O) = 0, we have k = -8, and hence the distance fallen (before striking the water) is given by' -2t x = 16t + 8e - 8. Since the point of release was 50 feet above the water, if we let x = 50 in this, it will determine the time at which the object hits the surface. Thus we must solve -2t 50 - 16t + Be - 8 for t. We write this in the form 58 - 16t = 8e- 2t . A little trial-and-error calculation with a hand calculator leads to the approximate solution t = 3.62. The velocity at this instant is then found by letting t = 3.62 in (*). The result is approximately 15.99 (ft/sec). We now turn to problem (B). Ye again take the positive x-axis vertically downward, but now we take the origin at the point where the object hits the surface of the lake. The forces now acting on the body are given by 110 Chapter 3 F 1 = 32, F 2 = -6v, and F3 = -8. The last two (the water resistance and the buoyancy) have negative signs since they act upward. Newton's Second Law leads to the D.E. = 32 - 6v - 8. By part (A), the velocity of the object at the surface of the lake is 15.99 (ft/sec). Ye take this as the I.C. here: v(O) = 15.99. Separating variables, the D.E. becomes 24 v6v = dt. Integrating, we find Inl24 - 6vl = -6t + c 1 or 124 - 6vl = -6t ce Applying the I.C. gives c = 71.94. Thus we obtain -6t -6t -(24 - 6v) = 71.94 e and hence v = 4 + 11.99 e . This is the velocity after the object passes beneath the surface. We want to know what this is 2 sec. after. Hence we let t = 2 in this to obtain v 4.00 (ft/sec). Section 3.3, Page 102 1. Let x be the amount of radioactive nuclei present after t years. Then x satisfies the D.E. = -kx, where k > O. Letting xo denote the amount initially present, we have the I.C. x(O) = xO. Also, since 10% of the original number have undergone disintegration ln 100 years, 90% remain, and so we have the additional condition x(100) = 9x O JR). The D.E. is separable; and separating variables and -kt integrating, we at once obtain x = c e Application of the I.C. immediately glves Xo = c. Hence we have Applications of First-Order Equations 111 -kt x = xOe Now apply the additional condition to 9x O -lOOk determine k. We have JR) = xOe , which reduces to e - k (1 9 0) 1/100 = Thus the solution takes the form = ( ) t/l00 x Xo 10 (*) To answer question (a), we let t = 1000 in (*). We find x(1000) = Xo b 9 0 fO 0.3487 xo' Thus the answer to x 34.87%. To answer question (b), we let x = 4 0 in (*). We xo = ( ) t/l00 ( ) ( ) have 4 Xo 10 From this, 10 In 1 9 0 = In ' In 1 4 9 In 10 from which t = 100 1315.28. Thus t 1315 years. 4. Let x = the amount satisfies the D.E. of the first chemical present. Then x dx dt = -kx, where k > O. Two conditions and x(4) = . The D.E. 1S separable; are glven: x(l) = and separating variables and integrating, we at once b . - k t A 1 . h d .. bt . o ta1n x = ce . pp Ylng t e two con 1t10ns, we o aln . -k 2 -4k 1 respectlvely ce = 3 and ce = 3 . These two equations will determine c and k. Dividing the first by the second gives e 3k = 2, from which e k = 21/3. Then from the second (1) 4k 2 4 / 3 equation for c and k, c 3 e = 3 . Thus the solution of the D.E. which satisfies the two given 112 Chapter 3 (2 4 / 3) k-t conditions is x = 3 (e) = 2(4-t)/3 3 (2 4 / 3) 1/3-t 3 (2) or x = (*) To answer question (a), let t = 7 in (*). This gives 2- 1 1 x(7) = 3 = 6 (kg) Now compare this wi th the orignial amount, which is x(O) = 24/3 3 (kg) Tie have x(7) x(O) 1 6 = = 24/3 3 1 3 0.1984. Thus 19.8% of the first chemical rema1ns 4 {2 at the end of seven hours. To answer question (b), we first note that one tenth (0) 24/3 of the first chemical is x 10 = 30 (kg). We thus let x 2 4 / 3 . 2(4-t)/3 2 4 / 3 = 30 ln (*) and solve for t. Tie have 3 = 30 · [4 - t] 24/3 From this 3 [In 2] = In 10 ; and sol ving for t, we find t 9.97. Thus the answer is 9 hours, 58 minutes. 7. Let x = the temperature of the body at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = k(x - 40). We also have the I.C. x(O) = 100 and the additional condition x(10) = 90. The D.E. is separable. We write it in the form x x40 = kdt. Integrating, we obtain Inlx - 401 = kt + Co and hence Ix - 401 = c e kt Since kt x 40, I x - 40 I = x - 40, so we have x = 40 + c e . Application of I.C. x(O) = 100 gives 100 = 40 + c or c = 60. Thus x = 40 + 60 e kt . Then application of the Applications of First-Order Equations 113 additional condition x(10) = 90 gives 90 = 40 + 60e 10k from which e 10k = or e k = ( )1/10 Thus e kt = ( )t/l0, and we have the solution (5) t/10 x = 40 + 60 6 (*) (a) Let t = 30 1n (*). e find x(30) - 40 + 60( )3 74.720 F. 5 t/10 (b) Let x = 50 in (*) . e have 50 = 40 + 60(6) , from which ( )t/l0 1 Then t - 10 In (1 / 6) 98.29. = 6. - In (5/6) Hence t 98 minutes, 17 seconds. 9. Let x = the temperature of the ple at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = k(x - 80). We also have the I.C. x(O) = 350 and the additional condition x(5) = dx we have x - 80 300. Separating variables in the D.E., = kdt; and integrating, we find Inlx - 801 kt = kt + cO' and hence I x - 80 I = c e Since x 80, this kt simplifies to x = 80 + c e Applying the I. C. x(O) = 350, we have 350 = 80 + c, so c = 270. Thus x = 80 + 270e kt . Application of the additional condition 5k 5k 220 x(5) = 300 gives 300 = 80 + 270 e , from which e = 270 k or e = (;)1/5 Thus we obtain the solution (22) t/5 x = 80 + 270 27 . (*) 114 Chapter 3 (22) 2 (a) Let t = 10 ln (*). We find x(10) = 80 + 270 27 259 . 26 ° . (b) Let x = 100 in (*). We have 100 (22)t/5 = 80 + 270 27 ' (22)t/5 2 Then t 5 In (2 / 27) from which 27 = N 27 . = N In(22/27) 63.54. Hence t 63 minutes, 32 seconds. 10. Let x. = the temperature of the coffee at time t. Assuming dx Newton's Law of Cooling, we have the D.E. dt = k(x - 70). We also have the I.C. x(O) = 180 and the additional condition x(10) = 160. Separating variables in the D.E., dx we have x _ 70 = k dt ; and integrating, we find In 1 x - 70 I kt = kt + cO' and hence Ix - 701 = c e Since x 70, this kt simplifies to x = 70 + c e Applying the I.C. x(O) = 180, we have 180 = 70 + c, so c = 110. Thus x = 70 + 110 e kt . Application of the additional condition x(10) = 160 gives 160 = 70 + 110 e 10k , from which e 10k = 90 k = (1 9 1) 1/10 110 or e Thus we obtain the solution (9) t/10 x = 70 + 110 IT (*) . (9) 3/2 (a) Let t = 15 ln (*). We have x(15) = 70 + 110 IT 151.41°. Applications of First-Order Equations 115 (b) Let x = 140 in (*). (1 9 1) t/l0 from which - 22.52. So x = 140 at (9) t /10 We have 140 = 70 + 110 II ' _ 10 In(7/11) N 11. Then t - In(9/11) N t 22 minutes, 31 seconds. Now (9) t/l0 We have 130 = 70 + 110 II ' 6 Th t _ 10 In(6/11) N II- en - In(9/11) N t 30 minutes, 12 seconds. let x = 130 in (*). (1 9 1) t/l0 from which - 30.20. So x = 130 at Thus the woman should have drunk the coffee between 22 minutes, 31 seconds past 10 and 30 minutes, 12 seconds past 10. 11. Let x = the population at time t. Then we at once have the D.E. = kx. Letting xo denote the population at the start of the given 40 year period, we have the I.C. x(O) = xO. Since the population doubles in 40 years, we have the additional condition x(40) = 2xO. The solution of the D. E. is x = c e kt Applying the I. C _ to this we at once have c = xo' and hence x = xoe kt . Now applying the additional condition to this, we have 2x O = x O e 40k , from which e 40k = 2 and k = In 4 0.0173. Thus the solution of the D.E. which satisfies the two given conditions lS x 0.0173t = xOe To answer the stated question, we let x = 3x O in this, from which eO.0173t = 3 and 0.0173t = In 3. From this we find t 63.5. Thus the population triples in approximately 63.5 years. 116 Chapter 3 12. Let x = the population at time t. We at once have the dx D.E. dt = kx. We take 1970 as the zeroth year of the problem and have the I.C. x(O) = 30,000. Then 1980 is the tenth year, and we have the additional condition x(10) = 35,000. The one-parameter family of solutions of the D.E. is x = c e kt Applying the I. C. to this, we have c = 30,000. Thus x = 30,000 e kt . We apply the additional 10k condition to this, obtaining 35,000 = 30,000 e . From . k (7) 1/10 thlS, e = 6 (7) t/l0 30,000 6 · We seek the population in 1990, which is Thus we obtain the solution x = the twentieth year on or scale. Letting t = 20 in the solution, we find x = 30,000( )2 40,833. 15. The population x satisfies the D.E. dx = dt 3x -- 100 3x 2 10 8 . The I.C. lS x(1980) = 200,000. The D.E. lS separable. We write it ln the form 10 6 dx 3 dt = 100 · 6 x(x - 10) To integrate the left member, we use partial fractions. Thus we obtain [x _1106 - :] dX = 3 100 dt Applications of First-Order Equations 117 6 In x - 10 = x -3t/l00 T.T e ce . " 3t - 100 + cO. 6 I 3t 10 ! - In!x = 100 + Co or x - 10 6 From this we obtain = Integrating we find In Ix x to this. apply the I.C. x(1980) = 200,000 = (2)(10 5) 1(2)(10 5) - 106 1 - -3(198)/10 C - - We have 5 - ce , or 2(10) _4e 59 . 4 . Thus we obtain the solution in the form x - 10 6 x = _4e59.4-3t/l00. We solve this for x, obtaining x + 4x e59.4-3t/l00 = 10 6 and hence 10 6 x = 1 + 4e59.4-3t/l00 (*) This 1S the answer to part (a). To answer part (b), 10 6 we let t = 2000 in (*). We find x(2000) = [1 + 4e 0 · 6J 312,966. To answer (c), simply find limx, where x is t-+CD glven by (*). Since lim e59.4-3t/l00 = 0, we have t-+CD lim x t-+CD 10 6 = -y- = 1,000,000. 16. dx 2 (a) We have the initial-value problem dt = k x - x , 2 x(t O) = xO' where we assume k x - x > o. The D.E. is separable. We separate variables and use partial fractions to obtain [+ k _"" x] dx = dt. Then integration gives [ln x - In(k - "x)] = t + In Co . x kt Simplifying, we obtain k _,\ x = c e Applying the 118 Chapter 3 -kt xOe 0 I.C. x(t O) = X o to this, we find c = Before k - x · o using this, we solve the one-parameter solutions for x. We have x = k c e kt - family of kt ,\ c x e , from k kt ce kt l+ce which (1 + c ekt)x = k c e kt and hence x = kc -kt. AC + e the value of c already determined from the initial condition. We find or finally x - We now substitute into this x = r -k t .. k xOe 0 k - A x 0 -k t xOe 0 -kt k- x 0 e 0 '- - We simplify this by multiplying numerator and denominator through by (k - ,\ Xo)e k to. "We obtain x - k Xo , (k ' Xo) e -k (t - t o). It Xo + - " 17. Let x = the population at time t. Since 100 people leave the island every year, the D.E. (3.58) is correctly modified by subtracting 100 from its right member. Thus we have the D.E. dx dt = (4O)x - 10- 8 x 2 - 100. We also have the I.C. x(1980) = 20,000. The D.E. is separable. We write it in the successive forms Applications of First-Order Equations 119 dx (10)6 x - 4x 2 - 4(10)10 dt = 4(10)8 ' dx = [x 2 _ 25(10)4 x + (10)10 J dt (10)8' dx = [x - 5(10)4 J [x - 20(10)4 J dt (10)8. We now apply partial fractions to the left member and multiply through by (15)(10)4 to obtain [x - 1 20(10)4 1] dx - 4 - x - 5(10) 15 dt (10)4. Integrating and simplification give In [x - 20(10)4 4] - x - 5(10) 15t 4 + Co (10) or x - 20(10)4 = ce- 15t /(10)4 x - 5(10)4 We apply the I.C. x(1980) = 20,000 = (20)(10)3 to this. With x = (20)(10)3, the left member reduces thus: (20)(10) 3 (1 - 10) = 3 6. (5) (10) (4 - 10) 4 Thus we find 6 = c e -(15) (1980)/(10) , and so c = 6 e(15) (1980)/(10)4. Thus the solution takes the form 120 Chapter 3 x - (20)(10}4 = 6 e 15 (1980-t)/(10)4 x - 5(10)4 We must solve this for x. After some algebraic manipulations, we obtain the desired result 4 x = (10)5[3 e 15 (1980-t)/(10) - 21 6 e 15 (1980-t)/(10)4 - 1 18. This falls into two problems: (A) Before the cats arrived, and (B) after the cats arrived. We let x = the number of mice on the island at time t and proceed to solve problem (A). During this time, in the absence of cats killing mice, the D.E. is simply = kx, with solution x = c e kt Choosing January 1, 1970 as time t = 0, the I.C. is x(O) = 50,000. Then, measuring t 1n years, Jan. 1, 1980 is time t = 10; and we have the additional condition x(10) = 100,000. Applying the I.C. x(O) = 50,000, we at once find c = 50,000 and hence kt x = 50,000 e . The additional condition now gives e 10k = 2, from which k = In ;0 ' Vith this, the solution takes the form x = 50,000 e t In(2/10). But this solution only holds until 1980 and so will not help us to answer the stated question. It is the number k, which represents the natural rate of population increase, which we need here to solve problem (B) and answer the stated question. Turning to problem (B), Slnce the cats kill 1000 mice/ month = 12,000 mice/year, the D.E. of problem (A) must be modified by subtracting 12,000 from its right member. Applications of First-Order Equations 121 dx 2 Thus we have dt = kx - 12,000, where k = In 10 was found in problem (A). The D.E. is separable, and we write it In dx the form kx _ 12 000 = dt. Integrating we obtain x = , kt 12,000 + ce Now we have an I.C. here, for on Jan. 1, 1980, there were 100,000 mice. Taking this date as time t = 0 for problem (B), we thus have the I.C. x(O) = 100,000. Application of this gives c = 100,000 k - 12,000. With this value of c and k = In 1 ' we obtain the solution of problem (B): x = (102) [12,000 + (10,000 In 2 - 12,000)e t In 2/10 J The answer to the stated question is now found by letting t = 1 in this expresslon. We have x(l) = (14.427)[12,000 + (6931 - 12,000)(1.0718)J = 94,742. 19. Let x = the amount of money at time t. We have the D.E. = kx. Since the annual rate is 6%, we let k = .06 and dx so have the D.E. dt = .06x. We also have the I.C. x(O) = 1000. Solving the D.E., we find x = c e. 06t The I.C. gl ves c = 1000, so we have the solut ion x = 1000 e. 06t . (a) We let t = 10 in the solution. We find x = 1000 e 0.6 1822.12. Thus there will be $1822.12. 122 Chapter 3 (b) Ye let t = 2000 in the solution. Ye obtain 2000 = 1000 e .06t .06t = 2. From this .06t = In 2 and t or e N 11.55. Thus the amount will double In approximately N 11.55 years. 20. From 19, have the D.E. dx kx, where k exerC1se we = glves dt the annual rate. Ye have the I.C. x(O) = xO' where Xo denotes the original amount. Solving the D.E., we find x = c e kt Applying the I. C. to this, we find c = Xo and so h h 1 . kt ave t e so ut10n x = xOe . (a) If the original amount doubles ln two years, then x = 2x O when t = 2. Applying this to the solution, we obtain 2x O 2k From this k = In 2 0.3466. = xOe 2 Thus the annual rate is a remarkable 34.66%. (b) Here we have the additional condition x(1/2) = 3xO/2. Ye apply this to the solution. Ye have 3xO/2 = k/2 k xOe , from which e = 9/4. Then the solution becomes x = xo( )t. We ask how long it will take the original amount to double, so we let x = 2x O in this solution and solve for t. We find ( )t = 2, from h . h In 2 0 8547 S h d bl w 1 c t = In (9/4) . . 0 team 0 un t 0 u e s 1 n approximately 0.85 years. Applications of First-Order Equations 123 21. Let x = the amount of salt at time t. We use the basic equation (3.63), = IN - OUT. We find IN = (3 lb./gal) (4 gal./min.) = 12(lb./min.); and OUT = (C lb./gal.) (4 gal./min.) where C lb./gal. is the concentration. Since the rates of inflow and outflow are the same, there x is always 100 gal. in the tank and so this is simply 100 . Thus we have OUT = lxO (lb./min.) Hence the D.E. of the problem IS = 12 - lxO ' The I.C. is x(O) = 20. The D.E. separable. We write it as dx 300 - X 1S = or dt 25 dx dt Integrating, find Inl300 - xl t = 25 . we = - - + 300 - x 25 Co or 1300 - xl -t/25 Applying the I.C., we find = c e . 280 = c; and since 300 - x > 0, we take 1300 - xl = 300 x. Thus we obtain the solution ln the form 300 - x = 280 e -t /25 or x = 300 - 280 e -t/25 (*) To answer question (a), let t = 10 in (*). 112.31 (lb.). To answer question (b), let x - (*). We have 280 e- t / 25 = 140 or e- t / 25 = 0.5. We find x 160 in t Then - 25 = In 0.5, from which we find t 17.33 (min.). 22. Let x = the amount of salt at time t. We use the basic dx equation (3.63), dt - IN - OUT. Ve have IN = 0, since pure water flows into the tank; and we find OUT = (C lb./ gal.)(2gal/min), where c lb./gal. is the concentration. Since water flows in at 5 gal/min. and the mixture flows 124 Chapter 3 out at 2 gal/min., there is a gaIn of 3 gal/min of brine in the tank. Since there was initially 100 gal. there, the amount of brine at time t 18 100 + 3t. Thus the .. x Th A UT 2x (I b / concentratlon lS 100 + 3t . us = 100 + 3t · min.). Hence the D.E. of the problem is = 100 2 : 3t ' The I.C. is x(O) = 10. The D.E. is separable. U . . dx 2 d t we wrIte lt as)(= - 3t + 100. 2 3 In (3t + 100) + In Co ' from Integrat ing, we find In x = c which x = Ye now apply the I.C., obtaining (3t + 100)2/3. c 2/3 2/3 or c = 10(100) Thus we have the (100) 10 (100) 2/3 solution x = - - (3t + 100)2/3. 10 = (a) Ye let t = 15 in the solution. 10 (100) 2/3 We find x = - - (145)2/3 7.81. So there is approximately 7.81 lb. The concentration is 7;4 8 ; 0.0539 lb./gal. (b) Since there was initially 100 gallons of brine in the tank and the amount is increasing at the rate of 3 gal./min., the number of minutes t needed to obtain a full tank of 250 gallons is given by 100 + 3t = 250; so t = 50. Ye let t = 50 in the solution and find 2/3 x = 10(100) 5.4279. The concentration is then (250)2/3 5.4279 250 0.021 7 lb. / gal. Applications of First-Order Equations 125 25. Let x = the amount of salt at time t. Ye use the basic equation = IN - OUT. We find IN - (30 gm/liter)(4 liters/min) = 120 (gm/min); and OUT = (C gm/liter) (liters/min), where C gm/liter is the concentration. The rate of inflow is 4 liters/min and that of outflow 1S liters/min, so there is a net gain of liters/min of fluid in the tank. Hence at the end of t minutes the f fl ° d 0 h k 0 300 3t l O t Th th amount 0 Ul In t e tan IS + 1 ers. us e concentration at time t is x 3t gm/liter, and so OUT - 300 + 2 5x 5x 3t J 3t + 600 (gm/min). 2 L 300 + 2"" = problem is = 120 - 3t x600 ' Hence the D.E. of the The I.C. lS x(O) = 50. The D.E. 1S linear. Ye write it ln the standard form dx 5 120. dt + x = 3t + 600 An I.F. is e J [(5/(3t+600)]dt = e(5/3)ln(3t+600) = (3t + 600)5/3. Multiplying through by this we have (3t + 600)5/3 + 5(3t + 600)2/3 x = 120(3t + 600)5/3 or d [(3t + 600)5/3x] = 120(3t + 600)5/3. Integration glves (3t + 600)5/3 x = 15(3t + 600)8/3 + c or x = 15(3t + 600) + c(3t + 600)-5/3. 126 Chapter 3 Applying the I.C. x(O) = 50 gives 50 = (15)(600) + c(600)-5/3, from which c = -8950(600)5/3. Thus the solution is given by x = 15(3t + 600) - (8950) (600)5/3(3t + 600)-5/3. (*) The stated question asks for x at the instant when the tank overflows. Since the amount of fluid increases at the rate of liter/min and the 500 liter tank originally had 300 liters in it, this time t is given by 3; = 200 or t = 40 . Ve thus let t = 40 ln (*) and obtain x (40) = ( ) 5/3 1500(1000) - (8950) (600)5/3(1000)5/3 = 15000 - 8950 11,179.96 (grams). 26. Let x = the amount of salt at time t. Ye use the basic equation = IN - OUT. Ve find IN = (50 gm/liter) (5 liters/min) = gm:n ; and OUT = (C gm/liter)(7 liter/min), where C gm/liter lS the concentration. The rate of inflow is 5 liters/min. and that of outflow 1S 7 liters/min., so there is a net loss of 2 liters/min of fluid in the tank. Hence at the end of t minutes the amount of fluiq in the tank is 200 - 2t liters. Thus the concentration at time t is 200 2t gm/liter, and so OUT = 7x . dx 7x 200 - 2t (gm/min). Hence the D.E. lS dt = 250 - 200 - 2t . The I.C. is x(O) = 40. The D.E. 1S linear. Ve write it ln the standard form Applications of First-Order Equations 127 dx 7 dt + 200 2t x = 250. An I.F. is e J [7/(200-2t)]dt = e-(7/2)ln(200-2t) - (200 - 2t)-7/2. Multiplying through by this, we have (200 - 2t)-7/2 + 7(200 - 2t)-9/2 x = 250(200 _ 2t)-7/2 or d [(200 - 2t)-7/2 x] = 250(200 2t)-7/2. Integration gives (200 - 2t)-7/2 x = 50(200 - 2t)-5/2 + c or x = 50(200 - 2t) + c(200 - 2t)7/2. Applying the I.C. x(O) = 40, we find 40 = 10,000 + c(200)7/2, from which c = -9960(200)-7/2. Thus the solution is given by (t) 7/2 x = 50(200 - 2t) - 9960 1 - 100 . (*) The tank will be half full when it contains 100 liters. Since it initially contained 200 liters and it loses 2 liters/min., it will contain 100 liters after 50 mln. Thus we let t = 50 in (*). Ve find 7/2 x(50) - 5000 - 9960() 4119.65 (gm). 128 Chapter 3 30. Let x = the number of people who have the disease after t days. Then 10,000 - x people do not have it, and we have dx lJe also have the I.C. x(O) the D.E. dt = kx(10,000 - x). = 1. The D.E. is separable; and separating variables, we h dx k d U . . 1 f . d ave x(lO,OOO _ x) = t. slng partla ractlons an integrating, we find 10OO [lnlxl-lnll0,OOO - xl] = kt + cO. Noting that x 0 and 10,000 - x 0, we x simplify and obtain In 10 000 _ x = 10,000 kt + c 1 . Then , x 10,000kt we have 10 000 _ x = c e . We apply the I.C. to , obtain c = 1/9999. 10 ,000 kt e 9999 Thus we obtain the solution x 10,000 - x = We solve this for x In terms of t. lJe find (1 el0,000 kt) _ 10,000 10,000 kt + 9999 x - 9999 e and hence 10,000 x = 1 + 9999 e -10 ,000 kt · Now besides the I.C., we also have the additional condition x(5) = 50. We apply this to the preceeding solution. We have 50 = 10,000 1 + 9999 e -50,000 k. Then (50) (9999) e -50,000 k = -10 ,000 kt (199) t / 5 so e = 9999 form 9950, -50,000 k e 199 = 9999 ' and Thus the solution takes the Applications of First-Order Equations 129 x = 10,000 [199] t/5. 1 + 9999 9999 We are asked how many people have the disease after 10 days. We let t = 10 ln this to obtain x(10) - 10 ,000 N N [199] 2 1 + 9999 9999 478.88. Thus about 479 people have the disease at that time. 31. Let x = the number of Ibs. of c 3 formed ln time t. We have the D.E. = k(lO -)(15 - 3 4 X), the I.C. x(O) = 0, and the additional condition x(15) = 5. Separating variables and simplifying, the D.E. takes the 16 dx form 3 (x _ 40) (x _ 20) - k d t . In(x partial fractions, we have l [X ! 40 - x ! 20]dX Integrating, we obtain l [In(x - 40) 20)J = kt + c o or In - 40 = 15 kt + C 1 . From - 20 4 Applying = k dt . this, x - x - 15 kt 40 4 = c e 20 (*) 130 Chapter 3 We apply the I.C. x(O) = 0 to this, and at once obtain c = 2. We set c = 2 in (*), and apply the additional condition x(15) = 5. 35 (1; k) 15 1; k We have 15 = 2 e and hence e = Thus we obtain the solution ( )1/15 x - x - = 2( )t/15 We solve this for x In terms of t, obtaining x = [()t/15 - 11 2 [r /15 - 1 This is the answer to part (a). To answer (b), let t = 60 in this. We have x(60) _ ()4 _ 11_ N 4 '" 2 [ ]- 1 40(0.8526) '" 2.7052 '" 12.6068 Thus we find x 12.61 (lb). Chapter 4 Section 4.1.B, Page 122 6. x 3x (a) Suppose c 1 e + c 2 e = 0 for all x on a x b. x 3x Differentiating, we obtain c 1 e + 3c 2 e = 0 for all x on a x b. Subtracting the first of these from the second, have 2c 2 e 3x = 0 for all x a < b. we on S . 2 3x f. 0 for all such x, we must have c 2 = o. lnce e Then c 1 e x 3x = 0 reduces to x = 0 for all x c 2 e c 1 e on a x b. Since eX f 0 for all such x, we must have x 3x c 1 = o. Thus c 1 e + c 2 e = 0 for all x on a x b - l - h x d 3x 1 - 1 lmp 1es t at c 1 = c 2 = 0, so e an e are ..1near y independent on a < x < b. x 3x (b) W (e , e) = x e 3x e 4x = 2e f 0 on a < x < b, so x e 3e 3x e x and e 3x 1 - 1 . d d th - - t 1 are lnear y 1n epen ent on 1S 1n erva . 7. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have W(e 2x ,e 3x) = 2x e 3x e = 3e 5x _ 2e 5x - e 5x f 0 2e 2x 3e 3x 131 132 Chapter 4 for all x, -CD < X < CD. Thus the two solutions are linearly independent on -CD < x < CD. (b) 2x 3x y = c 1 e + c 2 e (c) To satisfy the condition y(O) = 2, we let x = 0, y = 2 ln the general solution of part (b) . We have c 1 + c 2 = 2. (*) Now we differentiate the general solution, obtaining y' = 2c 1 e 2X + 3c 2 e 3x . To satisfy the condition y'(O) = 3, we let x = 0, y' = 3 in this derived equation. We have 2c 1 + 3c 2 - 3. (**) Solving the two equations (*) and (**) for c 1 and c 2 , - we find c = 3, c 2 = -1. Substituting these values of 1 c 1 and c 2 into the general solution of part (b) , we 3 2x 3x have the desired particular solution y = e - e . This is unique by Theorem 4.1; and it is defined on -CD < x < CD. 8. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have Higher-Order Linear Differential Equations 133 x e x xe W(ex,xe x) = x e x (x + l)e - e 2x f 0 for all x, -00 < x < 00. Thus the two solutions are linearly independent on the interval -00 < x < 00. (b) x x y = c e + c 2 xe . 1 (c) To satisfy the condition y(O) - 1 , we let x = 0, y = 1 - ln the general solution of part (b) . We have c 1 = 1 . Now we differentiate the general solution, obtaining y' = c 1 e x + c 2 (x + 1) eX. To satisfy the condition y'(O) = 4, we let x = 0, y' = 4 in the derived equation. We have c 1 + c 2 = 4. Since c 1 = 1, this yields c 2 = 3. Substituting these values of c 1 and c 2 into the general solution, we have the desired particular solution y = eX + 3xe x . This is unique by Theorem 4.1; and it is defined on --00 < x < 00. 10. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have 2 1 x 2 V(x2, :2) x 2 2 4 f 0 = = - - - - - 2 x x x 2x - - 3 x for all x, 0 < x < 00. Thus the two solutions are linearly independent on 0 < x < 00. 134 Chapter 4 (b) 2 Y = c x 1 c 2 + 2. x (c) To satisfy the condition y(2) = 3, we let x = 2, Y = 3 in the general solution of part (b). We have c 2 4c 1 + 4 = 3. (*) Now we differentiate the general solution, obtaining y' = c 1 e x + c 2 (x + l)e x . To satisfy the condition y'(O) = 4, we let x = 0, y' = 4 in the derived equation. We have c 1 + c 2 = 4. Since c 1 = 1, this yields c 2 = 3. Substituting these values of c 1 and c 2 into the general solution, we have the desired particular solution y = eX + 3xe x . This is unique by Theorem 4.1; and it lS defined on - CD < X < (I). 10. (a) One readily verifies by direct substitution into the D.E. that each of these functions is indeed a solution. To show that they are linearly independent, we apply Theorem 4.4. We have 2 1 x 2 x (v 2 :2) 2 2 4 f 0 x , = = = 2 x x x 2x 3 x for all x, 0 < x < (I). Thus the two solutions are linearly independent on 0 < x < 00. Higher-Order Linear Differential Equations 135 (b) Y 2 = c 1 x c 2 + 2"- x (c) To satisfy the condition y(2) = 3, we let x = 2, Y = 3 ln the general solution of part (b) . We have 4c 1 + c 2 3 (*) - = 4 Now we differentiate the general solution, obtaining -3 y' = 2c 1 x - 2c 2 x To satisfy the condition y'(2) = -1, we let x = 2, y' = -1 in this derived equation. We have C 2 4c - - = -1 1 4 (**) . Solving the two equations (*) and (**) for c 1 and c 2 , 1 we find c 1 = 4 ' c 2 = 8. Substituting these values of C 1 and c 2 into the general solution of part (b) we - _(4 1) X 2 have the desired particular solution y 8 + 2". x This is unlque by Theorem 4.1; and it lS defined on 0 < x < (I). 136 Chapter 4 12. We use Theorem 4.4. We have -x 3x 4x e e e W(-x 3x 4x) -x 3e 3x 4e 4x e ,e ,e = -e -x ge 3x 16e 4x e 1 1 1 -x 3x 4x -1 3 4 = e e e - - 1 9 16 6x { 3 4 -1 4 -1 3 } = e + 9 16 1 16 1 9 = e 6x [12 + 20 - 12J = 20e 6x f 0 for all x, -00 < x < 00. Thus the three solutions are linearly independent on -00 < x < 00. The general solution . -x 3x 4x lS y = c 1 e + c 2 e + c 3 e 13. We have 2 4 x x x 2 4 4x 3 W(x,x ,x) = 1 2x 0 2 12x 2 2x 4x 3 2 4 x x = x 2 12x 2 2 12x 2 4 4 6x 4 f 0 -= 16x - lOx = Higher-Order Linear Differential Equations 137 on 0 < x < 00. Thus by Theorem 4.4, the three glven solutions are linearly independent on 0 < x < 00. The 1 1 . . 2 4 h genera so ut10n 1S y = c 1 x + c 2 x + c 3 x , were c 1 ' c 2 and c 3 are arbitrary constants. Section 4.1.D, Page 132 L illi 1. et y = vx. Then y' = xv' + v and y = xv + 2v'. Substituting these into the glven D.E., we obtain x 2 (xv H + 2v') - 4x(xv' + v) + 4xv = 0 or x 3 v H - 2x 2 v' = 0 or xv" - 2v' = O. Letting w = v', we obtain x : - 2w = 0 or dw = 2 dx . Integrating, we find w x 2 lnlwl = 21n Ixl + Inlcl or w = cx. We choose c = 1, 3 x recall v' = w, and integrate to obtain v = . Now 4 x forming y = xv, we obtain y = . This or any nonzero constant multiple thereof serves as the desired linearly independent solution. Choosing multiple 3, we have y = x 4 . The general solution is then y = c 1 x + c 2 x 4 . 4. Let y = xv. Then y' = xv' + v and yH = xv H + 2v'. Substituting these into the given D.E., we obtain (x 2 - x + 1) (xv H + 2v') - (x 2 + x) (xv' + v) + (x + l)xv = o or x(x 2 - x + 1)v H + (_x 3 + x 2 - 2x + 2)v' = 0 or x(x 2 - x + 1)v H + (x 2 + 2)(1 - x)v' = O. Letting w = v', 2 dw 2 dw weobtalnx(x -x+1) dx +(x +2)(1-x)w=0,or w = (x -2 1)(x 2 + 2) dx. Using long division and partial x(x - x + 1) 138 Chapter 4 f . h " h f dw ractlons, we express t lS ln t e orm -- = w [1 - + 22x - 1] dX. Integration now yields x - x + 1 In I wi = x - 2 In I x I + In I x 2 - x + 11 + In I c I and so Inlwl 2 =x+ln J c (x - x + 2 x 1) L We choose c = 1 , recall w = y' and obtain , y' (1 - 1 1) x = + x 2 e · x x f 1 x Thus y x f : dx Integrating = e + 2" e dx. x f 1 x x x by obtain e + f : dx · Thus 2" e dx parts, we - - x x v = eX - or y = e X (l -). Now forming y = xv, we find the desired linearly independent solution y = (x - l)e x . x Thus the general solution is y = c 1 x + c 2 (x - l)e · 5. 2x 2x Let y = e v. Then y' = e v' + 2e v and H 2x H 4 2x, 4 2x S b " . h " Y = e v + e v + e v. u stltutlng t lS . D E ( )(2x u 2x glven . ., we obtain 2x + 1 e v' + 4e v' + into the 4e 2x v) Higher-Order Linear Differential Equations 139 4( )(2x 2 2x) 4 2x 0 - x + 1 e v' + e v + e v = or ( ) 2x u 2x 2x + 1 e v + 4xe VI = 0 or (2x + l)v H + 4xv' = O. Letting w = v', we obtain (2x + 1) : + 4xw = 0 or dw dx w = -4x 2x + 1 . Using long division on the right member, we rewrite this as d: = (-2 + 2x : 1)dx. Integrating, we have Inlw/ = -2x + Inl2x + 11 + Inlcl or -2x w = c(2x + l)e . We choose c = 1 and recall w = v' to obtain : = (2x + 1)e- 2x . Integrating this, we obtain V - - _ (x + 1)e -2x . N f 2x bt ow orming y = e v, we 0 ain y = -(x + 1). This or any nonzero constant multiple thereof serves as the desired linearly independent solution. Choosing multiple -1, we have y = x + 1. The 2x general solution is then y = c 1 e + c 2 (x + 1). 8. x x Let y = e v. Then y' = e v' + e v and " x" 2 x ' x Sb . . y = e v + e v + e v. u stltutlng thlS lnto the glven D E b . (2) (x" x x) . ., we 0 taln x + x e v + 2e v' + e v - (x 2 - 2) (exv' + eXv) - (x + 2) (exv) = 0 or 2 2 (x + x)v H + (x + 2x + 2)v' = O. Letting w = v', we obtain (x 2 + x) + (x 2 + 2x + 2)w = 0 or dw w = 2 x + 2x + 2 2 dx. Using long division and partial x + x fractions, we express this ln the form d: = [1 + ; - x : 1]dX. Integration now yields 140 Chapter 4 Inlwl = -x - 2 In Ix I + In Ix + 11 + Inlcl and so Inlwl = -x + In Jc(x ; 1}L x We choose c = 1, recall w = v', and obtain (1 1) -x y = x + 2 e · Thus -x -x y = f ex dx + f e 2 dx. x Integrating f e-xdx by parts, we obtain -x -x -x - f dx. Thus we have v = _ e x 2 x x y = eXv, we find the desired linearly independent solution y = -l/x. Thus the general solution is y = c 1 e x + c 2 /x. Now forming 11. We first apply Theorem 4.10 to obtain the solution of the D.E. H 1 y - 5y' + 6y = 2 - 2x. (*) From the first two glven D.E.'s, F 1 (x) = 1, F 2 (x) = x, with respective given particular integrals f 1 (x) = 1/6, f 2 (x) = x/6 + 5/36. The right member of (*) is 2F 1 (x) - 12F 2 (x). Then by the theorem, a particular integral of (*) is y = 2f 1 (x) - 12f 2 (x) = 1/3 - 2x - 5/3 = -2x - 4/3. Higher-Order Linear Differential Equations 141 We now apply the

2x , = 4Ax 2 e 2x + (8A + 4B)xe 2x + (2A + 4B)e 2x . " yp We ubstitute into the D.E., obtaining 2 2x 2x 2x 4[Ax e + (8A + 4B)xe + (2A + 4B)e J 2 2x 2x - 4[Ax e + Bxe J = 16xe or 2x 2x 2x 8Axe + (2A + 4B)e = 16xe . We equate coefficients of like terms on both sides of this to obtain 8A = 16, 2A + 4B = O. From this, we find A = 2, B = -1. Thus we obtain the particular integral y p 2x -2x 2 2x The G.S. is y = c 1 e + c 2 e + e 2 2 2x 2x x e - xe 2x xe 15. The corresponding homogeneous D.E. m 4y " + y' - 6y lS Y + - - o. The auxiliary equation 3 4m 2 - 6 O. By lS m + + m = inspection note that m = 1 lS a root. From this (by synthetic division or otherwise) we obtain the factored form (m - l)(m + 2)(m + 3). Thus the roots of the auxiliary equation are m = 1, -2, -3. The complementary f t .. x - 2x -3x Th NH t . unc 10n lS Yc = c 1 e + c 2 e + c 3 e e erm lS a iinear combination of the UC functions x 2 and 1. Higher-Order Linear Differential Equations 169 SteD 1 : Form the UC set of each of these two UC functions: 2 Sl = {x ,x,l}, S2 = {1}. SteD 2 : Set S2 is completely included ln 51' so S2 lS omitted, leaving just Sl. SteD 3 : An examination of the complementary function shows that none of the functions in Sl is a solution of the corresponding homogeneous D.E. Hence Sl does not need revision. SteD 4 : Thus the original set Sl remalns. We form a linear combination of its three members. SteD 5 : Thus we take p y 2Ax + B, y H = 2A, Y m p p obtaining = Ax 2 + Bx + C. Then y , = p = O. We substitute into the D.E., o + 4(2A) + (2Ax + B) - 6(Ax 2 + Bx + C) = -18x 2 + 1 or 2 2 -6Ax + (2A - 6B)x + (8A + B - 6C) = -18x + 1. We equate coefficients of like terms on both sides of this to obtain -6A = -18, 2A 6B = 0, 8A + B - 6C = 1 . From these, we find A = 3, B = 1 , C = 4. Thus we obtain the particular solution 2 Y = 3x + x + 4. p 170 Chapter 4 The G.S. of the D.E. lS x -2x -3x 2 Y = c 1 e + c 2 e + c 3 e + 3x + x + 4. 17. The corresponding homogeneous D.E. '" " + 3y' - 5y lS Y + Y = o. The auxiliary equation 3 2 + 3m - 5 = O. By lS m + inspection note that m = 1 lS a root. From this (by synthetic division or otherwise) we obtain the factored form (m 2 5) From this, the roots of l)(m + 2m + = o. the auxiliary equation are 1 and -1 % 2i. The x -x complementary function is Yc = c 1 e + e (c 2 sin2x + c 3 cos2x). The NH term lS a linear combination of the UC functions given by sin 2x, x 2 , x, and 1. Step 1 : Form the UC set of each of these four UC 2 functions: Sl = {sin 2x,cos 2x}, S2 = {x ,x, 1}, S3 = {x,l}, S4 = {1}. Step 2 : Sets S3 and S4 are completely included in S2; so 8 3 and 8 4 are omitted, leaving the two sets Sl and S2. Step 3 : An examination of the complementary function shows that none of the functions in Sl or S2 is a solution of the corresponding homogeneous D.E. Hence neither Sl nor S2 needs revision. Step 4 : Thus the original sets Sl and 52 remaln. Ye form a linear combination of their five members. Higher-Order Linear Differential Equations 171 SteD 5 : Thus we take y = A sin 2x + B cos 2x + Cx 2 + Dx + E p as a particular solution. Then Yp , = 2A cos 2x - 2B sin 2x + 2Cx + D, " -4A sin 2x 4B cos 2x + 2C, Yp = III -SA cos 2x + SB sin 2x. Yp = We substitute ln the D. E _ , obtaining (-SA cos 2x + SB sin 2x) + (-4A sin 2x - 4B cos 2x + C) + 3(2A cos 2x - 2B sin 2x + 2Cx + D) - 5(A sin 2x + B cos 2x + Cx 2 + Dx + E) = 5 sin 2x + 10x 2 + 3x + 7 or (-9A + 2B) sin 2x + (-2A - 9B) cos 2x - 5Cx 2 + (6C - 5D)x + (2C + 3D - 5E) - 5 sin 2x + 10x 2 + 3x + 7. We equate coefficients of like terms on both sides of this to obtain -9A + 2B = 5, -2A - 9B = 0, -5C = 10, 6C - 5D = 3, 2C + 3D - 5E = 7. From these, we find A - - ;7 ' B = 2 17 ' C = -2, D = -3, E = -4. Thus we obtain the particular integral y = p 9 sin 2x 17 2 cos 2x 2 2 9x 82 + 17 - x - 1r - 25 - 172 Chapter 4 The G.S. of the D.E. lS x -x y = c 1 e + e (c 2 sin 2x + c 3 cos 2x) 9 sin 2x 17 + 2 cos 2x 2 - 2x - 3x - 4. 17 19. The corresponding homogeneous D.E. The auxiliary equation is m 2 + m - -3. The complementary function is lS y" + y' - 6y = O. 6 = 0, with roots 2, 2x -3x Y c = c 1 e + c 2 e The NH term is a linear combination of the UC functions 2x 3x e , e , and x, and 1. SteD 1 : Form the UC set of each of these four functions: 2x 3x Sl = {e }, S2 = {e }, S3 = {x,l}, S4 = {1}. SteD 2 : Set S4 is completely included in S3; so S4 lS omitted, leaving the three sets Sl' S2' and S3. Step 3 : Observe that the only member e 2x of 8 1 = {e 2x } 1S included ln the complementary function and so is a solution of the corresponding homogeneous D.E. Thus we 2x multiply the member e of Sl by x to obtain the revised set S ' 1 2x = {xe }, whose only member lS not a solution of the homogeneous D.E. 2x SteD 4 : We now have the three sets Sl' = {xe }, 8 2 = {e 3x }, 8 3 = {x,l}. Ve form a linear combination of their four elements. Higher-Order Linear Differential Equations 173 Steo 5 : Thus we take 2x 3x Y p = Axe + Be + Cx + D as a particular solution. Then Y , 2Axe 2x A 2x 3Be 3x + C, = + e + p H 4Axe 2x 4Ae 2x + 9Be 3x , Y p = + We substitute 1n the D.E., obtaining 2x 2x 3x (4Axe + 4Ae + 9Be) + (2Axe 2x + Ae 2x + 3Be 3x + C) - 6(Axe 2x + Be 3x + Cx + D) = 10e 2x - 18e 3x - 6x - 11 5Ae 2x + 6Be 3x - 6Cx + (C - 6D) = 10e 2x - 18e 3x - 6x - 11. or We equate coefficients of like terms on both sides of this to obtain 5A = 10, 6B = -18, -6C = -6, C - 6D = -11. From this, we find A = 2, B = -3, C = 1, D = 2. We thus obtain the particular integral 2x 3x Y = 2xe - 3e + x + 2. p The G.S. of the D.E. 1S 2x -3x 2x 3x Y = c 1 e + c 2 e + 2xe - 3e + x + 2. 174 Chapter 4 22. The corresponding homogeneous D.E. is y" - 4y' + 5y = o. The auxiliary equation is m 2 - 4m + 5 = 0, with roots 2 % 1. The complementary function is 2x (.) Y c = e c 1 Sln x + c 2 cos x. The NH term lS a constant 2x multiple of the UC function glven by e cosx. 2x Steo 1 : Form the UC set of e cos x. 2x 2x . Sl = {e cosx, e slnx}. It lS Steo 2 : This step does not apply. St 3 B h b 2x d 2x. f S eo : ot mem ers e cos x an e SIn x 0 1 are included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of set Sl by x to obtain the revised S { 2x 2x. } h . h h b set 1 = xe co s x, xe Slnx, w lC as no mem ers which are solutions of the homogeneous D.E. S 4 T.T h h S { 2x 2x. } teo :" e now a ve t e set l' = xe co s x, xe s 1 n x , and we form a linear combination of its two members. Steo 5 Thus we take A 2x. B 2x Y = xe Sln x + xe cos x p as a particular solution. Then y , p 2x . 2x = (2A - B)xe Sln x + (A + 2B)xe cos x 2x . 2x +Ae slnx+Be cosx, = (3A - 4B)xe 2x sin x + (4A + 3B)xe 2 xcos x + (4A - 2B)e 2x sin x + (2A + 4B)e 2 xcos x. " yp Higher-Order Linear Differential Equations 175 Ye substitute 1n the D.E., obtaining [(3A - 4B)xe 2x sin x + (4A + 3B)xe 2x cos x + (4A - 2B)e 2x sin x + (2A + 4B)e 2x cos xJ 2x + (A + 2B)xe cos x - 4[(2A - B)xe 2x sin x 2x . 2x + Ae Slnx + Be cosxJ 2x . 2x 2x + 5 [Axe slnx+Bxe cosxJ=6e cosx or - 4B - SA + 4B + 5A)xe 2x sin x 2x -4A-8B+5B)xe cosx - 4A) e 2x sin x + (2A + 4B - 4B) e 2 xcos x or simply (3A + (4A + 3B + (4A - 2B 2x = 6e cos x 2x . 2x -2Ae Sln x + 2Ae cos x = 6e cos x. We equate coefficients of like terms on both sides of this to obtain -2B = 0, 2A = 6. From this, we find A = 3, B = o. Ye thus obtain the particular solution 3 2x. Yp= xe Slnx. The G.S. of the D.E. lS 2x (.) 3 2x. Y = e c 1 s1nx + c 2 cosx + xe Slnx. 176 Chapter 4 24. The corresponding homogeneous D.E. is ym 2yH _ y' + 2y = O. The auxiliary equation lS 322 m - 2m - m + 2 = 0, that is (m - l)(m - 2) = O. From this, the roots are 1, -1, 2. The complementary function x -x 2x lS Yc = c 1 e + c 2 e + c 3 e The NH term is a linear combination of the UC functions given by 2x and e 3x . SteD 1 : Form the UC set of each of these two functions: 2x 3x Sl = {e }, S2 = {e }. Ste o 2: Neither set is identical with nor included ln the other, so both are retained. Ste o 3: Observe that the only member 2x of Sl = {e 2x } e lS included 1n the complementary function, and so lS a solution of the corresponding homogeneous D.E. Thus we multiply the member e 2x of Sl by x to obtain the revised 2x set Sl' = {xe } whose only member lS not a solution of the homogeneous D.E. S u S { 2x } S {e 3x }. teo 4 : we now have the two sets l' = xe , 2 = We form a linear combination of their two elements. 2x 3x Steo 5 : Thus we take y = Axe + Be as a particular p 2x 2x 3x solution. Then y , = 2Axe + Ae + 3Be . p " 4Axe 2x yp = + m SAxe 2x yp = + 4Ae 2x + 9Be 3x , 12Ae 2x + 27Be 3x . Higher-Order Linear Differential Equations 177 Ve substitute 1n the D.E., obtaining (SAxe 2x + 12Ae 2x + 27e 3x) _ 2(4Axe 2x + 4Ae 2x + 9Be 3x) _ (2Axe 2x + Ae 2x + 3Be 3x) + 2(Axe 2x + Be 3x) = ge 2x _ Se 3x or 3Ae 2x + SBe 3x = ge 2x _ 8e 3x . Ve equate coefficients of like terms on both sides of this to obtain 3A = 9, SB = -8, and hence A = 3, B = -1. Thus 2x 3x we obtain the particular integral y = 3xe - e . The p G.S. of the D.E. is x -x 2x 2x 3x y = c 1 e + c 2 e + c 3 e + 3xe - e 25. The corresponding homogeneous D.E. is ym + y' = O. The auxiliary equation is m 3 + m = 0, with roots m = 0, % i. The complementary function is y c = c 1 + c 2 sin x + c 3 cos x. The NH term is a linear combination of the UC functions . b 2 d . glven y x an Sln x. SteD 1 : Form the UC set of each of these two UC functions: 2 Sl = {x ,x,l}, S2 = {sin x, cosx}. SteD 2 : Neither set is identical with nor included ln the other, so both are retained. SteD 3 : Observe that the member 1 of Sl is included in the complementary function (in the c 1 = c 1 · 1 term) and so is a solution of the corresponding homogeneous D.E. Thus we 178 Chapter 4 multiply each member of set Sl by x to obtain the revised 3 2 set Sl' = {x ,x ,x}, which has no members which are members of S2 are included in the complementary function. Thus we multiply each member of S2 by x to obtain the revised set S2' = {x sin x, xcosx}, which has no members which are solutions of the homogeneous D.E. 3 2 Step 4 : Ve now have the two sets S1' = {x ,x ,x}, S2' = {x sin x, xcosx}. We form a linear combination of their five elements. Step 5 : Thus we take yp cos x. Then = Ax 3 + Bx 2 + Cx + Dx sin x + Ex 2 - Ex s in x yp , = 3Ax + 2Bx + C + Dx cos x + D sin x + E cos x, " 6Ax + 2B - Dx sin x - Ex cos x yp = + 2D cos x - 2E sin x, III 6A - Dx cos x + Ex sin x - 3D sin x yp = - 3E cos x. Ve substitute ln the D.E., obtaining (6A - Dx cos x + Ex sin x - 3D sin x - 3E cos x) 2 + (3Ax + 2Bx + C + Dx cos x - Ex sin x + D sin x + E cos x) = 2x 2 + 4 sin x 3Ax 2 + 2Bx + (6A + C) - 2D sin x - 2E cos x = 2x 2 + 4sinx. or Higher-Order Linear Differential Equations 179 "We equate coefficients of like terms on both sides of this to obtain 3A = 2, 2B = 0, 6A + C = 0, -2D = 4, -2E = 0, and hence 2 B 0, C -4 D E o. Thus A = -, = = = -2 = we 3 , , - _ (2 3) X 3 obtain the particular integral yp The G.S. of the D.E. is 4x - 2x sinx. (2 3) X 3 - 4x - 2x S 1 " n X . Y = C 1 + c 2 s1n x + c 3 cos x + 26. The corresponding homogeneous D.E. is ylV ym + 2yH = o. Th . 1 . .. 4 3 3 2 2 0 e aUXl lary equatl0n lS m - m + m = , or m 2 (m - l)(m - 2) = 0, with roots 0,0 (real double root), 1, 2. The complementary function is x 2x Yc = c 1 + c 2 x + c 3 e + c 4 e The NH term is a linear - x 2x combination of the UC functions given bye, e , and x. Step 1 : Form the UC set of each of these three UC -x 2x functions: 51 = {e }, S2 = {e }, 53 = {x,l}. Step 2 : No set lS identical with nor included ln any other, so each lS retained. Step 3 : Observe that the member e 2x of S2 is included in the complementary function and so lS a solution of the corresponding homogeneous D.E. Thus we multiply each 2x member of S2 by x to obtain the revised set S2' = {xe }, whose member lS not a solution of the homogeneous D.E. Next observe that both members x and 1 of 53 are included in the complementary function. Thus we multiply each member of 53 by x 2 to obtain the revised set 180 Chapter 4 8 3 ' = {x 3 ,x 2 }, whose members are not solutions of the homogeneous D.E. (note that multiplication by x, instead of x 2 , is not sufficient here). -x SteD 4 : Ve now have the three sets 51' = {e }, 8 2 ' = {xe 2x }, 8 3 ' = {x 3 ,x 2 }. Ve form a linear combination of their four elements. x 2x 3 2 Step 5 : Thus we take Y p = Ae- + Bxe + Cx + Dx as a particular solution. Then -x + 2Bxe 2x 2x 2 Yp , = -Ae + Be + 3Cx + 2Dx, " -x 4Bxe 2x 4Be 2x + 6Cx + 2D, Yp = Ae + + III -x + SBxe 2x 12Be 2x + 6C, Yp = -Ae + . 2x 32Be 2x . 1V -x yp = Ae + 16Bxe + Ve substitut into the D.E., obtaining -x 2x -x 2x (Ae + 16Bxe + 32Be) - 3(-Ae + SBxe + 12Be 2x + 6C) + 2(Ae- x + 4Be2x + 4Be 2x -x 2x + 6Cx + 2D) = 3e + 6e - 6x or -x 2x 6Ae + 4Be + 12Cx + (-lSC + 4D) -x 2x = 3e + 6e - 6x. Ve equate coefficients of like terms on both sides of this to obtain 6A = 3, 4B = 6, 12C = -6, -18C + 4D = 0, and Higher-Order Linear Differential Equations 181 1 B 3 C 1 D 9 Thus obtain the hence A = 2 ' = 2 ' = - 2 ' = - 4 . we particular integral 1 -x 3 2x 1 3 9 2 y =-e + -xe - -x - -x . p 2 2 2 4 x The G.S. of the D.E. is y = c l + c 2 x + c 3 e 1 -x 3 2x 1 3 9 2 + 2 e + 2 xe - 2 x - 4 x · 2x + c 4 e 27. The corresponding homogeneous D.E. lS III - 6y " lly' - 6y = O. The auxiliary equation y + lS 3 2 11m - 6 = O. By inspection we find that m 1 m 6m + = lS a root. Then by synthetic division, we have (m 1)(m 2 - 5m + 6) = 0 or (m - l)(m - 2)(m - 3) = O. Thus the roots are m = 1, 2, 3. The complementary x 2x 3x function lS Yc = c l e + c 2 e + c 3 e The NH member lS a x 2x 4x linear combination of the UC functions xe , e , and e . SteD 1 : Form the UC set of each of these three UC x 2x 4x functions: Sl = {xe ,e }, S2 = {e }, S3 = {e }. SteD 2 : No set lS identical with nor included ln another, so each lS retained. x SteD 3 : Observe that member e of Sl lS included in the complementary function and so is a solution of the corresponding homogeneous equation. Thus we multiply each member of Sl by x to obtain the revised set S ' { 2 x X } h b t 1 t o f th 1 = x e ,xe , w ose mem ers are no so u 10ns 0 e homogeneous D.E. Similarly, member e 2x of S2 is included in Yc' so we multiply each member of 52 by x to obtain the revised set S2' = {xe 2x }. 182 Chapter 4 Step 4 : Ve now have the two revised sets Sl' = {x 2 e x ,xe x } 2x 4x and 52' = {xe } and also the original set S3 = {e }. We 2 x x 2x 4x form a linear combination Ax e + Bxe + Cxe + De of their four elements. 2 x x 2x 4x Step 5 : Thus we take y = Ax e + Bxe + Cxe + De as a p particular solution. Then 2 x + B)xe x x 2x y , = Ax e + (2A + B + 2Cxe p 2x 4x + Ce + 4De , " 2 x x x Y p = Ax e + (4A + B)xe + (2A + 2B)e + 4Cxe 2x 4Ce 2x + 16De 4x , + ", 2 x x x Y p = Ax e + (6A + B)xe + (6A + 3B)e + SCxe 2x 2x 64De 4x . + 12Ce + We substitute ln the D.E., obtaining 2 x x x 2x [Ax e + (6A + B)xe + (6A + 3B)e + 8Cxe 2x 4x 2 x x x + 12Ce + 64De] - 6[Ax e + (4A + B)xe 2x 2x 4x + (2A + 2B)e + 4Cxe + 4Ce + 16De] + 11 [Ax 2 e x + (2A + B)xe x + Be x + 2Cxe 2x + Ce 2x + 4De 4x] _ 6[Ax 2 e X + Bxe x C 2x D 4x] x 4 2x 6 4x + xe + e = xe e + e or x x 2x 4x 4Axe + (-6A + 2B)e - Ce + 6De x 4 2x 6 4x = xe - e + e . Higher-Order Linear Differential Equations 183 "We equate coefficients of like terms on both sides of this to obtain 4A = 1 , -6A + 2B = 0, -c = -4 6D = 6. From , these equations, we find A 1 B 3 C = 4, D 1 . Thus = 4 ' = 4 ' = we obtain the particular integral 1 2 x 3 x 2x 4x yP = 4 x e + 4 xe + 4xe + e The G.S. of the D.E. is x y = c 1 e 2x + c 2 e 3x + c 3 e 1 2 x + - x e 4 3 x 2x 4x + 4 xe + 4xe + e . 28. The corresponding homogeneous D.E. is ym 4yH + 5y' - 2y = O. The auxiliary equation is m 3 4m 2 + 5m - 2 = O. By inspection we find that m = 1 1S a root. Then by synthetic division, we have (m - 1)(m 2 - 3m + 2) = 0 or (m - 1)2(m - 2) = O. Thus the roots are m = 1, 1 (real double root), and 2. The 1 f .. ( ) x 2x comp ementary unctlon lS Yc = c 1 + c 2 x e + c 3 e The 2 x NH member is a linear combination of the UC functions x e and eX. SteD 1 : Form the UC set of each of these two UC functions: 2x x x x 51 = {x e ,xe ,e }, 52 = {e }. SteD 2 : "We have S2 (Sl' so we discard 52' leaving only Sl. 184 Chapter 4 x x Step 3 : Observe that the members e and xe of S1 are both included in the complementary function and so are solutions of the corresponding homogeneous equation. Thus we multiply each member of 51 by x 2 to obtain the revised 4 x 3 x 2 x set S1' = {x e ,x e ,x e }, whose members are not solutions of the homogeneous D.E. Step 4 : Ve now have the set S1' 4 x 3 x 2 x . = {x e ,x e ,x e }, WhlCh was revised in Step 3. Ve form the linear combination 4 x 3 x 2 x Ax e + Bx e + Cx e of the three elements of 51'. 4 x 3 x 2 x Step 5 : Thus we take y = Ax e + Bx e + Cx e as a p particular integral. Then y , p 4 x 3 x 2 x x = Ax e + (4A + B)x e + (3B + C)x e + 2Cxe , " Yp = Ax 4 e x + (SA + B)x 3 e x + (12A + 6B + C)x 2 e x x x + (6B + 4C)xe + 2Ce , III Yp 4 x 3 x 2 x = Ax e + (12A + B)x e + (36A + 9B + C)x e x x x + (24A + 1SB + 6C)xe + (6B + 6C)e . Ve substitute into the D.E., obtaining [A 4 x (A) 3 x (A B C)x 2 e x x e + 12 + B x e + 36 + 9 + x x 4 x + (24A + 1SB + 6C)xe + (6B + 6C)e] - 4[Ax e 3x 2x) x + (SA + B)xe + (12A + 6B + C)xe + (6B + 4C xe x 4 x 3 x 2 x + 2Ce] + 5[Ax e + (4A + B)xe + (3B + C)xe x 4 x 3 x 2 x + 2Cxe] - 2[Ax e + Bx e + Cx e] 2 x x = 3x e - 7e Higher-Order Linear Differential Equations 185 or 2 x x x -12Ax e + (24A - 6B)xe + (6B - 2C)e 2 x x = 3x e - 7e . We equate coefficients of like terms on both sides of this to obtain -12A = 3, 24A - 6B = 0, 6B 2C = -7. From these equations, we find A = -1/4, B = -1, C = 1/2. Thus we obtain the particular integral 1 4 x yP = - 4 x e 3 x - x e 1 2 x + 2 x e · The G.S. of the D.E. lS x 2x Y = (c 1 + c 2 x) e + c 3 e 1 4 x - x e 4 3 x e 1 2 x + 2 x e · 31. The corresponding homogeneous D.E. is yH + Y = O. The auxiliary equation is m 2 + 1 = 0, with roots *i. The complementary function is y c = c 1 Sln x + c 2 cos x. The NH term is the UC function x sin x. SteD 1 : Form the UC set of this UC function. It lS Sl = {x sin x, xcosx, sin x, cosx}. SteD 2 : This step does not apply. SteD 3 : Observe that the members sin x and cos x of Sl are included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of Sl by x to obtain the revised set S ' { 2. 2 } h b 1 = x slnx, x cosx, xSlnx, xcosx W ose mem ers are not solutions of the homogeneous D.E. 186 Chapter 4 Step 4 : Ve now have the set Sl'. We form a linear b . . f . f 1 2 . 2 . com lnatlon 0 ltS our e ements x Sln x, x cos x, x Sln x, and x cos x. Step 5 : Thus we take A 2 . B 2 C . D y = XSlnx+ xcosx+ XSlnx+ xcosx p as a particular solution. Then y , - Ax 2 cos x - Bx 2 sin x + (2A - D)x sin x p + (2B + C)x cos x + C sin x + D cos x and = -Ax 2 sinx - Bx 2 cosx + (-4B - C)xsinx " Yp + (4A - D)x cos x + (2A - 2D)sin x + (2B + 2C) cos x. Ve substitute into the D.E., obtaining -4Bx sin x + 4Ax cos x + (2A - 2D) sin x + (2B + 2C)cosx = XSlnx. Ve equate coefficients of like terms on both sides of this to obtain -4B = 1, 4A = 0, 2A - 2D = 0, 2B + C = O. From this, A = 0, B = - , C = , D = O. Thus we obtain the particular integral yp = _ x2cosx + xsinx. The G.S. of the D.E. is . 1 2 1. y = c 1 Sln x + c 2 cos x - 4 x cos x + 4 x Sln x · Higher-Order Linear Differential Equations 187 33. Th d " h D E " 1 V 2 III 3 II e correspon lng omogeneous . . lS Y + Y - Y = Th " I " "" 4 2 3 3 2 0 e aUXl lary equatlon lS m + m - m = or 2 m (m - l)(m + 3) = 0, with roots 0, 0 (real double root), 1, -3. The complementary function is x -3x Yc = c 1 + c 2 x + c 3 e + c 4 e The NH term is a linear " 2 x 3x combination of the UC functions glven by x , xe , e , and 1. Ste p 1: Form the UC set of each of these four UC functions: Sl 2 S - {xe x x = {x ,x,l}, ,e }, 2 - 3x S4 {1}. S3 = {e }, = Step 2 : Set S4 is completely included in Sl' so S4 lS omitted, leaving the three sets Sl' S2' 53. Step 3 : Observe that members x and 1 of 51 are included III the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of Sl by x 2 to obtain the revised set Sl' = 432 {x ,x ,x } , whose members are not solutions of the homogeneous D.E. (Note that multiplication by x, instead 2 of x , lS not sufficient here.) Next observe that the member eX of 52 is included in the complementary function. Thus we multiply each member of 52 by x to obtain the 2 x x revised set S2' = {x e ,xe }, whose members are not solutions of the homogeneous D.E. Step 4 : e now have the three sets Sl' = {x 4 ,x 3 ,x 2 } , 2 x x { 3x S2 = {x e ,xe }, S3 = e }. We form the linear 188 Chapter 4 combination x 4 + Bx 3 + Cx 2 + Dx 2 e x + Exe x + Fe 3x of their six elements. SteD 5 : Thus we take 4 3 2 2 x x 3x y = Ax + Bx + Cx + Dx e + Exe + Fe . p Then y , 4Ax 3 2 2 x = + 3Bx + 2Cx + Dx e p + (2D + E)xe x x 3Fe 3x , + Ee + " 2 D 2 x Yp = 12Ax + 6Bx + 2C + x e + (4D + E)xe x (2D + x 9Fe 3x , + 2E)e + III 24Ax + 6B + D 2 x (6D + E)xe x Yp = x e + x 27Fe 3x , + (6D + 3E)e + lV 2 x E)xe x Yp = 24A + Dx e + (8D + x 81Fe 3x . + (12D + 4E)e + We substitute into the D.E., obtaining 2 x x 24A + Dx e + (8D + E)xe + 3x + 81Fe + 2[24Ax + 6B + x 3x + (6D + 3E)e + 27Fe] 2 x x + Dx e + (4D + E)xe + 2 x 3x = 18x + 16xe + 4e - x (12D + 4E)e 2 x x Dx e + (6D + E)xe 2 - 3[12Ax + 6Bx + 2C x 3x (2D + 2E)e + 9Fe] 9 or -36Ax 2 + (48A - 18B)x + (24A + 12B - 6C) x x 3x + 8Dxe + (18D + 4E)e + 108Fe = 18x 2 + 16xe x + 4e 3x - 9. Higher-Order Linear Differential Equations 189 We equate coefficients of like terms on both sides of this to obtain -36A = 18, 48A - 18B = 0, 24A + 12B - 6C = -9 , 8D = 16, 18D + 4E = 0, 108F - 4. From this, we find A = - 1/2, B = -4/3, C = -19/6, D = 2, E = -9, F = 1/27. Thus we obtain the particular integral 1 4 4 3 19 2 2 x yp = - x x -x + 2x e 2 3 6 - 9xe x 1 3x + 27 e The G.S. of the D.E. is x -3x y = c 1 + c 2 x + c 3 e + c 4 e 1 4 - - x 2 4 3 - - x 3 19 2 2 x x 1 3x - lf x + 2x e - 9xe + 27 e 34. The corresponding homogeneous D.E. lS 1V 5 III 7" 5y' + 6y = O. The auxiliary equation y - y + y - lS 4 3 2 - 5m + 6 = O. By inspection find that m m - 5m + 7m we = 2 is a root. Then by synthetic division, we have (m - 2)(m 3 - 3m 2 + m - 3) = 0 or (m - 2)(m - 3)(m 2 + 1) = o. Thus the roots are m = 2, 3, % i. The complementary f .. 2x 3x Th unctlonlsYc=c1e +c 2 e +c 3 s1nx+c 4 cosx. e NH term is a linear combination of the UC functions glven by sin x and sin 2x. SteD 1 : Form the UC set of each of these two UC functions: S1 = {sin x, cosx}, S2 = {sin2x, cos2x}. SteD 2 : Neither set lS identical with nor included ln the other, so both are retained. 190 Chapter 4 Step 3 : Observe that both members of S1 are included in the complementary function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of Sl by x to obtain the revised set Sl' = {x sin x, x cos x}, whose members are not solutions of the homogeneous D.E. Step 4 : Ye now have the two sets S1' = {x sin x, x cos x}, S2 = {sin 2x, cos 2x}. Ye form a linear combination of their four elements. Step 5 : Thus we take y = Ax sin x + Bx cos x + C sin 2x p + D cos 2x as a particular solution. Then y , = p Ax cos x - Bx sin x + A sin x + B cos x + 2Ccos2x - 2Dsin2x, y" = -Ax sin x - Bxcosp p + 2A cos x - 2B sin x - 4C sin 2x - 4D cos 2x, Y III = -Ax cos x + Bx sin x - 3A sin x - 3B cos x p - 8Ccos2x + 8Dsin2x, y lV = Axsinx + Bxcosx p - 4A cos x + 4B sin x + 16C sin 2x + 16D cos 2x. Ye 9ubstitute into the D.E., obtaining (Ax sin x + Bx cos x - 4A cos x + 4B sin x + 16C sin 2x + 16Dcos2x) - 5(-Axcosx + Bxsinx - 3Asinx - 3Bcosx - 8Ccos2x + 8Dsin2x) + 7(-Axsinx - Bxcosx + 2Acosx - 2Bsinx - 4Csin2x - 4Dcos2x) - 5(Axcosx - Bxsinx + Asinx + Bcosx + 2Ccos2x - 2Dsin2x) + 6(Axsinx + Bx cos x + C sin 2x + D cos 2x) = 5 sin x - 12 sin 2x or (lOA - lOB) sin x + (lOA + lOB) cos x + (-6C - 30D) sin 2x + (30C - 6D) cos 2x = 5 sin x - 12 sin 2x. Ye equate coefficients of like terms Higher-Order Linear Differential Equations 191 on both sides of this to obtain lOA - lOB = 5, lOA + lOB = 0, -6C - 30D = -12, 30C - 6D = O. The equations in A and Bare equivalent to 2A - 2B = 1 , A + B 0, from which A = 1 B 1 The = 4' = 4. equations ln C and D are equivalent to C + 5D = 2, 5C - D = 0, from which C = 1 5 Thus we obtain the 13' D = 13 . particular integral 1 1 1 . 2 yP = 4 x Sln x - 4 x cos x + 13 Sln x + 5 13 cos 2x. The G. S. of the D. E. is 2x Y = c 1 e 3x + c 2 e + c 3 s1nx + c 4 cosx 1. 1 1 . 2 + 4 x Sln x - 4 x cos x + 13 Sln x. 5 + 13 cos 2x. 37. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 8m + 15 = 0, with roots 3, 5. The 3x 5x complementary function lS yc = c 1 e + c 2 e The UC set 2x 2x 2x of UC function xe lS Sl = {xe ,e }. This does not 2x 2x need revision, so we take y = Axe + Be as a p . l . 1 T A 2x (A 2B)e 2x d partlcu ar lntegra. hen y , = 2 xe + + an p y H = 4Axe 2x + (4A + 4B)e 2x . We substitute in the D.E. p obtaining 4Axe 2x + (4A + 4B)e 2x - 8[2Axe 2x + (A + 2B)e 2x] 2x 2x 2x + 15(Axe + Be = 9xe or 2x 2x 2x 3Axe + (-4A + 3B)e = 9xe . 192 Chapter 4 We equate coefficients of like terms on both sides of this to obtain 3A = 9, -4A + 3B = o. From these, we find A = 3, B = 4. Thus we obtain the particular integral 2x 2x y = 3xe + 4e . The G.S. of the D.E. is p 2x 5x 2x 2x Y = c 1 e + c 2 e + 3xe + 4e · We apply the I.C. y(O) = 5 to this. We find c 1 + c 2 + 4 = 5 or c 1 + c 2 = 1. (*) We next differentiate the G.S. 3x 5x 2x 2x to obtain y' = 3c 1 e + 5c 2 e + 6xe + l1e . Ye apply the I.C. y'(O) = 10 to this. We find 3c 1 + 5c 2 + 11 = 10 or 3c 1 + 5c 2 - -1. (**) From (*) and (**) we find c 1 - 3, c 2 = -2. Thus we obtain the particular solution 3 3x 2 5x 3 2x 4 2x Y = e - e + xe + e . 38. The corresponding homogeneous equation is yH + 7y' + 10y = 0; and the auxiliary equation lS m 2 + 7m + 10 = 0, with roots m = -2, -5. Thus the -2x -5x complementary function lS Yc = c 1 e + c 2 e The UC -3x -3x -3x set of UC function 4xe is {xe ,e }. Ye assume a -3x -3x particular integral y = Axe + Be . Then p -3x -3x y , = -3Axe + (A - 3B)e , p -3x -3x ypH = 9Axe + (-6A + 9B)e . Substituting ln the glven D.E., we obtain -3x -3x -3x [9Axe + (-6A + 9B)e] + 7[-3Axe -3x -3x -3x -3x + (A - 3B)e] + 10[Axe + Be] = 4xe . Higher-Order Linear Differential Equations 193 Simplifying, we find -3x -3x -3x -2Axe - (A + 2B)e = 4xe . From this, we have the equations -2A = 4, A - 2B = 0, with the solution A = -2, B = -1. Thus we find the particular -3x -3x integral y = -2xe - e and the general solution p -2x -5x -3x -3x y = c 1 e + c 2 e - 2xe - e We now apply the initial condition y(O) - 0 to this. Ye have c 1 + c 2 = 1. Differentiating the general solution, we find -2x y' = -2c e 1 -5x -3x -3x 5c 2 e + 6xe + e · Applying the initial condition y/(O) = -1 to this, we find 2c 1 + 5c 2 = -2. The solution of the two equations ln c 1 and c 2 lS c 1 = 1, c 2 = O. Thus the solution is -2x -3x -3x Y = e - 2xe - e 194 Chapter 4 39. The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 8m + 16 = 0 or (m + 4)2 = 0, with double real root -4. Thus the complementary function is -4x -2x . y = (C + C 2 x)e . The UC set of UC function 8e lS C 1 {e- 2x }. -2x Ye assume the particular integral y = Ae . p -2x H -2x Then yp' = -2Ae , yp = 4Ae . Substituting into the given D.E., we find 4Ae- 2x _ 16Ae- 2x + 16Ae- 2x = 8e- 2x 4Ae -2x -2x or = 8e . From this 4A = 8, and so A = 2. -2x we obtain the particular integral y = 2e and the general p -4x -2x general solution y = (c 1 + c 2 x)e + 2e . Thus Ye apply the initial condition y(O) = 2 to this, obtaining c 1 = o. Ye differentiate the general solution to obtain y' -2x -4x -2x = (-4c 1 + c 2 - 4c 2 x e) e - 4e . From this, (-4c 1 + c 2) - 4 = o. Since c 1 = 0, this gives c 2 = 4. Thus we obtain -4x -2x y = 4xe + 2e . 40. The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 6m + 9 = 0, with double real root -3. The -3x complementary function is y = (c 1 + c 2 x)e . The UC set -6x -6x of the UC function 27e is {e}. We assume the Higher-Order Linear Differential Equations 195 -6x -6x particular integral y = Ae . Then y I = -6Ae , p p Y p . = 36Ae- 6x . Substituting in the given D.E., we obtain 36Ae- 6x - 36Ae- 6x + 9Ae- 6x = 27e- 6x -6x or 9A = 27e . From this, 9A = 27, and so A = 3. Thus we obtain the particular integral y = 3e- 6x and the p -3x -6x general solution y = (c 1 + c 2 x)e + 3e . Now apply the I.C. y(O) = -2 to the general solution. Ye obtain c 1 + 3 = -2 so c l = -5. Differentiate the general , solution to obtain y' = (-3c 1 - 3c 2 x)e -3x 18e- 6x . = + c 2 - Apply the I.C. y'(O) = 0 to this. Ye obtain -3c 1 + c 2 - 18 = O. Since c l = -5, we find c 2 = 3. Thus -3x -6x we obtain the solution y = (3x - 5)e + 3e . 42. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 10m + 29 = 0, with roots 5 2i. The complementary function is Yc = e 5X (c l sin 2x + c 2 cos 2x). 5x . 5x The UC set of the UC function e lS Sl = {e }. This d d k _ - Ae 5x oes nt nee revlslon, so we ta e y as a p 5x 5x particular integral. Then y , = 5Ae and y H = 25Ae . p p Ye substitute into the D.E., obtaining 25Ae 5x 10(5Ae 5x) + 29Ae 5x = 8e 5x 5x 8 5x He at once see that A 2 and hence or 4Ae = e . = 196 Chapter 4 obtain the particular integral y = 2e 5x . The G.S. of the p D.E. is 5x . 5x y = e (c 1 s1n 2x + c 2 cos 2x) + 2e . Ye apply the I.C. y(O) = 0 to this. We find c 2 + 2 = 0, so c 2 = -2. Ye next differentiate the G.S. to obtain 5x. 5x y' = e [(5c 1 - 2c 2) Sln 2x + (2c 1 + 5c 2) cos 2x] + 10e · We apply the I.C. y'(O) = 8 to this. We find 2c 1 + 5c 2 + 10 = 8 or 2c 1 + 5c 2 = -2. Since c 2 = -2, this glves c 1 = 4. Thus we obtain the particular solution y = 2e 5x (2 sin 2x - cos 2x + 1). 44. The auxiliary equation of the corresponding homogeneous D.E. . 2 - m - 6 = 0 (m - 3)(m + 2) = 0, with roots lS m or - - 3, -2. The complementary function 3x -2x lS Y = c 1 e + c 2 e c The UC sets of the UC functions in the right member of the 2x 3x D.E. are Sl 2x 3x = {e }, S2 = {e }. Neither is completely contained ln the other so both are retained. The member 3x e of S2 lS contained ln the complementary function and so lS a solution of the corresponding homogeneous D.E. Thus we multiply the member of S2 by x to obtain the 3x revised set S2' = {xe }, whose member lS not a solution of the homogeneous D.E. Higher-Order Linear Differential Equations 197 2x , 3x Now we have the two sets Sl = {e }, S2' = {xe }. We take a linear combination of their two members as a 2x 3x particular integral. That is, we take y = Ae + Bxe . p 2x 3x 3x " 2x 3x Then y , = 2Ae + Be + 3Bxe and y = 4Ae + 6Be p p 3x + 9Bxe . We substitute into the D.E., obtaining 4Ae 2x + 6Be 3x + 9Bxe 3x _ (2Ae 2x + Be 3x + 3Bxe 3x) _ 6(Ae 2x + Bxe 3x) = Se 2x _ 5e 3x _4Ae 2x + 5Be 3x = Se 2x _ 5e 3x . or We equate coefficients of like terms on both sides of this to obtain -4A = S, 5B = -5. From these, we find A = -2, B 2x = -1. Thus we obtain the particular integral y = -2e - p 3x xe The G.S. of the D.E. is 3x -2x 2x 3x Y = c 1 e + c 2 e - 2e - xe Ve apply the I.C. y(O) = 3 to this. We find c 1 + c - 2 = 3 or c 1 + c 2 = 5 (*) . We next 2 differentiate the G.S. to obtain y' 3c 1 e 3x 2c 2 e -2x = _ 4e 2x - 3xe 3x 3x We apply the I.C. y' (0) = 5 to - e this. We find 3c 1 - 2c 2 - 4 - 1 = 5 or 3c 1 - 2c 2 = 10 (**) . 198 Chapter 4 From the two equations (*) and (**) in the unknowns c 1 and c 2 , we find that c 1 = 4, c 2 = 1. Thus we obtain the 4 3x -2x 2 2x 3x particular solution y = e + e - e - xe . 45. The auxiliary equation of the corresponding homogeneous D.E. is m 2 - 2m + 1 = 0 or (m - 1)2 = 0, with roots 1, 1 (double root). The complementary function is x x Yc = c 1 e + c 2 xe. The UC sets of the UC function ln the right member of the D.E. are Sl 2x 2x S2 x = {xe ,e }, S2 = {e }. Neither is completely contained in the other, so both are retained. The member x of S2 is included in the e complementary function and so is a solution of the corresponding homogeneous D.E. Thus we multiply the member of 52 by x2 to obtain the revised set 52' = {x 2 e x }, whose member lS not a solution of the homogeneous D.E. (note that multiplication by x, instead of x 2 , is not sufficient here). 2x 2x Now we have the two sets Sl = {xe ,e }, 52' = {x 2 e x }. We take a linear combination of their three members as a particular integral. That is, we take 2x 2x 2 x yp = Axe + Be + Cx e. Then yp' = 2Axe + (A + 4A the 2B)e 2x + Cx 2 e x 2x 2 x + 4B)e + Cx e D. E., obtaining x d H A 2x + 2Cxe an y = 4 xe + p + 4Cxe x + 2Ce x . We substitute into Higher-Order Linear Differential Equations 199 2x 2x 2 x x 4Axe + (4A + 4B)e + Cx e + 4Cxe + 2Ce 2x 2x 2 x x - 2[2Axe + (A + 2B)e + Cx e + 2Cxe] + Axe 2x + Be 2x + Cx 2 e x - A 2x (A) 2x C x xe + 2 + B e + 2 e = 2x X 2xe + 6e 2x x 2xe + 6e . or We equate coefficients of like terms on both sides of this to obtain A = 2, 2A + B = 0, 2C = 6. From these, we find A = 2, B = -4 C = 3. Thus we obtain the particular , integral 2xe 2x 2x 2 x The G.S. of the D.E. yp = 4e + 3x e . x x 2x 2x 2 x lS y = c 1 e + c 2 xe + - 4e + 3x e . We apply the I.C. y(O) = 1 to this. We find c 1 - 4 = 1 or c 1 = 5. We next differentiate the G.S. to obtain y' x x x x + 4xe 2x 2x 8e 2x = c e + c 2 e + c 2 xe + 2e - 1 2 x + 6xe x We apply the I.C. y' (0) = 1 to this. We + 3x e + . find c 1 + c 2 + 2 - 8 = 0 or c 1 + c 2 = 6. Since c = 5, 1 this glves c 2 = 1. Thus we obtain the particular solution 5e x x 2 2x 4 2x 3 2 x Y = + xe + xe + ( ) x 2 x 2 4 2x = x + 5 e + 3x e + xe - e . 47. The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 1 - 0, with roots *i. The complementary function is y c = c 1 sin x + c 2 cos x. The UC sets of the UC functions in the right member of the D.E. are Sl = {x 2 ,x,1}, S2 = {sin x, cosx}. Neither is completely contained in the other, so both are retained. The members sin x and cos x of 52 are included in the complementary 200 Chapter 4 function and so are solutions of the corresponding homogeneous D.E. Thus we multiply each member of S2 by x to obtain the revised set S2' = {x sin x, x cos x}, whose members are not solutions of the homogeneous D.E. Now we have the two sets Sl = {x,x,1}, S2' = {x sin x, xcosx}. We take a linear combination of their five members as a particular integral. That 1S, we take y = Ax 2 + Bx + C + Dx sin x + Excosx. then p yP , = 2Ax + B + Dx cos x + D sin x Ex sin x + E cos x and " Yp substitute into the D.E., obtaining = 2A - Dxsinx + 2Dcosx - Excosx - 2Esinx. We 2A - Dx sin x + 2D cos x - Ex cos x - 2E sin x + Ax 2 + Bx + C + Dx sin x + Ex cos x = 3x 2 4 sin x Ax 2 + Bx + (2A + C) + 2D cos x - 2E sin x = 3x 2 - 4sinx. or We equate coefficients of like terms on both sides of this to obtain A = 3, B From these we find Thus we obtain the cosx. The G.S. of the D.E. is y = c l sinx + c 2 cosx + 3x 2 - 6 + 2x cos x. = 0, 2A + C = 0, 2D = 0, -2E = -4. A = 3, B = 0, C = -6 D = 0, E = 2. , particular integral 3x 2 6 2x yp = - + Higher-Order Linear Differential Equations 201 We apply the I.C. y(O) = 0 to this. We find c 2 - 6 = 0 or c = 6. We next differentiate the G.S. to obtain 2 y' = c 1 cos x - c 2 sin x + 6x - 2x sin x + 2cosx. We apply the I.C" y'(O) = 1 to this. We find c 1 + 2 = 1 or c 1 = -1. Thus we obtain the particular solution y = 6 cos x - sin x + 3x 2 - 6 + 2x cos x. 48. The auxiliary equation of the corresponding homogeneous D.E. is m 2 + 4 = 0, with pure imaginary conjugate complex roots :2i. Thus the complementary function lS y C = c 1 sin 2x + c 2 cos 2x. The UC set of the UC function 8 sin 2x is S = {sin 2x, cos 2x}. Since each member of this set is a solution of the corresponding homogeneous equation, we multiply each by x and replace S by S' = {x sin 2x, x cos 2x}. We then form the particular integral y = Axsin2x + Bxcos2x. Then p y , = -2Bx sin 2x + 2Ax cos 2x + A sin 2x + B cos 2x, and p yp" = -4Ax sin 2x 4Bx cos 2x 4B sin 2x + 4A cos 2x. Substituting in the given D.E., we obtain (-4Ax sin 2x - 4Bx cos 2x - 4B sin 2x + 4A cos 2x) + (4Ax sin 2x +

which to determine v 1 '(x) and v 2 '(x): sln x v 1 '(x) + cos x v 2' (x) = 0, cos x v 1 '(x) - Sln x v 2' (x) = sec x . Solving, we find Higher-Order Linear Differential Equations 217 0 cos x sec x -Sln x v 1 '(x) = cos x 1 . = = = -1 Sln x cos x cos x -Sln x sln x 0 cos x sec x, Slnxsecx -tan x. v 2 = = = -1 Sln x cos x cos x -sec x Integrating, we find v 1 (x) = x, v 2 (x) = Inl cos xl. Substituting into yp(x) = v 1 (x)sinx + v 2 (x) cosx, we find y (x) = x sin x + cos x [lnl cos x IJ. The G.S. of the D.E. p 1S y = c 1 Sln x + c 2 cos x + x Sln x + cos x [In cos x I]. 6. The complementary function 1S defined by y c (x) = c 1 sin x + c 2 cos x. We assume a particular integral of the form y p (x) = v 1 (x) sin x + v 2 (x) cos x. (1) Then yp'(x) = v 1 (x)cosx - v 2 (x)sinx + v 1 '(x)sinx + v 2 ' (x)cos x. We impose the condition v 1 '(x)sinx + v 2 '(x)cosx = O. (2) 218 Chapter 4 leaving Yp' (x) = v 1 (x)cos x - v 2 (x)sin x. Then from this Yp" (x) = -v 1 (x)sin x - v 2 (x)cos x + v 1 '(x)cos x - v 2 '(x)sin x. (3) Substituting (1) and (3) into the given D.E., we obtain v 1 '(x)cos x - v 2 '(x)sin x = t an x s e C x. (4) We now have conditions (2) and (4) from which to determine v 1 '(x) and v 2 '(x): { sin x cos x v 1 ' (x) + cos x v 1 '(x) Slnx v 2' (x) = 0 v 2' (x) = t an x S e C x. Solving, we find 0 cos x v 1 '(x) tan x sec x -Sln x = -1 = tan 2 x x -Sln x -tan 2 1 2 = x = - see x. Higher-Order Linear Differential Equations 219 Integrating, we find vi (x) = -In I cos x I, v 2 (x) = x - tan x . Substituting into y p (x) = vi (x) sin x + v 2 (x) cos x, we find y p (x) = -sin x In I cos x I + x cos x - sin x. The G.S. of the D.E. 1S y = c 1 sin x + c 2 cos x - sin x In I cos x I + x cos x - sin x. This may be more simply written as y = cOsln x + c 2 cos x - sin x In I cos x I + x cos x, where Co = c 1 - 1. 7. The auxiliary equation 2 + 4m + 5 = 0 has the conjugate m complex roots -4 :i: i, so the complementary function is -2x -2x y c (x) = c 1 e sin x + c 2 e cos x. We assume a particular integral of the form -2x -2x y p (x) = v 1 (x)e sin x + v 2 (x)e cos x. (1) -2x -2x Then yp' (x) = vi (x)e cos x - 2v 1 (x)e sin x -2x -2x -2x + v 1 '(x)e sinx - v 2 (x)e sinx - 2v 2 (x)e cosx -2x + v 2 ' (x)e cos x. We impose the condition -2x -2x v 1 '(x)e sin x + v 2 '(x)e cos x = 0, (2) leaving ... differentiating this and collecting like terms, we find y "(x) p -2x -2x = 3v 1 (x)e sin x - 4v 1 (x)e cos x -2x -2x + vi' (x) [e cos x - 2e sin x] -2x -2x + 3v 2 (x)e cos x + 4v 2 (x)e sin x -2x -2x - v 2 '(x) [e sin x + 2e cos x] ... sin x -2x -2x + 2e cos xJ = e sec x. (5) We now have conditions (2) and (5) from which to determine vi' (x) and v 2 '(x): -2x . e Sln x vi' (x) + -2x e cos x v 2 ' (x) = 0, -2x -2x e (cos x - 2sin x) vi' (x) ... (x) = -2x . e Sln x -2x e cos x -2x e (cosx - 2sinx) -2x . e (-sln x - 2 cos x) = -4x -e cos x see x e- 4x (_1) = 1. -2x . e Sln x o -2x e (cosx - 2sinx) -2x e sec x v 2 '(x) = -2x . e Sln x -2x e cos x ... 1nto y p x =v i xe slnx+v 2 xe cosx, we find y (x) = xe- 2x sinx + [lnlcosxlJ e- 2 xeosx. The p G.S. of the D.E. 1S -2x . -2x . Y = e (c i s1nx + c 2 cosx) + xe Slnx I I -2x + [In cos x] e eos x ... Yc(x) = c 1 e + c 2 xe. We assume a particular integral of the form x x yp(x) = v 1 (x)e + v 2 (x)xe . (1) Then Y'p(x) = v l (x)e x + v 2 (x) [(x + l)e x] + v l '(x)e x + v 2 '(x)xe x . We impose ... [(x + l)e x J. (4) Then substituting (1), (3), and (4) into the glven D.E., we obtain x x x v 1 (x)e + v 2 (x) [(x + 2)e J + v 1 '(x)e x x + v 2 '(x)[(x + l)e J - 2{v 1 (x)e x X) x + v 2 (x) [(x + l)e J ... (5) from which to determine v 1 '(x) and v 2 '(x): x x v 1 '(x)e + v 2 '(x)xe = 0, x x x v l' (x) e + v 2' (x) [(x + 1) e J = xe In x. Dividing out eX f 0, these simplify to v 1 '(x) + xv 2 '(x) = 0, v ... tables or integration by parts), we find v 1 (x) 1 3 1 3 = -x 3 x In x 9 v 2 (x) 1 2 1 2 = 2 x ln x -x 4 224 Chapter 4 Substituting into Yp(x) = v l (x)e x + v 2 (x)xe x , we find Yp = (x 3 - ... complementary function 1S defined by y c (x) = c 1 sin x + c 2 cos x. We assume a particular integral of the form y p (x) = v 1 (x) sin x + v 2 (x) cos x. (1) Then y p' (x) = vi (x) cos x - v ... -vi (x) sin x - v 2 (x) cos x + vi' (x) cos x - v 2' (x) sin x. (3) Substituting (1) and (3) into the glven D.E., we obtain 3 vi' (x) co s x - v 2' (x) s in x = t an x. (4) We now have conditions (2) and (4) ... Solving, we find 0 cos x 3 3 tan x -Sln x v 1 '(x) -tan x cos x = = -1 Sln x cos x cos x -Sln x _ _ (1 - cos 2 x) sin x _ _ [( ) -2 1J 2 cosx - Slnx, cos x Sln x 0 3 3 . 4 cos x tan x v 2' (x) tan x Sln ... tables), we find v l (x) = (cosx)-l + cosx, v 2 (x) = 1 2 tan x sec x + Inlsecx + tanxl - sinx. 226 Chapter 4 Substituting into y (x) p = vi (x)sin x + v 2 (x)cos x, we find -1 Y (x) = [(cos x) - ... + 2 cos x In sec x + tan x . 13. 2 The auxiliary equation m + 3m + 2 = 0 has roots -1, -2, -x -2x so the complementary function is y (x) = c e + C 2 e c 1 We assume a particular integral ... 0 (2) leaving y ,(x) p -x -2x = -v 1 (x)e - 2v 2 (x)e · (3) From this, Higher-Order Linear Differential Equations 227 y "(x) p -x -2x -x = v 1 (x)e + 4v 2 (x)e - v 1 '(x)e -2x - 2v 2 '(x)e (4) Subs... = 1 1 x + e or -x -2x -v 1 '(x)e - 2v 2 '(x)e 1 (5) = x 1 + e We now have conditions (2) and (5) from which to obtain v 1 '(x) and v 2 '(x): -x -2x e v 1 '(x) + e v 2 '(x) = 0, -x -2x -e V 1 '(x) - 2 ... + e e e -x -2e -2x -e Integrating, we find v 1 (x) x = In(1 + e) v 2 (x) = f _e 2x x dx = f [eX X - ex] dx 1 + e 1 + e x x = In(1 + e) e -x -2x Substituting into Yp(x) = v 1 (x)e + v 2 (x)e , we ... Differential Equations 229 or -x -2x -x -2x x Y = c 3 e + c 2 e + (e + e)[In(l + e)], where c 3 = c 1 - 1. 15. The complementary function 1S defined by y c (x) = c 1 sin x + c 2 cos x. We a... condition v l' (x) sin x + v 2' (x) cos x = 0, (2) leaving yp' (x) = v 1 (x)cos x - v 2 (x) sin x. Then from this yp" (x) = -v 1 (x)sin x - v 2 (x) cos x + v l' (x) cos x - v 2' (x) sin x. (3) Substituting ... 230 Chapter 4 sin x v 1 '(x) + cosx v 2 '(x) = 0, cosx v 1 '(x) - sin x v 2 '(x) = 1 1 + Sln x Solving, we find o cos x 1 1 + Sln x -Sln x v 1 '(x) = COS x = 1 + sin x ' Sln x COS x COS x -Sln x ... COS x = -x - 1 + Sln x Substituting into Yp(x) = v 1 (x)sinx + v 2 (x)cosx, we find y (x) p 2 cos x = sin x [In(1 + sin x)] - x cos x - 1 + Sln x Higher-Order Linear Differential Equations 2... root), so the complementary function is Yc(x) = (c 1 + c 2 x)e x , which we rewrite slightly as x x y c (x) = c 1 e + c 2 x e. We assume a particular integral of the form y (x) p x x - v 1 (x) ... (x)e x + v 2 (x) [(x + l)e x J. (3) Then Y " (x) ( ) x ( ) [(2) X] ( ) x p = v 1 x e + v 2 x x + e + v 1 ' x e x + v 2 '(x)[(x + l)e J. (4) 232 Chapter 4 Then substituting (1), (3), and (4) into the ...

y" = Ln(n n=2 obtain n-2 l)c x . Substituting into the D.E., we n Series Solutions of Linear Differential Equations 311 CD CD 2 n-2 n-2 x n(n - l)c n x + n(n - l)c n x n=2 n=2 CD CD + x
fourth summations so that x has the exponent n in each. We have: CD CD Ln(n - l)C n x n + L (n + 2) (n + 1)C n + 2 x n n=2 n=O CD CD + Lncnx n + Lc n _ l x n = o. n=l n=l The comm
)x CD + L {(n n=2 n + c n _ 1 }x + 2)(n + l)c 2 + [n(n - 1) + nJc n+ n = O. 312 Chapter 6 Equating to zero the coefficients of each power of x, we obtain 2c 2 = 0, Co + c 1 + 6c 3 = 0,
> 2. Using this, we find c 4 = - 4c 2 + c 1 c 1 = -. 12 12' 9c 3 + c 2 (io) (cO + c 1) · Substituting these c =- = 5 20 values into the assumed solution, we have y = Co + c 1 x - 3 (cO +
y' = ncnx , n=l CD y" = Ln(n n=2 obtain n-2 l)c x . Substituting into the D.E., we n Series Solutions of Linear Differential Equations 313 00 00 3 n-2 n-2 x n(n - l)c n x - n(n - l)c n x n=2
second summation so that x has the exponent n + 1 in it (since this lS the exponent of x 1n all the other summations). Thus we have: 00 n+l n(n - l)c n x - n=2 00 L n=-l n+l (n + 3)(n +
do not belong to this range. Thus we have: 314 Chapter 6 2 -2c 2 + (cO - 6c 3)x + (2c 1 - 12c 4)x CD + L {-en + 3) (n + 2)C n + 3 n=2 + [n(n - 1) + n + 1]c }x n = O. n Equating to ze
c 2 = 0, c 3 find Co c 1 = 6' c 4 - 6. From (2), we c n+3 2 (n + 1)c n (n + 2)(n + 3)' n > 2. Using this, we find c 5 5c 2 = 20 = 0; c 6 - 10c 3 30 = c 3 3 Co = -- 18' 17c 4 and c 7 = 42 17c
...) + c (X + x 4 + 17x 7 + ...) . 6 18 1 6 252 Series Solutions of Linear Differential Equations 315 CD 15. \Ie assume y = L cnx n · n=O CD Then y' = L nc n x n - l , n=l CD y" = Ln(n
L (-cn)x n = o. n=2 n=l n=O We rewrite the first summation so that x has an exponent n. Thus we have: CD CD CD LCn+ l)(n + 2)c n + 2 x n + L(-ncn)x n + L(-cn)x n - o. n=O n=l n=O
CD + [en + 1) (n + 2)c 2 + (-n - l)c Jxn = O. n+ n n=l Equating the coefficients of each power of x to zero we obtain 316 Chapter 6 2c 2 - Co = 0, (1) (n + l)(n + 2)c 2 + (-n - l)c = 0,
c 1 ; c 6 the D.E. is 1 = 6 c 4 = 1 48 cO. Thus the G.S. of y = C O (l 1 2 + - x 2 1 4 + - x 8 1 6 + 48 x + ...J + ...J. (1 3 1 5 + c 1 x + 3 x + 15 x We now apply the I.C. y(O) = 1 to this, obta
solution 1 2 Y = 1 + - x 2 1 4 + - x 8 1 6 + 48 x + ... Writing this as 1 1 2 1 4 1 6 y = + x + x + x + . . . 2 1 1! 22 2 ! 2 3 3! and observing that for n > 1, c 2n = 2 · 4 · 6 ... = (2n) 1 , 2 n ,
l , n=l m y" = L,n(n n=2 obtain n-2 l)c x . Substituting into the D.E., we n m m 2 n-2 n-2 x n(n - l)c n x + n(n - l)c n x n=2 n=2 m m n-l n + xncnx + 2xcnx = 0 n=l n=O or m m L,n(n n=
Thus we have: 318 Chapter 6 CD CD Ln(n - l)C n x n + L (n + 2) (n + 1)C n + 2 x n n=2 n=O CD CD + Lncnx n + L2Cn_1Xn = O. n=l n=l The common range of these four summations
(n + 2)(n + 1)c n + 2 + 2c n _ 1 }x = O. Equating to zero the coefficients of each power of x, we obtain 2c 2 = 0, 2c O + c 1 + 6c 3 = 0, (1) [n(n - 1) + nJc + (n + 2)(n + l)c 2 n n+ + 2c
12 c 1 = - 6' c - 5 - 9c 3 + 2c 2 20 = 3(2c O + c 1) 40 Substituting these values into the assumed solution, we have Series Solutions of Linear Differential Equations 319 y = Co + c 1 x
this, obtaining Co = 2. Differentiating we find y' Co (_X2 3x 4 .. .) = + -+ 4 c 1 (1 2 2x 3 3x 4 ...). x + - - - -+ -+ 2 3 8 Applying the I.C. y'(O) = 3 to this, we have c 1 = 3. Thus we have t
L,cnxn. n=O CD n-l Then y' = ncnx , n=l CD y" = Ln(n n=2 obtain n-2 l)c x . Substituting into the D.E., we n CD CD 2 n-2 n-2 x n(n - l)c n x - n(n - l)c n x n=2 n=2 CD CD + 4x Lnc n x
that x has the exponent n, as ln the other three summations. We have: CD CD Ln(n - l)c n x n - L (n + 2) (n + 1)C n + 2 x n n=2 n=O CD CD + 4Lnc n x n + 2Lc n x n = O. n=l n=O The
Differential Equations 321 (-2c 2 + 2c O) + (-6c 3 + 6c 1)x CD + LHn(n - 1) + 4n + 2Jc n n=2 (n + 2)(n + l)c 2 }xn = O. n+ Noting that n(n - 1) + 4n + 2 = (n + l)(n + 2), we equate to
Co and c 3 = c 1 . From (2), we obtain c 2 = c , n > 2. From this we observe that n+ n C 2n = Co and c 2n + 1 = c 1 for all n 1. Thus we obtain the G.S. C O (l + 2 + x 4 + ...) + c l (x +

whose roots are 1 = 1, 2 = 5. Setting = = 1 ln (1) , we obtain 2A + B = 0, 1 4A + 2B = 0, a nontrivial solution of which lS A = 1 , B = -2. With these values of A, B, and , we find the nontrivial solution t t x = e , y = -2e . (2) Setting = 2 = 5 in (1), we obtain -2A + B = 0, 4A 2B = 0, a nontrivial solution of which is A = 1, B = 2. With these values of A, B, and , we find the nontrivial solution 5t x = e y = 2e 5t . (3) 410 Chapter 7 The G.S. of the glven system is a linear combination of (2) and (3). Thus we find t x = c 1 e 5t + c 2 e t Y = -2c e 1 5t + 2c 2 e 6. At We assume a solution of the form x = Ae , y = Substituting into the given system, we obtain BeAt. AAe At = 6Ae At B At e , BAe At = 3Ae At + 2Be At , which leads at once to the algebraic system { (6 - A)A - B = 0, 3A + (2 - A)B = o. (1) For nontrivial solutions of this, we must have 6 - A -1 = o. 3 2 - A This leads at once to the characteristic equation A 2 - 8A + 15 = 0, whose roots are A 1 = 5, A 2 = 3. Setting A = A 1 = 5 in (1), we obtain A - B = 0, 3A - 3B = 0, a nontrivial solution of which is A = 1, B = 1. With these values of A, B, and A, we find the nontrivial solution 5t 5t x = e y = e (2) Systems of Linear Differential Equations 411 Setting = 2 = 3 in (1), we obtain 3A - B = 0 (twice), a nontrivial solution of which is A = 1, B = 3. With these values of A, B, and , we find the nontrivial solution 3t x = e y - 3e 3t . (3) The G.S. of the glven system lS a linear combination of (2) and (3). Thus we find 5t x = c 1 e 3t + c 2 e 5t Y = c 1 e 3t + 3c 2 e 7. We assume a solution of the form x = Aet, y = Bet. Substituting into the given system, we obtain Aet = 3Aet 4Bet, Bet = 2Aet 3Be At , which leads at once to the algebraic system { (3 - A) A - 4B = 0, 2A + (-3 -)B = o. (1) For nontrivial solutions of this, we must have 3 - -4 - o. - -3 - A 2 This leads to the characteristic equation A 2 - 1 = 0, whose roots are Ai = 1, "2 = -1. 412 Chapter 7 Setting = 1 = 1 in (1), we obtain 2A - 4B = 0 (twice), a nontrivial solution of which is A = 2, B = 1. With these values of A, B, and , we find the nontrivial solution t t x = 2e , y = e . (2) Setting = 2 = -1 in (1), we obtain 4A - 4B = 0, 2A - 2B = 0, a nontrivial solution of which is A = B = 1. With these values of A, B, and , we find the nontrivial solution -t -t x = e y = e (3) The G.S. of the glven system lS a linear combination of (2) and (3). Thus we find t x = 2c 1 e -t t + c 2 e , y = c 1 e -t + c 2 e 8. . t t We assume a Solutl0n of the form x = Ae , y = Be . Substituting into the given system, we obtain Aet = 2Aet B t e , B ' e t t t " = 9Ae + 2Be , which leads at once to the algebraic system { (2 -)A - B = 0, 9A + (2 -)B = o. (1) Systems of Linear Differential Equations 413 For nontrivial solutions of this, we must have 2 - " -1 = o. 92- " This leads to the characteristic equation ,,2 - 4" + 13 = 0, whose roots are" = 2 3i. Letting" = 2 + 3i in (1), we obtain -3iA - B = 0, 9A - 3iB = 0, a nontrivial solution of which is A = 1, B = -3i. With these values of A, B, and ", we find the complex solution _ (2+3i)t - e , - 3 - (2+3i)t y - - le . Using Euler's formula this takes the form x = e 2t (cos 3t + i sin 3t) , y = e 2t (3 sin 3t - 3i cos 3t). Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions 2t cos 3t 2t sin 3t x = e x - e - and 3 2t - 3 2t y = e Sln t y = -3e cos 3t 414 Chapter 7 The G.S. of the system lS thus 2t x = e (c 1 cos 3t + c 2 sin 3t) , y = e 2t (3c l sin 3t - 3c 2 cos 3t) . 11. We assume a solution of the form x = Aet, y = Bet. Substituting into the glven system, we obtain Aet = Aet - 4Bet, B ' e t Ae t At " = + Be , which leads at once to the algebraic system { (1 -)A - 4B = 0, A + (1 -)B = o. (1) For nontrivial solutions of this, we must have 1 - -4 = o. 1 1 - This leads to the characteristic equation 2 - 2A + 5 = 0, whose roots are 1 2i. Letting = 1 + 2i in (1), we obtain -2iA - 4B = 0, A - 2iB = 0, a nontrivial solution of which is A = 2i, B = 1. With these values of A, B, and , we find the complex solution Systems of Linear Differential Equations 415 x = 2. (1+2i)t, le y (1+2i)t. = e Using Euler's formula this takes the form x = 2iet(cos2t + i sin 2t) , t i sin 2t) . y = e (cos 2t + Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions -2e t sin 2t 2e 2t cos 2t x = x = and t co s 2t . t sin 2t y = e y = e The G.S. of the system lS thus x = 2e t (-c l sin 2t + c 2 cos 2t) , t c 2 sin 2t) · Y = e (c 1 cos 2t + 15. At We assume a solution of the form x = Ae , y = Substituting into the given system, we obtain BeAt. AAe At = 4Ae At - 2Be At , BAe At = 5Ae At + 2Be At , which leads at once to the algebraic system { (4 - A)A - 2B = 0, 5A + (2 - A)B = o. (1) 416 Chapter 7 For nontrivial solutions of this, we must have 4 - " -2 = o. 52- " This leads to the characteristic equation ,,2 - 6" + 18 = 0, whose roots are" = 3 3i. Letting " = 3 + 3i ln (1) , we obtain (1 - 3i)A - 2B = 0, 5A - (1 + 3i)B = 0, a nontrivial solution of which lS A = 2, B = 1 - 3i. With these values of A, B, and ", we find the complex solution x = 2e(3+3i)t, y = (1 - 3i)e(3+3i)t. Using Euler's formula this takes the form x = 2e 3t (cos 3t + i sin 3t) , y = (1 - 3i) e 3t (cos 3t + i sin 3t) . Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions 3t x = 2e cos 3t 3t + 3 sin 3t) , Y = e (cos 3t and 2 3t . 3 x = e Sln t 3t (. 3 - 3 cos 3t) . Y = e Sln t Systems of Linear Differential Equations 417 The G.S. of the system lS thus x = 2e 3t (c l cos 3t + c 2 sin 3t), y = e 3t [c l (COS3t + 3sin3t) + c 2 (sin3t - 3cos3t)J. 17. Ve assume a solution of the form x = Aet, y = Bet. Substituting into the given system, we obtain Aet = 3Aet 2Be At , B ' e '\t '\t At A = 2Ae + 3Be , which leads at once to the algebraic system { (3 - '\)A - 2B = 0, 2A + (3 - '\)B = o. (1) For nontrivial solutions, we must have 3 - ,\ -2 = O. 2 3 - ,\ This leads to the characteristic equation ,\2 - 6,\ + 13 = 0, whose roots are ,\ = 3 % 2i. Letting'\ = 3 + 2i in (1), we obtain -2iA - 2B = 0, 2A 2iB = 0, a nontrivial solution of which is A = i, B = 1. Vith these values of A, B, and '\, we find the complex solution 418 Chapter 7 x . (3+2i)t = le , y (3+2i)t = e . Using Euler's formula this takes the form . 3t (2 .. 2) x = le cos t + 1 sln t , y = e 3t (cos 2t + i sin 2t) . Both the real and imaginary parts of this are solutions. Thus we obtain the two linearly independent solutions 3t sin 2t 3t cos 2t x = -e x = e and 3t co s 2t , 3t sin 2t y = e y = e The G.S. of the system lS thus 3t (. 2 + c 2 co s 2t) , x = e -c 1 Sln t 3t c 2 sin2t). y = e (c 1 cos 2t + 18. At Ve assume a solution of the form x = Ae , y = Substituting into the given system, we obtain BeAt. AAe At = 6Ae At _ 5Be At , BeAt = Ae At + 2Be At , which leads at once to the algebraic system { (6 - A)A - 5B = 0, A + (2 - A)B = o. (1) Systems of Linear Differential Equations 419 For nontrivial solution of this, we must have 6 - -5 = o. 1 2 - This leads to the characteristic equation A 2 - 8 + 17 = 0, whose roots are = 4 % i. Letting = 4 + 1 ln (1) , we obtain (2 - i)A - 5B = 0, A - (2 + i)B = 0, a nontrivial solution of which is A = -5 B = 1 - 2. Vith these values of A, B, and , we find , the complex solution x = _5e(4+i)t , y = (i - 2)e(4+i)t. Using Euler's formula this takes the form 5 4t (. .) x = - e cos t + 1 Sln t , (.) 4t (. .) y = 1 - 2 e cos t + 1 Sln t . Both the real and imaginary'parts of this are solutions. Thus we obtain the two linearly independent solutions 4t 5 4t . x = -5e co s t , x = - e Sln t and 4t sin t) 4t 2sint). y = e (-2 cos t - Y = e (cos t - 420 Chapter 7 The G.S. of the system lS thus x = -5e 4t (c l cos t + c 2 sin t), y = e 4t [(c 2 - 2c l)cost - (c l + 2c 2)sintJ. 19. Ve assume a solution of the At Bet. form x = Ae , y = Substituting into the glven system, we obtain Aet 3Aet B At = e , Bet 4Aet B t = e , which leads at once to the algebraic system r 3 -)A - B = 0 (1) 4A + (-1 -)B = o. For nontrivial solutions of this, we must have 3 - -1 = o. 4 -1 - This leads to the characteristic equation 2 - 2 + 1 = 0, whose roots are 1, 1 (real and equal). Setting = 1 in (1), we obtain 2A - B = 0, 4A - 2B = 0, a nontrivial solution of which is A = 1, B = 2. Vith these values of A, B, and , we find the nontrivial solution t t x = e , y = 2e . (2) Systems of Linear Differential Equations 421 Ye now seek a second solution of the form x = t t (A 1 t + A 2)e , y = (B 1 t + B 2)e. Substituting these into the glven system, we obtain (A 1 t + A 2 + t 3(A 1 t t (B 1 t t A 1)e - + A 2)e + B 2)e , - t t t (B 1 t + B 2 + B 1)e = 4(A 1 t + A 2)e - (B 1 t + B 2)e · These equations reduce to (2A 1 - B 1)t + (2A 2 - A 1 - B 2) = 0, (4A 1 - 2B 1)t + (4A 2 - B 1 - 2B 2) = o. Thus we must have 2A 1 - B = 0, 2A - A 1 - B = 0, 1 2 2 4A 1 - 2B = 0, 4A 2 B 1 2B 2 = o. 1 A simple nontrivial solution of these is A 1 = 1, B 1 = 2, A 2 = 0, B 2 = -1. This leads to the solution x = te t , y = (2t - l)e t . (3) The G.S. of the glven system lS a linear combination of (2) and (3) . Thus we

[illegible]

(**) are linearly independent, and a G.S. lS -t 2e 2t 0 e -t 0 3e 2t x = c 1 e + c 2 + c 3 -t 2t 2t -e -e -e where c 1 , c 2 , c 3 are arbitrary constants. In component form, this is -t 2c 2 e 2t xl = c 1 e + , -t 3c 3 e 2t x 2 = c 1 e + , -t 2t 2t x 3 = -c e + c 2 e c 3 e 1 16. We assume a solution of the form x At At At xl = ale , x 2 = a 2 e , x 3 = a 3 e - At = a e , that is, Ye know that A must Systems of Linear Differential Equations 529 be a solution of the characteristic equation of the coefficient matrix 1 A = 0 o 9 19 9 8 18 10 . The characteristic equation lS 1 - ,\ 9 9 0 19 ,\ 18 = o. 0 9 10 - ,\ Expanding the determinant and simplifying, it reduces to ,\3 _ 30,\2 + 57,\ - 28 = 0 or (,\ - 1)2(,\ - 28) = o. Its roots are the characteristic vectors '\1 = 28, '\2 = '\3 = 1. We use A a - ,\ a to find the corresponding characteri s- tic vectors. With,\ = 28 this is 1 9 9 a 1 a 1 0 19 18 a 2 = 28 a 2 0 9 10 a 3 a 3 . Thus ai' a 2 , a 3 must satisfy -27a + 9a 2 + 9a 3 = 0, 1 -9a + 18a 3 = 0, 2 9a 2 18a 3 = O. 530 Chapter 7 A nontrivial solution of this is a 1 = 1, a 2 = 2, a 3 = 1. Thus a characteristic vector corresponding to A = 28 is 1 a - 2 1 and a corresponding solution lS 1 2 e 28t 1 28t e that lS, 2e 28t 28t e (*) With A = 1 , Aa = Aa is 1 9 9 a 1 a 1 0 19 18 a 2 = 1 a 2 0 9 10 a 3 a 3 . Thus a 1 , a 2 , a 3 must satisfy 9a 2 + 9a 3 = 0, 18a 2 + 18a 3 = 0, 9a 2 + 9a 3 = o. Each of these three relationships lS equivalent to the one relation a 2 + a 3 = 0, Systems of Linear Differential Equations 531 and this the only relationship which a 1 , a 2 , a 3 must satisfy. Note that it allows a 1 to be arbitrary. Two linearly independent solutions of it are a 1 = 1, a 2 = a 3 = 0, and a 1 = 0, a 2 = 1, a 3 = -1. That is, corresponding to the double characteristic value A = 1, we have the two linearly independent characteristic vectors 1 o o and o 1 -1 . Respective corresponding solutions are 1 0 0 t and 1 t e , e , 0 -1 that lS, t 0 e 0 and t (**) e 0 t -e The three solutions glven by (*) and (**) are linearly independent, and a G.S. lS 28t t 0 e e 2e 28t 0 t x = c 1 + c 2 + c 3 e 28t 0 t e -e 532 Chapter 7 where c 1 , c 2 , c 3 are arbitrary constants. In component form, this is 28t t xl = c 1 e + c 2 e , 2c 1 e 28t t x 2 = + c 3 e , 28t t x 3 = c 1 e c 3 e . 17. We assume a solution of the form x At At At xl = ale x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the - At = a e , that is, We know that A must coefficient matrix -5 -12 6 A = 1 5 -1 -7 -10 8 . The characteristic equation lS -5 - A -12 6 1 5 - A -1 = o. -7 -10 8 - A This reduces to A 3 - 8A 2 + 19A - 12 = 0, that is, (A l)(A - 3)(A - 4) = O. Its roots are A 1 = 1, A 2 = 3, A3 = 4. We use A a = A a to find the corresponding characteri s- tic vectors. With A = 1 this is Systems of Linear Differential Equations 533 -5 -12 6 a 1 a 1 1 5 -1 a 2 = (1) a 2 -7 -10 8 a 3 a 3 Thus a 1 , a 2 , a 3 must satisfy -6a 12a 2 + 6a 3 = 0, 1 a 1 + 4a 2 a 3 = 0, -7a 10a 2 + 7a 3 = o. 1 A nontrivial solution of this is a 1 = 1, a 2 = 0, a 3 = 1. Thus a characteristic vector corresponding to A = 1 1S 1 a = 0 1 and a corresponding solution is t 1 e 0 t that 0 e , lS, 1 t e (*) With A = 3, Aa = Aa lS -5 -12 6 a 1 a 1 1 5 -1 a 2 = 3 a 2 -7 -10 8 a 3 a 3 . 534 Chapter 7 Thus ai' a 2 , a 3 must satisfy -8a 12a 2 + 6a 3 = 0, 1 a 1 + 2a 2 a 3 = 0, -7a 10a 2 + 5a 3 - O. 1 - A nontrivial solution of this is a 1 = 0, a 2 = 1, a 3 = 2. Thus a characteristic vector corresponding to A = 3 lS o a = 1 2 and a corresponding solution lS o o 1 e 3t 2 that lS, 3t e (**) 2e 3t · With A = 4, Aa - Aa lS - -5 -12 6 a 1 a 1 1 5 -1 a 2 = 4 a 2 -7 -10 8 a 3 a 3 . Thus ai' a 2 , a 3 must satisfy Systems of Linear Differential Equations 535 -9a 12a 2 + 6a 3 = 0, 1 a 1 + a 2 a 3 = 0, -7a 10a 2 + 4a 3 = O. 1 A nontrivial solution of this is a 1 = 2, a 2 = -1, a 3 = 1. Thus a characteristic vector corresponding to A = 4 is 2 a - -1 1 and a corresponding solution lS 2 2e 4t -1 4t that 4t e lS, -e 1 4t . e (***) By Theorem 7.16, the solutions (*), (**), and (**) are linearly independent, and a G.S. is t 0 2e 4t e 0 3t 4t x = c 1 + c 2 e + c 3 -e t 2e 3t 4t e e where c 1 , c 2 , c 3 are arbitrary constants. In component form, this is 536 Chapter 7 t 4t xl = c 1 e + 2c 3 e 3t x 2 = c 2 e t x 3 = c 1 e 4t - c 3 e 3t + 2c 2 e 4t + c 3 e 4t 18. We assume a solution of the form x t t t xl = ale x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the - '\t = a e , t hat is, We know that ,\ must coefficient matrix -5 -3 -3 A = 8 5 7 -2 -1 -3 . The characteristic equation 15 -5 - ,\ -3 -3 8 5 - 7 = o. -2 -1 -3 - ,\ Expanding the determinant and simplifying, it reduces to 3 2 (,\ 1)('\ + 2)2 Its roots ,\ + 3 - 4 = 0, or - = o. are the characteristic values '\1 = 1, '\2 = '\3 = -2. We use A a = ,\ a to find the corresponding characteri s- tic vectors. With,\ = 1 this is -5 -3 -3 a 1 a 1 8 5 7 a 2 = (1) a 2 -2 -1 -3 a 3 a 3 . Systems of Linear Differential Equations 537 Thus ai' a 2 , a':) must satisfy v -6a 3a 2 3a 3 = 0, 1 8a 1 + 4a 2 + 7a 3 = 0, -2a a 2 4a 3 = o. 1 Adding the second equation to four times the third equation, we find -9a 3 = 0, from which a 3 = O. With a 3 = 0, each equation is equivalent to a 3 = -2a 1 . Hence a nontrivial solution of the algebraic system lS a i = 1, a 2 = -2, a 3 = o. Thus a characteristic vector corresponding to ,\ = 1 lS 1 a = -2 0 , and a corresponding solution lS 1 t e t -2 e, that lS, t -2e (*) o o With ,\ = -2 Ali = '\a is , -5 -3 -3 a 1 a i 8 5 7 a 2 = -2 a 2 -2 -1 -3 a 3 a 3 . 538 Chapter 7 Thus a 1 , a 2 , a 3 must satisfy -3a 3a 2 3a 3 = 0, 1 8a 1 + 7a 2 + 7a 3 = 0, -2a a 2 a 3 = o. 1 Adding the first equation to (-3) times the third, we find 3a 1 = 0, from which a 1 = o. With a 1 = 0, each equation lS equivalent to a 3 = -a 2 . Hence a nontrivial solution of the algebraic system is a 1 = 0, a 2 = 1, a 3 = -1. Thus a characteristic vector corresponding to = -2 is o a = 1 -1 and a corresponding solution lS o o -2t 1 e that lS, -2t e (**) -1 -2t . -e A second solution corresponding to = -2 is of the - - -2t - - - - - form (at + p)e , where p satisfies (A + 2I)P = a. This lS -3 -3 -3 {31 0 8 7 7 {32 = 1 -2 -1 -1 {33 -1 . Systems of Linear Differential Equations 539 Multiplying this out yields -38 3{32 3{33 = 0, , 1 8{31 + 7{32 + 7{33 = 1 , -2/3 /3 2 {33 = -1. 1 a solution of which lS {31 = 1, {32 = -1, {33 = O. Thus we obtain 1 13 = -1 0 and the second solution (at 13)e -2t + lS -2t 0 1 e 1 -1 -2t that (t - l)e -2t + e lS, -1 0 -2t -te (***) The three solutions (*), (**), and (***) are linearly independent. Thus a G.S. is t 0 -2t e e t -2t (t l)e -2t x = c 1 -2e + c 2 e + c 3 - 0 -2t -2t -2e -te where c 1 , c 2 , c 3 are arbitrary constants. In component form, this is 540 Chapter 7 t -2t xl = c 1 e + c 3 e -2c e t -2t c 3 (t l)e -2t x 2 = + c 2 e + - , 1 -2c e -2t c 3 te -2t x 3 = . 2 22. Ve assume a solution of the form x t t t xl = ale x 2 = a 2 e x 3 = a 3 e be a solution of the characteristic equation of the - t = a e , that is, We know that must coefficient matrix 3 -2 -1 A = -4 2 4 5 -3 -3 . The characteristic equation is 3 - -2 -1 -4 2 - 4 = o. 5 -3 -3 - It reduces to 3 - 2A 2 = o. Its roots are the characteristic values 1 = 2, 2 = 3 = O. We use A a = a to find the corresponding characteris- tic vectors. With = 2 this is 3 -2 -1 a 1 a 1 -4 2 4 a 2 = 2 a 2 5 -3 -3 a 3 a 3 . Systems of Linear Differential Equations 541 Thus ai' a 2 , a 3 must satisfy a 1 - 2a 2 a 3 = 0, 4a 1 + 4a 3 = 0, 5a 1 - 3a 2 5a 3 = o. A nontrivial solution of this is a 1 = 1, a 2 = 0, a 3 = 1. Thus a characteristic vector corresponding to = 2 lS 1 a = 0 1 and a corresponding solution lS 2t 1 e 0 2t that 0 e lS, 1 2t e (*) We now consider the repeated characteristic value 2 = '\3 = o. With = 0, A a = a becomes 3 -2 -1 a 1 a 1 -4 2 4 a 2 = 0 a 2 5 -3 -3 a 3 a 3 . 542 Chapter 7 Thus ai' a 2 , a 3 must satisfy 3a 1 2a 2 a 3 = 0, -4a + 2a 2 + 4a 3 = 0, 1 5a 1 3a 2 3a 3 = O. Adding the first two equations, we obtain -ai' + 3a 3 = O. 3 From this a 1 = 3a 3 . Then 2a 2 - 3a 1 - a 3 - 8a 3 , so a 2 = - 4a 3 . Thus a simple nontrivial solution lS a 1 = 3, a 2 = 4, a 3 = 1. A characteristic vector corresponding to = 0 lS 3 a = 4 1 and the corresponding solution lS 3 3 4 Ot that 4 (**) e lS, 1 1 .

of (1), it becomes s2 y (s) - 7 + 3sY(s) + 2Y(s). By Table 9.1, number 4, the right member becomes ;os . Thus (1) becomes s + 1 (s2 + 3s + 2)Y(s) - 7 = 10s s2 + 1 Step 2. Solving for yes), we obtain yes) = 7 10s 2 2 s (s + 3s + 2)(s + 1) + s2 + 3s + 2 Step 3. Ye must now determine -1{ 7 10s +l] y = [, 2 + 2 2 s + 3s + 2 s (s + 3s + 2)(s -1{ 2 1}} · 7s + 10s + 7 = [, 2 (s + 1)(s + 2)(s + Ve first employ partial fractions. Ye have 7s 2 + 10s + 7 (s + 1)(s + 2)(s2 + 1) A B = s + 1 + s + 2 Cs + D + s2 + 1 Then 7s 2 + 10s + 7 - A(s + 2)(s2 + 1) + B(s + 1)(s2 + 1) + (Cs + D)(s + 1)(s + 2). Letting s = -1, we have 2A = 4, so A = 2; and letting s = -2, gives -5B = 15, so B = -3. Letting s = 0, we obtain 2A + B + 2D = 7. Yith A = 2, B = -3, this gives D = 3. Finally, letting s = 1, we find 6A + 6B + 6(C + D) = 24. Yith A = 2, B = -3, D = 3, this gives C - 1. Thus we have 702 Chapter 9 7s 2 + 10s + 7 (s + 1)(s + 2)(s2 + 1) = 2 s + 1 3 s + 3 2 + 2 s + S + 1 Thus y = 2£-1{ S 1 } - 3£-1{ S 2 } + £-1{ S2 : 1 } + 3.c- 1 { 21 } . s + 1 Then uSlng Table 9.1, numbers 2, 2, 4, and 3, we have -t -2t Y = 2e - 3e + cos t + 3 sin t. 20. Step 1. Taking the Laplace Transform of both sides of the D .E., we have .c{yH} + 5.c{y'} + 4.c{y} = 6.c{te- t } + S.c{e- t }. (1) Denoting .c{y} by yes) and applying Theorem 9.4 we have .c{yH} = s2 y (s) - sy(O) - y'(O), l{y'} = sY(s) - y(O). Applying the I.C.'s, these become .c{y"} - .c{y'} = s2 yes) - s - 1, sY(s) - 1. Substituting in the left member of (1), it becomes 2 2 s yes) - s - 1 + 5sY(s) - 5 + 4Y(s) or (s + 5s + 4)Y(s) - s - 6. Using Table 9.1, numbers 8 and 2, the right member of (1) becomes 6 8 l ' Thus (1) becomes (s + 1)2 s + (S2 + 5s + 4)Y(s) - s - 6 = 6 + (s + 1)2 8 s + 1. The Laplace Transform 703 Step 2. Solving for yes), we obtain yes) = s + 6 8 + (s2 + 5s + 4)(s + 1) 6 2 s + 5s + 4 + (s2 + 5s + 4)(s + 1)2 Adding fractions and observing that s2 + 5s + 4 = (s + l)(s + 4), we find yes) = s3 + 8s 2 + 21s + 20 (s + 1)3(s + 4) Step 3. Thus we must determine y = £-1 { 8 3 + 88 2 ; 218 + 20 } . (s + 1) (s + 4) (2) Ve employ partial fractions, writing s3 + 8s 2 + 21s + 20 (s + 1)3(s + 4) A B 1 + 2 s + (s + 1)2 + (s + 1)3 + s + 4. 322 Then s + 8s + 21s + 20 = A(s + 1) (s + 4) 3 + B(s + l)(s + 4) + C(s + 4) + D(s + 1) . Letting s = -1, we find 3C = 6, so C = 2; and letting s = -4, we find D = o. Then letting s = 0, we obtain 4A + 4B + 4C + D = 20. Yith C = 2, D = 0, this glves A + B-3. Finally, letting s = 1, we obtain 20A + lOB + 5C + 8D = 50, which, with C = 2 and D = 0, reduces to 2A + B = 4. From the two equations in A and B, we find A = 1, B = 2. Thus 704 Chapter 9 3 2 s + 8s + 21s + 20 3 (s + 1) (s + 4) 2 + 3. (s + 1) = 1 2 1 + 2 s + (s + 1) (3) Alternatively, an astute observer might have noticed that 322 s + 8s + 21s + 20 = (s + 4s2 + 4s + 5)(s + 4) and reduced the fraction in (2) to 2 s + 4s + 5 3 · (s + 1) Applying partial fractions to this, one writes 2 s + 4s + 5 3 (s + 1) ABC = s + 1 + (s + 1)2 + (s + 1)3' Then s 2 + 4s + 5 = A(s + 1)2 + B(s + 1) + C 2 (2A + B)s + (A + B + C). From this, A = 1, = As + 2A + B = 4, A + B + C = 5, and we find A = 1, B = 2, C = 2. So . obtain (3) , and this time with an we aga1n easier use of partial fractions. Therefore y = [l{ s 1 } + 2[l Ls +11)2 } + £ Ls +2 1)3 }' By Table 9.1, numbers 2, 8, and 8, respectively, we find t t 2 -t 2 -t Y = e- + 2te- + t e = (t + 1) e . 21. Step 1. Taking the Laplace Transform of both sides of the D . E., we have £{ym} _ 5£{yH} + 7£{y'} - 3£{y} = £{20 sin t}. (1) The Laplace Transform 705 Denoting £{y(t)} by yes) and applying Theorem 9.4, we have the following expressions for £{yffl}, £{yH}, and £{y'}: £{y"'} 3 2 - y"(O), = s yes) s y(O) - sy' (0) £{y"} 2 sy(O) - y'(O), = s yes) - sy(O) - y'(O), £{y'} = sY(s) - y(O). Applying the I.C.'s, they become £{yffl} 3 2, = s yes) - s, £{y"} 2 = s yes), £{y'} = sY(s). Substituting these expressions into the left member of (1), this left member becomes s3 y (s) + 2- 5s 2 y(s) + 7sY(s) - 3Y(s) or 3 2 (s - 5s + 7s - 3)Y(s) + 2. By Table 9.1, number 3, the right member of (1) becomes 20 2 s + 1 Thus (1) reduces to 3 2 (s - 5s + 7s - 3)Y(s) + 2 20 = 2 s. + 1 Step 2. Solving the preceding for yes), we have yes) = 2 18 - 2s 322 (s - 5s + 7s - 3)(s + 1) or yes) = -2(8 2 - 9) (s - 1)2(8 - 3)(8 2 + 1) 706 Chapter 9 or finally yes) = -2 (s + 3) (s - 1)2(s2 + 1) Step 3. Ve must now determine (t) - .c- 1 { -2s - 6 y - 2 2 (s - 1) (s + 1) } · Ve employ partial fractions. Ve have -2s - 6 A = + 2 2 s - 1 (s - 1) (s + 1) (s B 1)2 Cs + D + s2 + 1 or -2s - 6 = A(s - 1)(s2 + 1) + B(s2 + 1) + (Cs + D)(s - 1)2 or -2s - 6 = (A + C)s3 + (-A + B - 2C + D)s2 + (A + C - 2D)s + (-A + B + D). From this, we obtain { A + C = 0, A + C - 2D = -2, -A + B - 2C + D = 0, -A + B + D = -6. The first and third of these give C = -A, D = 1. second and fourth reduce to A + B = -1, -A + B = Then the -7 , respectively, from which A = 3, B = -4. Hence A = 3, B = -4, C = -3, D = 1; and we have -1 { -2s - 6 } = 3.c-1 { 1 } .c 2 2 s - 1 (s - 1) (s + 1) - 4.c- 1 { 1 2 } - 3.c- 1 { 2 s } (s - 1) s + 1 -1 { 1 } + .c 2 . s + 1 The Laplace Transform 707 Then by Table 9.1, numbers 2, 8, 4, and 3, respectively, we find y = 3e t - 4te t - 3cost + sint. Section 9.41, Page 527. 1 . The glven function is 5u 6 (t). -6s 5e Using formula (9.75), we have £{5u 6 (t)} = s 4. We may express the values of f in the form { 2 - 0 f(t) = ' 2 - 2 , o < t < 5, t > 5. Thus f(t) can be expressed as 2 - 2u 5 (t). Then uSlng formulas (9.2) and (9.75) we find £{f(t)} 2 2e- Ss = 2£{1} - 2£{u 5 (t)} = s - s -Ss = 2(1 - e) s 6. We may express the values of f in the form 0 + 0, 0 < t < 3, f(t) = -6 + 0, 3 < t < 9, -6 + 6, t > 9. Thus f can be expressed as -6u 3 (t) + 6u 9 (t). Then uSlng formulas (9.75), we find 708 Chapter 9 £{f(t)} = -6£{u 3 (t)} + 6£{u 9 (t)} -3s -9s -6e 6e = + s s 6 -9s -3s = . (e - e). s 7. We may express the values of f in the form f(t) = 1 + 0 + 0 - 0, 0 < t < 2, 1 + 1 + 0 - 0, 2 < t < 4, 1 + 1 + 1 - 0, 4 < t < 6, 1 + 1 + 1 - 3, t > 6. Thus f(t) can be expressed as 1 + u 2 (t) + u 4 (t) - 3u 6 (t). Then using formulas (9.2) and (9.75) we find £{f(t)} = £{1} + £{u 2 (t)} + £{u 4 (t)} - 3£{u 6 (t)} 1 -2s -4s 3e -6s e e = - + + s s s s 9. We express the values of f in the form 2 - o + 0, 0 < t < 3, f(t) = 2 2 + 0, 3 < t < 6, 2 - 2 + 2, t > 6. Thus f can be expressed as 2 - 2u 3 (t) + 2u 6 (t). Then uSlng formulas (9.2) and (9.75), we find The Laplace Transform 709 L{f(t)} = 2£{1} - 2£{u 3 (t)} + 2£{u 6 (t)} 2 -3s 2e -6s 2e = + s s s = 2 (1 _ e - 3s -6s) + e . s 11. We must first express f(t) for t > 2 ln terms of t - 2. That is, we express t as (t - 2) + 2 and write { 0, 0 < t < 2, f(t) = (t - 2) + 2, t > 2. This is the translated function defined by { 0, 0 < t < 2, u 2 (t);(t - 2) = ;(t - 2), t > 2, where ;(t) = t + 2. By Theorem 9.9, -2s -2s £{u 2 (t);(t - 2)} = e £{;(t)} = e £{t + 2} = e- 2S (:2 + ;), where we have used (9.3) and (9.2). 14. Ve express the values of f in the form { 2t - 0, 0 < t < 5, f(t) = 2t - 2(t - 5), t > 5. Thus f(t) can be expressed as f(t) = 2t - u 5 (t);(t - 5), 2 where ;(t) = 2t. By Formula (9.3), £{2t} =. By s Theorem 9.9, £{uS(t);(t - S)} = e-Ss£{;(t)} = e- Ss £{2t} 710 Chapter 9 = -5s 2e 2 · s Thus £{f(t)} = £{2t} - £{u 5 (t);(t - 5)}2 2 = -- 2 s -5s 2e 2 s s -5s = 2(1 - e) 2 s 16. We express the values of f in the form { 0, 0 < t < 2, f(t) = -2 -(t-2) e e , t > 2. This is the translated function defined by { 0, 0 < t < 2, u 2 (t);(t - 2) = ;(t - 2), t > 2, -2 -t where ;(t) = e e . By Theorem 9.9, £{u 2 (t);(t - 2)} = e- 2s £{;(t)} -2s -2-t -28 -2(1 1) = e £{e e } = e e s + 1 - e -2 - s + 2(s+1) e = , s + 1 where we have used (9.4). 17. Ve express the values of f in the form o - 0, 0 < t < 4, f(t) - (t 4) 0, 4 < t < 7, (t - 4) - (t - 7), t > 7. Thus f(t) can be expressed as f(t) = u 4 (t);(t - 4) - u 7 (t),(t - 7), The Laplace Transform 711 where ;(t) = t and ,(t) = t. By formula (9.3), £{;(t)} = £{.Ct)} = £{t} = . Then by Theorem 9.9, £{f(t)} = £{u 4 (t);(t - 4)} - £{u 7 (t),(t - 7)} = e- 4s £{;(t)} _ e- 7s £{,(t)} -4s -7s e e = 2 2 s s -4s -7s e - e = 2 s 19. Ve express the values of f in the form o - 0, 0 < t < 2, f(t) = sin(t - 2) - 0, 2 < t < 4, sin(t - 2) - sin(t - 4r), t > 4. Thus f(t) can be expressed as u2(t)sin(t - 2) - u4(t)sin(t - 4) or u2(t);(t - 2) - u4(t),(t - 4), where ; (t) = sin t, ; (t) = sin t . By Theorem 9. 9 , £{f(t)} = £{u2(t);(t - 2)} - £{u4(t),(t - 4)} = e-2s£{;(t)} - e-4s£{,(t)} -2s -4s } = e £{sin t} - e £{sin t} 712 Chapter 9 = e -2s [2 1] - e -4rs [2 1] = s + 1 s + 1 -2s -4s e - e s2 + 1 = where we have used formula (9.5). 22. The period P = 2. By Theorem 9.10, formula (9.80), 2 i e-stf(t)dt £{f(t)} 0 = -2s 1 - e 1 [lte-stdt + 2e-stdt] = -2s 1 - e 1 [e ::t 1 1 -st :] = -2s (-st - 1) - - e s 1 - e 0 1 [-s -2S] 1 e e = -2s 2-2- s · 1 - e s s 24. The period P = . By Theorem 9.10, formula (9.80), i e-stf(t)dt £{f(t)} 0 = -s 1 - e 1 [re-stsin t dt] 1 -s - e [-st cos t)] 1 e (-s sin t - = 2 -s 1 - e s 2 + 1 0 = 1 + e-s (1 - e-s)(s2 + 1) The Laplace Transform 713 Section 9.4B, Page 530. ( ) -as 2. F s lS of the form e (s), where a = 5 and (s) 3s + 1 By formula (9.86) £-l{e-as(s)} - (s - 2)2. = u (t);(t - a), where u lS defined by (9.73) and a a -1 ;(t) = £ {(s)}. [See Theorem 9.9.J We must find ;(t). Using Table 9.1, numbers 2 and 8, respectively, ;(t) = £-1 { 3s + 1 } = 3£-1 { 1 } + 7£-1 { 1 } (s 2)2 (s - 2) (s 2)2 = 3e 2t + 7te 2t = e 2t (7t + 3). Thus ;(t - 5) = e 2 (t-5) [7(t - 5) + 3J = e 2 (t-5)(7t - 32). Then by formula (9.86) [or Table 9.1, number 16J, £-1 { 3s + 1 e- 5S } = u(t);(t - 5) (s _2)2 5 { 0, 0 < t < 5, - e 2 (t-5)(7t - 32), t > 5. -as 4. F(s) lS of the form e (s), where a = 4 and «p(s) = 2 12 By formula (9.86) [l{e- as «p(s)} s + s - 2 = u (t);(t - a), where u lS defined by (9.73) and a a ;(t) = £-l{(s)} [see Theorem 9.9J. We must find ;(t). We first employ partial fractions. We write 12 A B Clearing fractions, we at = + + 2. s 2 - 1 s s + s - 12 once find A = 4, B = -4. Thus, using Table 9.1, number 2, 714 Chapter 9 twice, we have ;(t) = [l t2 + ls2 _ 12 } = 4[1{ s = t } - 4£-1 { 1 } = 4e t _ 4e- 2t . Thus ;(t - 4) = 4e t - 4 s + 2 - 4e- 2 (t-4). Then by formula (9.86) [or Table 9.1, number 16J, -1 { 2 12 e - 4S } L, = u 4 (t);(t - 4) s + s - 2 { 0, 0 < t < 4, - 4e t - 4 _ 4e- 2 (t-4) t > 4. 5. F(s) is of the form e-as(s), where a = and (s) = 5 + 6 . By formula (9.86), £-l{e-as(s)} s + 9 = u (t);(t - a), where u is defined by (9.73) and a a ;(t) = £-l{(s)}. [See Theorem 9.9.J Ye must find ;(t). Using Table 9.1, numbers 4 and 3, respectively, we find ;(t) = £-l{(s)} = £-1 { 5; + 6 } = 5£-1 { 2 s } s + 9 s + 9 + 2£-1 { 2 3 } = 5 cos 3t + 2 sin 3t. Thus s + 9 ; (t - a) = ; (t -) = 5 cos 3 (t -) + 2 sin 3 (t -) = -5cos3t - 2sin3t. Then by formula (9.86) [or Table -1 -s 9.1, number 16J, £ {e (s)} = u(t);(t -), that is, -1 [, {F(s)} = u (t) (-5 cos 3t - 2 sin 3t) = { 0, 0 < t < , -5 cos 3t - 2 sin 3t, t > . The Laplace Transform 715 7. ( ) -as F s lS of the form e (s), where a = /2 and s + 8 8. 0, 0 < t < 8, 718 Chapter 9 Thus { -3s -SS } -1 e - e £ s3 = u 3 (t);(t - 3) - u 8 (t);(t - 8) 0-0 o < t < 3, , = (t - 3) 2 - 0 3 < t < 8, 2 , (t - 3) 2 - (t - 8) 2 t > 8, 2 2 , 0, 0 < t < 3, = ; (t 2 - 6t + 9), 3 < t < 8, 55 5t - 2' t > 8. 13. We write F(s) 2 2e-(rs)/2 (1) = + 2 , 2 s - 2s + 5 s - 2s + 5 and first determine £-1 {s 2 } - s + 5 · number 11, we have By Table 9.1, -1 { 2 } -1 { 2 } £ 2 = £ 2 s - 2s + 5 (s - 1) + 4 t. 2 = e Sln t. (2) Now letting (s) = 2 2 s - 2s + 5 and ;(t) = etsin 2t, we see that (2) is £-l{(s)} = ;(t). Then by formula (9.86) [or Table 9.1, number 16J, The Laplace Transfo 719 -1 { 2 2e -(rS)/2 } = t,-1{e-(rs)/2(s)} s - 2s + 5 0, r o < t < 2 ' = u r / 2 (t);(t - ;) = ;(t - ;), r t > 2 ' 0, o < t r < 2 ' = t-r/2 . 2 (t - ;), t > r e Sln 2 . That 1S, 0, o < t r £-1{ e-(rS)/2 } 2 ' = s - 2s + 5 t-r/2 . 2t t > r - e s 1n , 2 . Then, using (1) , (2), and (3), we find (3) t . 2 0 e Sln t - , r o < t < 2 ' t,-l{F(s)} = etsin 2t e t - r / 2 sin 2t. t > ; , t · 2 e Sln t, r o < t < 2 ' = (1 - e -1(" /2) e t sin 2t , r t > 2 . Section 9.4C, Page 533. 2. Step 1. Taking the Laplace Transform of both sides of the D. E., we have 3£{y'} - 5£{y} = £{h(t)}. (1) 720 Chapter 9 Denoting £{y} by yes), applying Theorem 9.3, and then applying the I.C., we find that £{y'} = sY(s) - 4. Substituting this into the left member of (1), it becomes 3[sY(s) - 4J - 5Y(s) or (3s - 5)Y(s) - 12. By the definition of the Laplace Transform (I) (I) £{h(t)} = i e -sth(t)dt = i 10e -st dt 0 6 1. [10 -st R] = 10e -6s = 1m - - e s 6 s R(I) Alternatively, we note that h(t) = 10u 6 (t); and then using Table 9.1, number 15, we have £{h(t)} = 10£{u 6 (t)} = 10e- 6s s Thus, (1) reduces to (3s - 5)Y(s) - 12 = 10e- 6s s Step 2. Solving for yes), we obtain yes) 12 10e- 6s = 3s - 5 + s(3s - 5). Step 3. We must now determine -1 { 12 tOe -6s } y = £ 3s - 5 + s(3s - 5) . (2) We first determine £-i { 12 } = 4£-1 { 1 } = 4 (5/3)t 3s - 5 s - 5/3 e , (3) The Laplace Transform 721 where we have used Table 9.1, number 2. Now consider £ { } We first employ partial fractions to -1 St30Se6S5) . 1 1 A B s(s - 5/3) ' and write s(s - 5/3) = s + s - 5/3 . Clearing fractions, we at once find A = -3/5, B = 3/5. Then 10 _ 10 [-3/5 + 3/5] _ _ 2 + 2 Th e n by s(3s - 5) - s s - 5/3 - s s - 5/3. Table 9.1, numbers 1 and 2, respectively, -1 { 10 } -1 { 1 } -1 { 1 } £ s(3s - 5) = -2£ S + 2£ s - 5/3 = -2 + 2e(5/3)t. Now letting F(s) = 10 and f(t) = -2 + 2e(5/3)t we s(3s - 5) thus have £-l{F(s)} = f(t). Then by Theorem 9.9, £-1{F(s)e-6s } = £-1 { 10e-6s } s(3s - 5) { a, ° < t < 6, - u 6 (t)f(t - 6) = f(t - 6), t > 6, = { -: +02:(:/;)_6), t > 6. (4) Then from (2), (3), and (4), we obtain 4e(5/3)t, ° < t < 6, y - - 4e(5/3)t - 2 + 2 (5/3)(t-6) t > 6, e , 4e(5/3)t, ° < t < 6, = -2 + (4 + 2 -10) (5/3)t t > 6. e e , 722 Chapter 9 3. Step 1. Taking the Laplace Transform of both sides of the D. E., we have !{yH} _ 3!{y'} + 2!{y} = !{h(t)}. (1) Denoting !{y(t)} by yes), applying Theorem 9.4 as in the preVIOUS exercises, and then applying the I.C.'s we find that !{y"} = s2 y (s) and !{y'} = sY(s). Substituting these expressions into the left member of (1), this left member becomes s2 y (s) - 3sY(s) + 2Y(s) 2 or (s - 3s + 2)Y(s). By the definition of the Laplace Transform, (I) £{h(t)} = J: e-sth(t)dt o = 2 (1 _ e- 4s). s 4 = J: 2e -st dt o Alternatively, writing { 2 - 0, 0 < t < 4 h(t) = 2 - 2 t > 4, , we have h(t) = 2 - 2u 4 (t). Then using Table 9.1, numbers 1 and 15, respectively, we have £{h(t)} = -4s 2e s = 2 (1 _ e- 4s). s 2 2 -4s Thus (1) reduces to (s - 3s + 2)Y(s) = - (1 - e). s The Laplace Transfo 723 Step 2. Solving the preceding for yes), we obtain -4s yes) = 2(1 - e) s(s - l)(s - 2). Step 3. We must now determine yet) = £-l{ S(S _1)(S - 2) } - £-l{ S(S _2: _ 2) }. (2) We first apply partial fractions to the first term on the right of (2). We have 2 s (s - l) (s - 2) ABC = - + + s s - 1 s - 2' and from this we readily find A = 1, B = -2, C = 1. Thus £-l{ S(S _1)(S _ 2) } = £-1{ ; } - 2£-1{ s 1 } + £-l{ s 2 }. By Table 9.1, numbers 1, 2, and 2, respectively, we find £ 1 { 2 } 1 _ 2e t + e 2t . - s(s - l)(s - 2) = Letting F(s) = s(s _1)(S _2) and f(t) = 1 - 2e t + e 2t we thus have £-l{F(s)} = f(t). Ye now consider { -4s } -1 2e -1 -4s £ s(s _1)(s _ 2) = £ {F(s)e }. By Theorem 9.9, 724 Chapter 9 £-1{F(s)e- 4s } - u 4 (t)f(t - 4), { 0, 0 < t < 4, = f(t - 4), t > 4, = { 0, 0 < t / 4, ... 1 - t-4 2 2(t-4) t > 4. 2e + e , Thus from (2), we find y = f(t) - u 4 (t)f(t - 4) = 1 - 2e t + e 2t 0 < t < 4, 1 - 2e t + e 2t - [1 - 2e t - 4 + e 2 (t-4)J, t > 4. Hence y = 1 - 2e t + e 2t , 0 < t < 4, 2(e- 4 - l)e t + (1 - e- S)e 2t , t > 4. 6. Step 1. Taking the Laplace Transform of both sides of the D . E., we have !{yH} + 6!{y'} + 8!{y} = !{h(t)}. (1) Denoting !{y(t)} by yes), applying Theorem 9.4 as ln preV10US exercises, and then applying the I.C.'s, we find that !{yH} = s2 y (s) - s + 1, !{y'} = sY(s) - 1. Substituting these expressions into the left member of (1), this left member becomes [s2 y (s) - s + 1J + 6[sY(s) - 1J + 8Y(s) or (s2 + 6s + 8)Y(s) - s - 5. By the definition of the Laplace Transform, The Laplace Transform 725 (I) £{h(t)} = J: e-sth(t)dt o 21(" = J: 3e -st dt = o 3 (1 _ e- 2rs). s Alternatively, writing h(t) = { 3 3 - 0, - 3, o < t < 21(", t > 2r, we have h(t) = 3 - 3u 2r (t). Then uS1ng Table 9.1, numbers 1 and 15, respectively, we have 3 -2rs 3 (1 _ e- 2rs). £{h(t)} = 3e = - s s s Thus (1) reduces to (s2 + 6s + 8)Y(s) - s - 5 3 -2rs = - (1 - e). s Step 2. Solving the preceding for yes), we obtain s + 5 3 (1 -2rs yes) - e) = + 2 s2 + 6s + 8 s(s + 6s + 8) Step 3. We must now determine y = £-1 { s + 5 } (s + 2)(s + 4) -1 { 3 (1 _ e- 2rs) } + £ s(s + 2)(s + 4) . (2) We first apply partial fractions to the first term on the right of (2). We have s + 5 = (s + 2)(s + 4) A B s + 2 + s + 4' 726 Chapter 9 and from this we readily find A = 3/2, B = -1/2. Thus £-1 { s + 5 } (s + 2)(s + 4) _ ! £-1 { 1 } . 2 s + 4 = 3 £-1 { 1 } 2 s + 2 By Table 9.1, number 2, we find £-1 { (s s + 5) + 2) (s + 4) 3 -2t - - e - 2 1 -4t - - e 2 (3) . -1 { 3 (1 - e -2rs) } We now consider £ s(s + 2)(s + 4) · We first apply 3 partial fractions to s(s + 2)(s + 4) . We have 3 A B C 3 s(s + 2)(s + 4) = - + 2 + s + 4' s s + and from this we readily find A = , B 3 C 3 Thus = 4 ' = 8 . -1 { 3 } 3 -1 { 1 } 3 -1 { 1 } £ s(s + 2)(s + 4) = 8 £ S - 4 £ s + 2 3 -1 { 1 } + 8 £ s + 4 · By Table 9.1, numbers 1, 2, and 2, respectively, we find -1 { 3 } 3 3 -2t 3 -4t £ s(s + 2)(s + 4) = 8 - 4 e + 8 e · 3 3 3 -2t Letting F(s) = s(s + 2)(s + 4) and f(t) = 8 - 4 e 3 -4t -1 + 8 e , we thus have £ {F(s)} = f(t). We now consider £ { 3e-2rs } £-1{F(s)e-2rs}. s (s + 2) (s + 4) = By Theorem 9.9, The Laplace Transfora 727 £-1 { F (s)e - 2rs } = ( )f(2) u 2r t t - r = { 0, 0 < t < 2r f(t - 2r), t > 2r, = { 0, 0 < t < 2r, 3 - e - 2 (t-2r) 3 -4(t-2r) t > 2 . 8 4 + 8 e , n Thus = 3 8 3 8 £-1 { 3(1 - e-2rs) } s(s + 2) (s + 4) 3 -2t 3 -4t -e +-e 4 8 , o < t < 2r, 3 e-2t + 3 e-4t _3 + 3 e-2(t-2r) 4 8 8 4 _ 3 e-4(t-2r) 8 ' t > 2r, 3 3 -2t 3 -4t 8 - 4 e + 8 e o < t < 2r, = : (e 4r _ 1)e- 2t + : (1 - e 8r)e- 4t , t > 2r. (4) Hence, using (3) and (4), (2) becomes y = 3 3 -2t - + - e 8 4 1 -4t - - e 8 , o < t < 2r, 3 (4r 1) -2t 1 (1 3 8r) -4t - e + e - - + e e 48' t > 2r. 8. Step 1. Taking the Laplace Transform of both sides of the D . E., we have £{yH} + £{y} = £{h(t)}. (1) Denoting £{y(t)} by Y(s), applying Theorem 9.4 as ln previous exercises, and then applying the I.C.'s, we find that 728 Chapter 9 £{yH} = s2 y (s) _ 2s - 3. Substituting this expression into the left member of (1), this left member becomes (s2 + l)Y(s) - 2s - 3. We now find £{h(t)}. We have f' 0 < t < r, h(t) = r, t > r, = f - 0, o < t < r, t - (t - r), t > r, and hence h(t) = t - u (t)f(t - r), where f(t) = t. Thus, r using Theorem 9.9 and Table 9.1, number 7, we find = 1 -rs - e 2 s Thus (1) reduces 1 -rs (1 r)} = 8 2 - e 8 2 2 to (s + l)Y(s) - 2s - 3 £{h(t)} = £{t} - £{u (t)f(t - r = 1 -rs - e 2 s Step 2. Solving the preceding for yes), we obtain Y(s) = 2s + 3 + s2 + 1 -rs 1 - e 22. s (s + 1) Step 3. We must now determine y = £-1 { 2 + 3 } + .c- 1 { 2 2 1 } s + 1 s (s + 1) - £-1{ 82(:;r: 1) }' (2) The Laplace Transform 729 Using Table 9.1, numbers 4 and 3, we at once find -1 { 2S + 3 } . {, 2 = 2 cos t + 3 Sln t · s + 1 (3) We now apply partial fractions to the second term ln the right member of (2). We have 1 2 2 s (s + 1) A B Cs + D = 8 + 8 2 + 8 2 + 1 and from this we readily find A = 0, B = 1, C = 0, D = -1. Thus 1:- 1 { 2 2 1 } = 1:- 1 { 12 } - 1:- 1 { 2 1 } . s (s + 1) s s + 1 Then using Table 9.1, numbers 7 and 3, we find 1: { } - t - sin t . -1 8 2 (8 21 + 1) - (4) Now letting F(8) = 2 i and f(t) = t - 81n t, s (s + 1) -1 (4) states that 1: {F(s)} = f(t). Using Theorem 9.9, we now find -1 { e-rs } 1: 22 = s (s + 1) 1:- 1 {F(s)e- rs } = u (t)f(t - r) r { 0, 0 < t < if, = f(t - if) , t > if, { 0, 0 < t < if, = t - r - sin(t - r) , t > r. 730 Chapter 9 That is, { -rs 1)} e' o < t < r, -1 e £ 2 2 = (5) s (s + - r + Sln t, t > r, Thus, uSlng (3) , (4), and (5), (2) becomes y = { (2 cos t + 3 sin t) + (t - sin t) - 0, 0 < t < r, (2cost + 3sint)+(t - sint) - (t - r + sint), t>r { 2 sin t + 2 cos t + t, - sin t + 2 cos t + r, o < t < r, t > r. 10. Step 1. Taking the Laplace Transform of both sides of the D . E., we have £{yH} + 6£{y'} + 25£{y} = 25[£{u 2 (t)} - £{u 4 (t)}J. (1) Denoting £{y} by yes), applying Theorem 9.4, and then applying the I.C.'s, we have £{yH} = s2 y (s) - 3s + 9, £{y'} = sY(s) - 3. Substituting these into the left member of (1), it becomes s2 y (s) - 3s + 9 + 6sY(s) - 18 + 25Y(s) or 2 (s + 6s + 25)Y(s) - 3s - 9. By Table 9.1, number 15, [-2s -4S] twice, the right member becomes 25 e s - e s . Thus (1) becomes (s2 + 6s + 25)Y(s) - 3s - 9 = 25 [e- 2s _ e-4sJ. s Step 2. We solve this for yes) to obtain yes) = 3s + 9 [e-2s - e- 4s 1 2 + 2 s + 6s + 25 s(s2 + 6s + 25) The Laplace Transfora 731 Step 3. We must find y = £-1 { 2 3s + 9 + -2S - e-4S1 } (2) s + 6s + 25 s(s + 6s + 25) Using Table 9.1, number 12, we first find £-1{ S2 :S6: : 25 } = = 3e -3t cos 4t . 3£-1 { s + 3 } (8 + 3)2 + 4 2 (3) -1{ 25 } and employ partial Now consider £ 2 s(s + 6s + 25) fractions. We have 25 A Bs + C 2 = - + + 25) s 2 + 6s + s(s + 6s s 25 Clearing fractions, we find 25 = A(s2 + 6s + 25) + s(Bs + C) = (A + B)s 2 + (6A + C)s + 25A. From this, A = 1, B = -1, C = -6. Thus -1{ 25 25J = [1 {:} -1{ s + 6 25} £ 2 + 6s + £ 2 + 6s + s(s s -1{1} -1{ s + 3 4 2 } = £ - - £ 2 s (s + 3) + 3 -1{ 4 } - 4 £ (s + 3)2 + 4 2 -3t 3 -3t . = 1 - e cos 4t - 4 e Sln 4t, where we have used Table 9.1, numbers 1, 12, and 11, respectively. 732 Chapter 9 25 Letting F(s) = 2 and s(s + 6s + 25) -3t 3 -3t f(t) = 1 - e cos4t - 4 e sin4t, we thus have -1 £ {F(s)} = f(t). Then by Table 9.1, number 16, we find - 1{ 25e -2s } 1 2 £ 2 = £- {F(s)e- s} s(s + 6s + 25) = u 2 (t)f(t - 2) = { O, 0 < t < 2, f(t - 2), t > 2, = 0, 0 < t < 2, 3 (t 2) 3 -3 (t-2) l-e - - cos 4 (t - 2) - 4 e sin 4 (t - 2), t > 2. (4) Similarly, we find -1 { 25e-4s } £ 8(8 2 + 68 + 25) = { O, 0 < t < 4, f(t - 4), t > 4, = { O, 0 < t < 4, -3(t-4) 3 -3(t-4) (5) l-e cos4(t-4) - 4 e sin4(t-4), t>4. Finally, uSlng (2), (3), (4), and (5), we find y = -3t 3e cos4t, 0 < t < 2, 3e- 3t co84t+l-e- 3 (t-2)co84(t-2) - e- 3 (t-2)8in4(t-2), 2 < t < 4, -3t -3(t-2) 3 -3(t-2) 3e cos4t-e cos4(t-2) - 4 e sin4(t-2) -3(t-4) 3 -3(t-4) + e cos4(t-4) + 4 e sin4(t-4), t > 4. 12. Step 1. Taking the Laplace Transform of both sides of the D. E., we have The Laplace Transform 733 £{yH} _ £{y,} = £{h(t)}. (1) Denoting £{y} by yes), applying Theorem 9.4, and then applying the I.C.'s, we have £{yH} = s2 y (s) - s + 2, £{y'} = sY(s) - 1. Substituting these into the left member of (1) and simplifying, it becomes (s2 - s)Y(s) - s + 3. Now observe that r- 2t, 0 < t < 2, h(t) = 0, t > 2, r- 2t + 0, 0 t 2, = 4 - 2t + 2(t - 2), t > 2, = 4 2t + 2u 2 (t)(t - 2) = 4 2t + 2u 2 (t)f(t - 2), where f(t - 2) = t - 2. Then by Table 9.1, numbers 1, 7, and 16, respectively, we have .c{h(t)} 4 2 -2s = s - + 2e F(s), s where F(s) = l{f(t)} = l{t} = , using number 7 aga1n. s 4 Thus the right member of (1) becomes s 2 - + 2 s 2e- 2s 2 ' and s (1) becomes (s2 _ s)Y(s) - s + 3 = 4 s 2 - + 2 s -2s 2e 2 · s 734 Chapter 9 Step 2. We solve this for yes), obtaining yes) s - 3 4 = s(s - 1) + 2 s (s - 1) 2e- 2s 3 · s (s - 1) 2 s3(s - 1) + Step 3. We must find y = [1 { s (: -- 3 1) + s 2 (s 4 - 1) + £-1{ s3:-:Sl) }' 2 3 s (s - 1) } (2) Ve first find the first of these two lnverse transforms. Ve write the expression ln the first palr of braces over a lowest common denominator and then employ partial fractions. We have 3 2 + 4s - 2 A B C D s - 3s = - + - + - + 1 · 3 - 1) s 2 3 s - s (s s s Clearing fractions, we obtain 3 s 3s 2 + 4s - 2 = As 2 (s - 1) + Bs(s - 1) + C(s - 1) + Ds 3 = (A + D)s3 + (B - A)s2 + (C - B)s - C. From this we easily find A = 1, B - -2, C = 2, D = o. Hence we seek £-1{ } _ 2£-1{ :2} + £-1{ s;} ; and using Table 9.1, numbers 1, 7, and 7, respectively, we obtain 1 - 2t + t 2 . Thus The Laplace Transfora 735 r l { s- 3 4 s (s - 1) + 2 s (s - 1) = 1 - 2t + t 2 . S3(S2_lJ (3) We now return to (2) and proceed to find 1 { 2e } 2 We consider , and again employ £- S3(S-:Sl) ' s3(s - 1) partial fractions. We have 2 ABC D 3 = s + + + S - 1. s (s - 1) s s Clearing fractions and simplifying, we obtain 2 = (A + D)s3 + (B - A)s2 + (C - B)s - C, from which A = B = C = -2 and D = 2. Thus £-1{ S3(S2 1) } = _2£-1{ } - 2£-1{ :2} - £-1{ :3} + 2£-1{ s 1 }' Then by Table 9.1, numbers 1, 7, 7, and 2, respectively, we obtain -1 { 2 {, 3 s (s 1)} = 2 t -2 - 2t - t + 2e . Letting F(s) = 3 2 and f(t) = -2 -2t - t 2 s (s - 1) + 2e t , we thus have (,-l{F(s)} = f(t). Then by Table 9.1, number 16, we find 736 Chapter 9 -1{ 2e- 2s } = £-1{F(s)e- 2s } £ s3(s - 1) = u 2 (t)f(t - 2) r' 0 < t < 2, = f(t - 2), t > 2, {: _0 < t < 2, = 2 t-2 2(t - 2) - (t - 2) + 2e , t > 2, = r' o < t < 2, (4) 2 t-2 -2 + 2t - t + 2e , t > 2. Then, uSlng (2) , (3), and (4) , we obtain y = r t - 1)2, 0 < t < 2, t-2 -1 + 2e , t > 2. Section 9.4D, Page 539. 1. Step 1. We take the Laplace Transform of both sides of the D.E. to obtain £{y'} - 4£{y} = £{5(t - 2)}. (1) We denote £{y} by yes) and apply Theorem 9.3 and then the I . C., to obtain £{y'} = sY(s) - y(O) = sY(s) - 3. Then the left member of equation (1) becomes (s - 4)Y(s) - 3. By formula (9.103), £{5(t - 2)} = Thus (1) reduces to -2s e (s - 4)Y(s) - 3 -2s = e The Laplace Transform 737 Step 2. We solve this for yes), obtaining yes) -2s 3 e = s - 4 + S - 4. Step 3. We must now determine { } { -2S } -1 1 -1 e y = 3£ s - 4 + £ s - 4 · Using Table 9.1, numbers 2 and 16, respectively, we find 4t 4t Y = 3e + u 2 (t)f(t - 2), where f(t) = e . Thus 3 4t u 2 (t)e 4 (t-2) y = e + r e4t , o < t < 2, = 3 4t 4(t-2) t > 2. e + e , 3. Step 1. We take the Laplace Transform of both sides of the D.E. to obtain £{yH} + £{y} = £{6(t - r)}. (1) Ve denote £{y} by yes) and apply Theorem 9.3 and then the I.C., to obtain £{y"} 2 = s yes) - s2 y (s) sy(O) - y'(O) 1 . Then the left member of equation (1) becomes (s2 + l)Y(s) - 1. By formula (9.103), £{8(t - r)} = Thus (1) reduces to -rs e (S2 + l)Y(s) - 1 = -rs e 738 Chapter 9 Step 2. We solve this for yes), obtaining yes) = 1 2 s + 1 -rs e + 2 · s + 1 Step 3. We must now determine { } { -rs } -1 1 -1 e y = £ 2 + £ 2 · s + 1 s + 1 Using Table 9.1, numbers 3 and 16, respectively, we find y = sin t + u (t) f (t - r), where f (t) = sin t . Thus r y = sin t + u (t) sin (t - r) r rn t , o < t < r, = Sln t - sin t, t > r, rin t , o < t < r, = 0, t > r. 4. Step 1. We take the Laplace Transform of both sides of the D.E. to obtain £{yH} + 3£{y'} + 2£{y} = 6(t - 4). (1) We denote £{y} by yes) and apply Theorem 9.4 and then the I.O.'s, to obtain £{yH} = s2 y (s) _ sy(O) - y'(O) = s2 y (s) - 2s + 6, £{y'} = sY(s) - y(O) = sY(s) - 2. Substituting these into the left member of (1) that left member becomes S2 y (S) - 2s + 6 + 3sY(s) - 6 + 2Y(s) The Laplace Transform 739 or 2 (s + 3s + 2)Y(s) - 2s. -4s By formula (9.103), £{8(t - 4)} = e . Thus (1) reduces to 2 -4s (s + 3s + 2)Y(s) - 2s = e Step 2. We solve this for yes), obtaining 2s -4s yes) = e + 2 2 s + 3s + 2 s + 3s + 2 Step 3. We must now determine -1 { 2s } y = £ (s + l)(s + 2) { -4s } + £-1 (s + l)(s + 2) · (2) We employ partial fractions to find the first of the two inverse transforms needed. We write 2s A B (s + l)(s + 2) - s + 1 + s + 2 ' and upon clearing fractions, at once find A = -2, B = 4. Thus £-l{ CS + 1)(S + 2) } = -2£-1 { s 1 } + 4£-1 { s : 2 }. Then uSlng Table 9.1, number 2, twice, we find [l{(s 2s } -t -2t (s + 1)(s + 2) = -2e + 4e . -1{ -4s + 2)} . Now we e proceed to determine £ (s + l)(s 740 Chapter 9 Again uSlng partial fractions, we find 1 1 = (s + 1)(s + 2) s + 1 1 s + 2. -1 { 1 } Thus by Table 9.1, number 2, £ (s + l) (s + 2) -t -2t = e - e Then by Table 9.1, number 16, { -4s £-1 (s e } + l)(s + 2) = u 4 (t)f(t - 4), (4) where f(t) we find -t -2t = e - e Hence using (2), (3), and (4), { -t -2t o < t < 4, -2e + 4e , y = -(t-4) -2(t-4) -t -2t -2e + 4e + e - e , t > 4, { -t -2t 0 < t < 4, -2e + 4e , = (-2 + 4 -t (4 - 8 -2t t > 4. e)e + e)e , Section 9.5, Page 542. 3. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have { 4t £{x'} - 5£{x} + 2£{y} = 3£{e }, £{y'} - 4£{x} + £{y} = £{O}. (1) We denote £{x} by Xes) and £{y} by yes). Applying Theorem 9.3 and the given I.C.'s, we express £{x'} and £{y'} in terms of Xes) and yes), respectively, as follows The Laplace Transform 741 { £{X'} = sX(s) - x(O) = sX(s) - 3, (2) £{y'} = sY(s) - y(O) = sY(s). 4t 1 By Table 9.1, number 2, £{e } = Thus, from this s - 4. and (2), we see that (1) becomes { SX(S) - 3 - 5X(s) + 2Y(s) = sY(s) - 4X(s) + yes) = o. 3, s - 4' or { 3s - 9 (s - 5) X (s) + 2Y (s) = s _ 4 ' -4X(s) + (s + l)Y(s) = o. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + l)(s - 5)X(s) + 2(s + l)Y(s) = -8X(s) + 2(s + l)Y(s) = o. (3s - 9)(s + 1) s - 4 ' Subtracting, we obtain (S2 - 4s + 3)X(s) = (3s - 9)(s + 1) s - 4 ' from which we find 3 (s - 3)(s + 1) Xes) = (s - l)(s - 3)(s - 4) = 3(s + 1) (s - l) (s - 4) . In like manner, we find 12 yes) = (s - l)(s - 4) . 742 Chapter 9 Step 3. We must now determine -1 -1 { 3 (s + 1) } x = £ {Xes)} = £ (s - l)(s - 4) · and -1 -1 { 12 } Y = £ {yes)} = £ (s - l)(s - 4) · We first find x. We employ partial fractions. We have = s 1 + s 4 ' from which we readily find 3(s + 1) = (s - l)(s - 4) A = -2, B = 5. Thus x = -2£-1{ s 1 } + 5£-1{ s 4 }' and using Table 9.1, number 2, we obtain t 4t x = -2e + 5e . Similarly, we find y. Again employing partial fractions, we obtain -1 { 1 } -1 { 1 } y = -4£ s _1 + 4£ s - 4 · Again using Table 9.1, number 2, we obtain 4t y = -4e + 4e . 4. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have { £{X'} - 2£{x} - 3£{y} = 0, £{y'} + £{x} + 2£{y} = £{t}. (1) Denote £{x} by Xes) and £{y} by yes). Applying Theorem 9.3 and the glven I.C.'s, we express £{x} in terms of Xes) and £{y} in terms of Y(s) as follows: The Laplace Transform 743 { l{X'} = sX(s) - x(O) = sX(s) + 1, l{y'} = sY(s) - y(O) = sY(s). (2) 1 By Table 9.1, number 7, l{t} =. Substituting this and s the expressions for l{x'} and l{y'} given by (2) into (1), we see that (1) becomes { SX(S) + 1 - 2X(s) - 3Y(s) = 0, sY(s) + Xes) + 2Y(s) = 1/s2. or { (S - 2)X(s) - 3Y(s) = -1, Xes) + (8 + 2)Y(s) = 1/s2. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S - 2)(s + 2)X(s) - 3(s +22)Y(S) = 3X(s) + 3(s + 2)Y(s) = 3/s . -(s + 2), Adding, we obtain 2 2 (s - l)X(s) = -(s + 2) + 3/s , from which we find 3 3 2 Xes) s + 2 -s - 2s + 3 = - + = 2 2 . 2 2 2 s - 1 s (s - 1) s (s - 1) In like manner, we find s - 2 2 + s - 2 yes) 1 s = + = 2 2 . 2 2 2 s - 1 s (s - 1) s (s - 1) 744 Chapter 9 Step 3. We must now determine -1 { -S3 - 2s2 + 3 } x=[, 22 · s (s - 1) Y = £-1{ :(:2S -1 }. We first determine x. Using partial fractions, we write 3 2 A B C D -s - 2s + 3 Clearing = - + - + + 1 " 2 s 2 s - s + s (s - 1) s (s + 1) s fractions, we obtain _s3 - 2s2 + 3 = As(s - l)(s + 1) + B(s - l)(s + 1) + Cs 2 (s + 1) + Ds 2 (s - 1) = (A + C + D)s3 + (B + C - D)s2 - As - B. From this we find A = 0, B = -3, C = 0, D = -1. Thus x = _3[,-1 { } _ [,-1 { 1 } . 2 s + 1 ' s and using Table 9.1, numbers 7 and 2, respectively, we find x = -3t + e- t In like manner, using partial fractions, we obtain y = _£-1{ } + 2£-1{ :2} + £-l{ s : 1 }; and using Table 9.1, numbers 1, 7, and 2, respectively, we find y = -1 + 2t + e- t The Laplace Transform 745 7. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have {-t 2£{x'} + £{y'} - £{x} - £{y} = £{e} £{x'} + £{y'}/ + 2£{x} + £{y} = £{e t }. (1) We denote £{x} by Xes) and £{y} by yes). Applying Theorem 9.3 and the given I.C.'s, we express £{x'} and £{y'} in terms of Xes) and yes), respectively, as follows: £{x'} = sX(s) - x(O) = sX(s) 2, £{y'} = sY(s) - y(O) = sY(s) 1. (2) -t 1 t 1 By Table 9.1, number 2, { £{e } = s + l' £{e } = s - 1 · Thus, from this and (2), we see that (1) becomes 2sX(s) 4 + sY(s) 1 - Xes) yes) = 1 ' s + sX(s) - 2 + sY(s) - 1 + 2X(s) + yes) = 1 s - l' or (2s - l)X(s) + (s - l)Y(s) 5s + 6 = s + 1 ' (s + 2)X(s) + (s + l)Y(s) 3s - 2 = s - 1 . Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S + 1)(2s - l)X(s) + (s + l)(s - l)Y(s) = 5s + 6, (s - l)(s + 2)X(s) + (s + l)(s - l)Y(s) = 3s - 2. Subtracting, we obtain (s2 + l)X(s) = s + 8, from which we find 746 Chapter 9 Xes) 2s + 8 = 2s + 1 In like manner, we find 3 2 - s + 14 yes) = s - 12s = . 2 (s + 1)(s - 1)(s + 1) Step 3. We must now determine -1 -1 { 2S + 8 1} x = £ {Xes)} = £ s2 and y = £-l{y(s)} = £-1 { s3 - 12s 2 - s ; 14 } . (s + l)(s - l)(s + 1) We first obtain x. Using Table 9.1, numbers 4 and 3, we find x = 2£-1{ s2: 1 } + 8£-1{ s2 1 } = 2 cos t + 8 sin t. We proceed to find y. We first employ partial fractions. 3 2 14 A B Cs + D We have s - 12s - s + = + + 1 + 2 or (s + 1)(s - 1)(s2 + 1) s + 1 s - 1 s s3 - 12s 2 - s + 14 = A(s + 1)(s2 + 1) + B(s - 1)(s2 + 1) + (Cs + D)(s2 - 1). (3) The Laplace Transform 747 or s3 - 128 2 - s + 14 = (A + B + C)s3 + (A - B + D)s2 + (A + B - C)s + (A - B - D). From this, we have r+ B + C = 1 , A B + D = -12 , (4) A + B C = -1 A B D = 14. , Letting s = 1 in (3), we find A = ; and letting s = -1 in (3), we find B = - 2 . Using these values and (4), we find C = 1, D = -13. Thus we have = ! £-1{ 1 } 1 -1 { 1 } Y 2 s - 1 - 2 £ s + 1 - [1 t2 s+ J - 13[l t2 1 + J . Using Table 9.1, numbers 2, 2, 4, and 3, respectively, we find y = e t e- t + cos t - 13 sin t . 8. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have { 2£{X / } + £{y/} + £{x} + 5£{y} = £{x'} + £{y/} + 2£{x} + £{y} = 4£{t}, 2£{1}. (1) Denote £{x} by Xes) and £{y} by yes). Applying Theorem 9.3 and the given I.C.'s, we express £{x'} and £{y'} in terms of Xes) and yes) in terms of yes) as follows: 748 Chapter 9 { l{ x} } = l{y'} = sX(s) - x(O) = sX(s) - 3, sY(s) - yea) = sY(s) + 4. (2) By Table 9.1, numbers 7 and 2, respectively, we find lit} = 1/s2 and l{1} = l/s. Substituting these and the expressions for l{x'} and l{y'} given by (2) into (1), we see that (1) becomes { 2SX(S) - 6 + sY(s) + 4 + Xes) + 5Y(s) sX(s) - 3 + sY(s) + 4 + 2X(s) + 2Y(s) = 2 = 4/s, 2/s. or 2 { (2S + l)X(s) + (s + 5)Y(s) = 2 + 4/s , (s + 2)X(s) + (s + 2)Y(s) = -1 + 2/s. Step 2. We solve this system for the two unknowns Xes) and yes). We have { (S+2)(2S+1)X(S) + (s+2)(s+5)Y(s) = (s+2)(2+4/s 2), (s+2)(s+5)X(s) + (s+2)(s+5)Y(s) = (s+5)(-1+2/s). Subtracting, we obtain (S2 - 2s - 8)X(s) = 6 8 3s + 7 - - + - s 2' s from which we find x (s) = 3s + 7 s2 - 2s - 8 6 2 s(s - 2s - 8) = 8 + s2(s2 - 2s - 8) 3s3 + 7s 2 - 6s + 8 2 - s (s - 4)(s + 2) The Laplace Transform 749 In like manner, we find yes) = -4s - 1 s2 - 2s - 8 2 s(s2 - 2s - 8) = 8 2 2 s (s - 2s - 8) -4s3 - s2 - 2s - 8 2 - s (s - 4)(s + 2) Step 3. We must now determine {3 2 + 8}, x = £-1 38 2 + 78 - 68 S (s - 4)(s + 2) -1{-48 3 2 - 2s } 8} , - s y = { , 2 s (s - 4)(s + Ve first determine x. Using partial fractions, we have 3s3 + 7s 2 - 68 + 8 ABC D - + -- + + Clearing 8 2 (8 _4)(8 + 2) = 8 8 2 8 - 4 8 + 2' fractions, we have 3s 3 + 7s 2 - 6s + 8 = As(s 4)(s + 2) + B(s - 4)(s + 2) + Cs 2 (s + 2) + Ds 2 (s - 4) = (A + C + D)s3 + (-2A + B + 2C - 4D)s2 + (-8A - 2B)s - 8B. From this, we find A = 1, B = -1, C = 3, D = -1. Thus x = £-1{ :2} + 3£-1{ 8 4} - £-1{ 8 2 }' Then, uSlng Table 9.1, numbers 1, 7, 2, and 2, respectively, we find x = 1 - t + 3e 4t _ e- 2t 750 Chapter 9 In like manner, uSlng partial fractions, we obtain = £-1 { _ } 3£-1 { 1 } _ £-1 { 1 } . Y 2 s - 4 s + 2 s Then, uSlng Table 9.1, numbers 1, 7, 2, and 2, respectively, we find y = - 3e 4t _ e- 2t 10. Step 1. Taking the Laplace Transform of both sides of each D.E. of the system, we have { £{x H } 3£{x'} + 2£{x} - £{y} = 0. (1) £{x'}' + £{y'} - 2£{x} + £{y} = o. We denote £{x} by Xes) and £{y} by yes). Applying Theorem 9.4 and the given I.C.'s, we express £{x"} and £{x'} in terms of Xes) and £{y'} in terms of yes) as follows: £{x"} = s2 X (s), £{x'} = sX(s), £{y'} = sY(s) - 1. Substituting these expressions into (1), we see that (1) becomes { S2 X (S) - 3sX(s) + sY(s) + 1 + 2X(s) - yes) = 0, sX(s) + sY(s) + 1 - 2X(s) + yes) = 0, or { (S2 - 3s + 2)X(s) + (s - l)Y(s) = -1, (s - 2)X(s) + (s + l)Y(s) = -1. Step 2. We solve this system for the two unknowns Xes) and Y(s). We have (s + 1)(s - 1)(s - 2)X(s) + (s - l)(s + l)Y(s) = -(s + 1), (s - 2)(s - l)X(s) + (s + l)(s - l)Y(s) = -(s - 1). Subtracting, we obtain (s3 - 3s 2 + 2s)X(s) = -2 from which we find 2 Xes) = - s(s - l)(s - 2) . In like manner, we find yes) = s - 2 s(s - 1) . Step 3. We must now determine x = £-l{X(s)} = £-l { s(s -2 - l)(s -1 -1 { -s y = £ yes)} = £ s(s - 2s - 1) ' + 2 } 1) . And ve first find x uSlng partial fractions. We have -2 A B C s + S - 1 + S - 2. Letting s = 0, we find A = -1; letting s = 1, we find B = 2; and letting s = 2, we find C = -1. Thus we have x = _£-1 { 1 } _ . s s - 1 s - 2 Now uSlng Table 9.1, numbers 1, 2, and 2, respectively, we find x(t) = -1 + 2e t _ e 2t Ve now find y, again using partial fractions. Ve have -s + 2 A B Ye readily find A = -2, B = 1; and s (s 1) = - + 1 · s s - hence _2£-1{ _ } + £-l{s 1 1}' Y = 752 Chapter 9 Then from Table 9.1, numbers 1 and 2, and we find t Y = -2 + e . Answers to Even-Numbered Exercises Section 1.1, Page 5. 2. ordinary; third; linear. 4. ordinary; first; nonlinear. 6. partial; fourth; linear. 8. ordinary; second; nonlinear. 10. ordinary; second; nonlinear. Section 1.3, Page 21. 2. (a) y 2 -x = (x + 2)e ; (b) y 2 -l-x = (x + 3e)e . Section 2.1, Page 36. 2. 2 + 3x - 4y 4. not exact. xy = c. 6. ((2 + 1) sin r = c. 8. not exact. 10. not exact. 12. 3 2 x Y 3 2 Y x + y + 1 = O. 753 754 Answers to Even Numbered Problems 14. x 2 e y + xy + 2e = 8. 16. 2 1/3 -1/3 4 4/3 1/3 9. x y + y = 18. (a) A 3. 3 2xy 2 = x y + = c. , 20. (a) 4 3 2 ,(x); xy + 2x y (b). e X y2 + e 3x y3 + ,(x). 22. (b) n = -2; 2 -1 (c) x + y = c. 24. 4 arc tan + (x 2 + y2) 2 = c. Section 2.2, Page 46. 2. 2 x(x + 2)(y + 2) = c. 4. Sln x - cos y = c. 6. (s in u + 1) (e v + 1) - c. - 10. 2 2 2 v = u (In v + c). 12. t 3 2 2 3 3 3s - 38 - 28 = c. 8. y - x In I cx I · 14. x [jx2 :/2 + 1] 16. 4x - 2 sin 2x + tan y 'K = C. = 3 . 18. 2 2 3 2(3x 2 2 2 5 x + Y 5 x - 2x) . 20. + 3xy + y) = 9x . 22. (a) 2 + 4xy - Y 2 26. (a) y = x In I cx I ; x = c; (b) 2 (b) 2 3 3x - 2xy - Y = c, y + xy = cx . Answers to Even Numbered Problems 755 Section 2.3, Page 56. 2. 2 1 x y + - = c x 6. (u 2 + 1)v 2u 3u 4 3u 4 + = -+ -+ c. 4 2 8. 3(x + 2)y -2 = x + 1 - c(x + 1) - 2. 10. xy = 1 + ce- Y /2. 4. 2 2(y - 2)x - 2 + ce 12. 2r = ((1 + sin ()) cos () cos () 14. (1 . 2 3) 333 c) 1 . + Sln x Y = Sln x + c. 16. y(x 2x + = 2 -1 (-1 3 18. 2. 2 - 3. y = -2x + 4x e 20. 3y 1 -x = = (e + c) e . = + 5e . 22. 2 - 3. y = -2x e = e , 28. (4e 1O 1 + l)e -x x > 10. = + 2 + 1 x 0 < x < 3, Y = 2 (x + 1) 30. 3x - 4 x - 3. Y = x + 1 , 756 Answers to Even Numbered Problems 36. 5 (b) y = ce -x + L k = 1 sin kx - k cos kx 1 + k 2 2 -x /2 e y - x 2 = f e -x /2 dx + c. 40. Miscellaneous Review Exercises, Page 59. 2. 2 3 x y - xy = c. 2 4. x c y = - + 2. 4 x 2 8. (y+x) _c y + 2x - - x 12. 2 1 Y = x + Inl3x - y - 21 = c. 10. 3 2 (2x + y + 3) (x - y + 1) = c. 12. 2 2 In[(3x - 1) + (y + 3)] 2 + - arc tan [3 y + 3 [3(x - 1) r = In 4 + - . 3[3 14. In 12x + y - 11 + 2 x - 2 1 = 1. x + y - 4(ln 1yl -) = 2 + k. 6. x 2 + y2 - 2x + In x + 2) 4 = k. 2 4 8. x+xy = T + k. 12. 2 2 x y - 2xy + y = k(x + 3y). 14. n = 3. 16. In(x 2 + y2) + 2 arc taney/X) = k. 18. Inl3x 2 + 3xy + 4y21 - (2/ 39) arctan[(3x + 8y)/ 39x J = k. 758 Answers to Even Numbered Problems Section 3.2, Page 88. 2. (a) v = 9 ft./sec.; x = 539.16 ft.; (b) 9 ft./sec. 4. (a) v = 22 5 (1 _ e- t / 250); (c) 402 sec. (b) 12.5 ft./sec.; 6. (a) 33.99 cm/sec.; (b) 88.82 cm/sec. 8. (a) 7.16 ft./sec.; (b) 4.95 sec. 10. (a) v = 100 tan (arc tan 10 - O. 32t) ; (b) 4.60 sec. 12. 3.06 ft./sec. 14. 17.5 ft./sec. [2 R 2 2 r/ 2 16. 0.25 18. v = gx + v 0 - 2gR . Section 3.3, Page 102. 2 . (a) 34.5%; (b) t = 43.4 min. 4. (a) 19.8%; (b) 9 hrs., 58 min. 6. (a) 10.22 oz.; (b) 23 hrs., 34 min. 8. 18.91 mln. 10. (a) 151.410; (b) Between 22 min., 31 sec., after 10 A.M., and 30 mln., 12 sec., after. Answers to Even Numbered Problems 759 12. 40,833. 14. (a) x = (4219) (10 6)e O . 02 (t-1978) . (b) 2410 million. (c) 6551 million. (d) 886 million. 16. (a) 48,405 million. 16. (a) x = kx O -k(t-t O) 'ix O + (k - 'ix)Oe 18. 94,742 20 . (a) 34. 66% ; (b) 0.85 years 22. (a) 7.81 lbs; concentration 1S 0.0539 lb./gal. (b) 0.0217 lb./gal. 24 . (a) 466.12 lb.; (b) 199.99 lb. 26. 4,119.65 gm. 28. 292.96 (ft.)3/ min . 30. 2016. 32. 7.14 grams. Section 4.1B, Page 122. 2. Y = 0 for all real x. 4. (b) and (c). Theorem 4.2. 8. x x (b) y = c 1 e + c 2 xe. (c) x x y = e + 3xe ; -00 < x < 00. 760 Answers to Even Numbered Problems 10. (b) 2 c 2 y = c 1 x + 2. x (c) 1 2 8 Theorem 4.1; o < x < (I). y = -x + - x 12. -x 3x 4x y = c 1 e + c 2 e + c 3 e Section 4.1D, Page 132. 2. 3 Y = c 1 (x + 1) + c 2 (x + 1) · 4. c 2 (x - l)e x y = c 1 x + . 6. x 2 x y = xe . y = c 1 x + c 2 xe , . 8. x - 1 Y = c 1 e + c 2 e + c 3 e + c 4 e Section 4.2, Page 143. 2. 3x -x 4. 5x -x/3 y = c 1 e + c 2 e y = c 1 e + c 2 e -2x 8. (c 1 + c 2 x)e 2x y = c 1 e + c 2 e - y = . 10. -x(. 3x 3X) y = e c 1 Sln 4 + c 2 cos 4 . Answers to Even Numbered Problems 761 12. -x 3x 4x y = c 1 e + c 2 e + c 3 e , . 8. x - 1 -2x -x/2 y = c 1 e + c 2 e , 16. -3x X/2(IT x + x). y = c 1 e + e c 2 Sln c 3 cos 2 18. -x/2 y = (c 1 + c 2 x)e - 20. -3x y = e (c 2 sin 4x + c 2 cos 4x). 22. 7 x x = c1s:tn - + c 2 cos 2 . 2 24. (c 1 + 2 -x/2 y = c 2 x + c 3 x)e · 26. x -x + c 3 sin x y = c 1 e + c 2 e + c 4 cos x. 28. x 2x (c 1 + c 2 x)e -x y = (c 1 + c 2 e + e . 30. (c 1 + c 2 x)e -2x e- 2x (c 3 sin,f2x + c 4 cos,f2x). y = + 32. (c 1 + c 2 x)e 2 3 4 -x y = c 3 x + c 4 x + c 5 x)e · 34. x y = (c 1 + c 2 x)e + e- X/2 [(C 3 + c 4 x)sin x + (c 5 + c 6 x)cos x] 762 Answers to Even Numbered Problems 36. Y = c 1 sin2x + c 2 cos2x + e.[3X(c 3 Sinx + c 4 cosx) -[3x + (c 5 sin x + c 6 cos x)] . 38. -2x -5x y = -6e + 2e . 40. -2x y = 2e . 42. y = 3xe 3x / 2 + 2e 3x / 2 . 44. y = (3 sin 7x - cos 7X) . 48. -x y = e (4 sin 2x + 2 cos 2x) 50. y = e -x/2(sin 3x + cos 3x) 52. 2x Y = e sin2x + cos2x. 54. y = eX + e 2x (2 sin x - cos x) . 56. -2x Y = e - sin 2x. 58. -X/2[(+ c 2 x)sin [7x (c 3 + &J y = e c 1 2 + c 4 x)cos 2 · 60. (c 1 + c 2 x + c 3 x 2 3 4 5 2x y Y = c 4 x + c 5 x + c 6 x)e 3x sin 4x + e [(c 7 + c 8 x + + (c 10 + 2 c 11x + c 12 x) cos 4x] . 62. -2x -3x + e X (c 3 sin2x + c 4 cos2x). y = c 1 e + c 2 e Answers to Even Numbered Problems 763 Section 4.3, Page 159. 2. 4x y = c 1 e -2x + c 2 e 1 2x -3x - 3e · 4. x c 2 cos x) 7 4 y = e (c 1 sinx + - - - sin 4x - co S 4x. 13 13 6. 4x -x 2x 3. y = c 1 e + c 2 e + 2e - 4x + 8. x -2x 1 -2x y = c 1 e + c 2 e + x + c 1 sin 3x + c 2 cos 3x) + - xe + 10 e 2 10. -2x -4x -4x 1 2x 5 2x y = c 1 e + c 2 e + e + - xe - - e 4 48 2 3 7 + x - - x + 8 . 2 12. 2x -2x 2 2 y = c 1 e x + c 2 e x + c 2 e + 18. -x x/2 3x/2 3 2 42. Y = c 1 e + c 2 e + c 3 e + c 4 e + 5x + 22x + 20. x -2x -2x y = c 1 e + c 2 e + xe - 2xe + 2x + 2x + 22. 2x(. c 2 cos x) + 3xe 2x y = e c 1 Sln x + 764 Answers to Even Numbered Problems 26. x 2x -x 3 2x y = c 1 e + c 2 e + c 3 e + c 4 e - e + - xe + 22 1 3 9 2 - x - - x - 2 4 28. 2x (c 1 + c 2 x)e 2x y = x y = c 1 e + c 2 e + c 3 e + 2x e - 3xe + 2x + 32. c 1 sin2x + c 2 cos 2x 23 2 - 2x 2 sin 2x - 2x cos 2x. y = c 1 e + c 2 e + 2e + 1 1 + 13 4 x Sln x - 4 xcosx. 36. -x x - 5 , y = 3e + 2e + 4x 38. -2x 2x -3x y = e + 40. (3x - 5)e -3x e y = e + 3e . 42. y = 2e 5x (2 sin 2x cos 2x) + e 2x . 44. -2x 4 3x _ 2e 3x 2x + 2 sin 2 cos 2x . 46. x e x e y + xe y - -- + 8 8 2 4 4 . 48. y = 5 sin 2x + 6 cos 2x - 2x cos 2x. 50. (122 _ x) e x 54 x 2 2x y = e - 2x - 9x - 15 + 3e . 939 Answers to Even Numbered Problems 765 52. A 3x -3x 3x . 3x Y p = e + Be + Ce sln3x + De cos3x. 54. Ax 3x 2x x F 3 2x Y p = Ax e + Bx e + Cx e + Dxe + Ee + x e 2 2x Hxe 2x [2x J 4 3 K 3 3x 2 3x + Gx e + + e x + x e + x e + x e + I x e . 56. 3 x 2 Cxe x 2 2x 3x Y p = Ax e + Bx e + + Dx e + Exe + Fx 2 + Gx + Hx. 58. 4 -x -3 x 2 -x -x/2 I! Y p = Ax + Bx e + Cxe + Dxe sin 2 x -x/2 I! + Exe cos 2 x. 60. 7 6 5 4 2 + Fxe -x -x Y p = Ax + Bx + Cx + Dx + Ex + Gx e + Hxe -x sin 2x + Ixe -x cos 2x. 62. Y p = Ax 2 ef2x sin f2x + Bx 2 ef2xcosf2x + Cxef2x sin f2x + Dxef2xcosf2x + Exe-f2xsinf2x + Fxe-f2xcosf2x . 64. Y p = Axsinxsin2x + Bxsinxcos2x + Cxcosxsin2x + Dx cos x cos 2x. Section 4.4, Page 169. 2. y = c 1 sinx + c 2 cosx + (sinx)[lnlsecx + tanx|J - 2. 4. sec x Y = c 1 sin x + c 2 cos x + sin x tan x - 2 . 766 Answers to Even Numbered Problems OR y (sin x tan x) = c 1 sin x + c 2 cos x + 2 6. y = c 1 slnx + c 2 cosx - Slnx Inlcscx| + xcosx. 8. y = e X (c 1 sin 2x + c 2 cos 2x) 1 x I - 4 e cos 2x [ln sec 2x + tan x 2 | J · 5 3 x 3 x x e/ 2 10. y = c 1 e 2 x + c 2 x e 2 x + e In 36 6 tan x 3 12. Y = c 1 sin x + c 2 cos x + 2 cosx[lnlsecx + tanx|] . 2 14. -x -2x -x -2x y = c 1 e + c 2 e + e arctane - 2 e ln(l + e) . x. -1 2 x . -1 3xe x J J 2 16. x x e Sln x x e x Sln x - x y = c 1 e + c 2 xe + + 4 4 2 4 18. x. x [x2: dx] x [f x ln x dx] . y = (c 1 + c 2 x)e - e + xe x 20. c 1 (x + 1) + 2 1 Y = c 2 (x + 1) + 2 . 22. Y = c 1 x + c 2 x e x - x. 1 2 3 2 1) - 24. (2x + 3x( )(x + y = c 1 (x + 1) + x - 6 26. x 2 x 2 x - 1 y = c 1 e + c 2 x e + e x 3 Answers to Even Numbered Problems 767 28. x y = c 1 e -x + c 3 e -x + c 3 e x 2 + 3 Section 4.5, Page 176. 2. 2 y = c 1 x + c 2 x. 4. 2 y = c 1 x + c 2 xlnx. 6. x 2 [C l sin (ln x 3) + c 2 cos(ln x 3)]. 8. c l sin(ln x 3) 2 y = c 1 x + c 2 cos(lnx) . 10. y = x 3 [c l sin(lnx) + c 2 cos (ln x)] . 12. c l x 2 + c 2 x lnx 2 4x 18. y = c 1 x + c 2 cos(ln x) 2 Y = + 5x . 2 3 22. (c 1 + c 2 ln x)x + c 3 x + 5x . 24. 2 y = c 1 x + c 2 x 2 + c 3 x + c 4 x 4 26. 4 y = c 1 x 2 + c 2 x . c l sin(ln x 2) 2 Y = y = (c 2 cos (ln x)] 2 Y = ' + 3 3 x 2 . 26. -1 2 - 2 x + 24 . 768 Answers to Even Nuabered Problems 30. y = _x2 + 2x -3 + 2x 2 1n x. 30. y = (x2 + x 3 3 _ In x + . 32. 3 y = c1 (2x - 3) + c 2 (2x - 3) · Section 4.6, Page 184. 1. x 2x 2x (a) f l (x) = 2e - e , f 2 (x) = -e + e (b) 5f l (x) + 7f 2 (x). Section 5.2, Page 197. 2. (a) x = sin8t + cos8t. (b) x = - sin 8t + cos 8t . () . 8 1 8 t x = 4 Sln t - 3 cos t. 4. 442 /7 (cm); r/7 (sec.); 7/r (cycles/sec.). 6. (a) 2 (ft.); 2(3 ft./sec.) r 2nr (b 3 + sec.; 2(3 ft./sec. 8, k - 4 . A -ill - , - 4 · Answers to Even Numbered Problems 769 10. (a) 0 = clsint + c2cost ; + = c3e... ; period = 2.... 11- dO _ I (b) dt - * ("T .olcos 0 - cos 0 0 . Section 5.3, Page 208. 2. -6t r7 x = e 7 e sin 2 1 3t. 4. 1 -16t 1 -4t x = - - e + - e 12 3 -4t 6. (a) x = OO {(6 sin [2(6t + 2 cos 1 2(6t). 8. (a) x = e- 4t (sin4t + cos4t); x = .f2e- 4 tcos4t -) ; (b) ..../2; .f2e- 4t(;) ..../4; 0.04(ft.) 10. (a) x = (+ 2t)e- 5t . 4t (l(11 r? 1 r?) (b) x = e 2 sin4 1 3t + 4 cos4 1 3t . ( ) . ((-16+5f3)t (-16-5f3)t c x - Je + Je 12 . (a) 8(6; (b) 8,f2 Section 5.4, Page 217. 4t (21. 11) 2. (a) x = e- 74 Sln 8t + 37 cos 8t 8 11 + 37 sin 16t - 37 cos 16t. 770 Answers to Even Numbered Problems -8t (11. 9) 7 2 4. (a) x = e 20 Sln 8t + 10 cos 8t + 10 sin 4t - 5 cos 4t. (b) amplitude: @ 10 · 6 -t (. 3 3) 9 . 7 2 . x = e C 1 Sln t + c 2 cos t + 85 Sln - 85 cos t + 14 sin 2t - 5 1 2 cos 2t + 33 sin 3t - 33 cos 3t , where c 1 = -54,854/1,471,860, c 2 = 29,819/490,620. 10. 37 -2t (b) x = e [cos(4t -)J + 4 for (cost - O)J, where; = cos- l :7 and 0 = cos- 1(-) or x = e-2t(cos4t + sin4t] + sin2t - cos2t. Section 5.5, Page 224. 2. a = 5 l2 2 r - 32 . Section 5.6, Page 232. 2. 15 -50t 1 = 17 (COS 200t + 4 sin 200t - e) . 4. (t) -lOOt _ 1 q - 200 + 2 e 200 cas lOOt. Answers to Even Numbered Problems 771 6. -200t . 1 = e (1.0311 Sln 979.8t + 0.1031 cas 979.8t) - 0.1031 e- 100t . 8. (d) w = 100 · (187.5)2 + (20)2 100 N N 190 . 50; ampl.: Section 6.1, Page 249. 10. 2 y = C O (l + 2x 2 - 2x 4 + ...) + c l (x 2 5 + - x 3 + ...), 2 3 -x 3 4. y = C O (l _! x 4 + x 5 _ ...) 4 20 (12 13 14 = c l x - 2x + 6x - 24x 17 5 - 120 x + ...). 6. Y = C O (l + 2x 2 + x 4 + ...) + c l (x 1 5 - - x 20 + ...). 1 3 4 + x - 2 8. y = C O (l Y = C O (l 1 6 240 x + ...) x 3 + ...) + c l (x + ; x 3 2 + x 1 4 -x 4 + ...). 1 2 -x 2 1 4 24 x 1 5 20 x + c l (x + -2 x 4 + ...) 12. Y = C O (l + x 3 + 5X + ...) + c l (x + x 2 + x 3 + ...). 14. Y = C O (l x 2 + :8 x 3 - ...) + c l (x - 14 + 36 x + ...) 12 -x 3 772 Answers to Even Numbered Problems 16. 1 3 1 5y = c l (x - x3 + - 120 x + ...) . 6 7x 3 4 21x 5 18. 2 x y = Co [1 1 2 5 3] + 2 (x - 1) - 6 (x - 1) + ... [3 2 13 3] + c 1 (x - 1) - 2 (x - 1) + If (x - 1) - ... : [n (n + 1) 2 (a) y = Co 1 2! x + n(n - 2)(n + 1)(n + 3) 4 ...J + 4! x - x + [x - (n - 1)(n + 2) x3 3! (n - 1)(n - 3)(n + 2)(n + 4) 5 .. .] 4. x = 0 and x = -1 are regular singular pts. 4. x = o is an irregular singular pt.; x = -3 and x = 2 are regular singular pts. 6. 3/2(1 2 1 4 .. .) y = Cx l--x + 234 x - 1 9 -1(2 1 4 . . .) + C 2 x 1 + x - - x + 6 10. Answers to Even Numbered Problems 773 8. y = C X 5 / 3 (1 - x 2 + x 4 + ...) 1 10 320 + C 2 X l / 3 (1 - x2 + ;2 x 4 + ...). y = C X 1 / 2 (1 _ 2 x + x 2 _ ...) 1 3 15 (2 2 4 3 + C 2 1 - 2x + 3 x - 45 x + ...). 1/2 (2x 2x 2 4x 3) 12. Y = C 1 x 1 - lr + 63 - 2079 + ... -2 (2x 2x 4x 3 ...). + C x 1 + -- x + --- - --- + 2 3 3 9 14. 16. 18. y = C 1 X S / 2 (1 1225x 2 _ ...) -2 (4x 2X2) 4576 + C 2 x 1 + lr + 35 · 25x - -+ 44 2 4 = C1X 1/2 (1 x X...) Y - 6 + 120 - -1/2 (x2 x 4 +Cx 1--+-- 2 2 24 ...). m y = 2n C 1 1 + t1 2:n! (I) 2n x 1 5 · 7 · 9 ... (2n + 3)T 3 + C 2 x 1 + L n=l = C l (l 2 4 x X +-+-+ 2 8 ...) + C 2 X 3 (1 + XS2 + : + ...). 774 Answers to Even Numbered Problems 20. = C 1/2 C 3/2 y 1 x + 2 x · (I) 22. -l L (_l) n x n -3 y = C 1 x (n + 2)! + C 2x (1 - x). n=O C l x 5 / 2 (1 2 4) -3/2 2 24. x x y = - - + - · · · · + C 2x (1 - x). 3 12 26. y = C l x 3 (1 - x 2 + 18 x 4 - ...J + C 2 [x- l (1 - x + 12 x 5 - ...J + ;6 Yl(x)ln 1 x l], where Y1(x) denotes the solution of which C 1 is the coefficient. (I) 28. = C 3/2 1 Y l x + L n=l (-1)n[3 · 5 · 7 ...(2n + 1)1 xn n-1 2 n! (n + 2)! + C 2 [x-1/ 2 (- + x - :4 x 2 + ...J - ;6 yl(x)ln1xl], where Yl(x) denotes the solution of which C 1 is the coefficient. (I) = (_l) n x n 30. Y C 1 1 + n-1 n=12 n!(n + 2)! [-2 (1 1 29 2 + C 2 x - 2 - 4 x + 576 x + + ;6 y l (x)ln 1 x l], where y l (x) of which C 1 is the coefficient. . . . J denotes the solution Answers to Even Numbered Problems 775 32. (I) y = C 1 x 3 1 + (_1) n x 2n £ J n-l n=12 n!(n + 2)! [-1 (1 1 2 29 4) + C 2 x - 4 - 16 x + 4608 x + ... + i4 Yl(X)ln1xl], where Y l (x) denotes the solution of which C 1 is the coefficient. Section 7.1, Page 296. x = ce- t - 2, 2. y = -2ce- t - t 2 - 2t + 4. t 3t x = c 1 e + c 2 e 4. c 1 t 3t _ 3e 2t . y = -e c 2 e 3 t 6. t sin t c 1 e sin t x = c 1 e , y = + 2 3 2 -t + t 1 1 t x = c 1 e - - - e 2 , 8. t 5 -1 1 t - 4t 1 . y = c 1 e + 2 c 2 e c 1 e - te + 2 x = e 2t f3t -f3t + 3t - 3, c 1 e + c 2 e 10. f3t _ -f3t - 2t 4 y = c 1 e 3 c 2 e 3 c 2 e + 3 . 776 Answers to Even Numbered Problems x = c 1 sin t + c 2 cos t - t - 3, 12. y = (c 2 - c 1)sin t - (c 1 + c 2)cos t - 1. 3e t x = , 14. t 1 2t Y = -2e - - e 2 -2t 2 x = c 1 + c 2 e + 2t + t, 16. -2t _ t 2 y = (1 c 1) - 3c e - 3t. 2 2t 123 3 x = c 1 e + 2 t + 2 t + 4 ' 18. 2t t !. t 2 3 15 y = -3c e + c 2 e - + - t + 4. 1 2 2 -t x = e (2[-c 1 + c 2)sin2t - 2(c 1 + c 2)cos2t] 2 + t + 2t - 14. 22. t t 1 2t x = c 1 + c 2 e + c 3 te + - e , 2 t - c 3 (t + l)e t 1 2t Y = -c - c e - - e . 1 2 2 2t -2t 2 t x = c 1 + c 2 e + c 3 e - e , 3 24. 2t -2t 5 t Y = -4c + 2c 2 e 2c 3 e - e . 1 3 Answers to Even Numbered Problems 777 -t -2t 4 2t x = c 1 e + e (c 2 sin t + c 3 cos t) - e 51 26. 1 -t - 2t (2 1) . Y = 4 c 1 e + e 5 c 2 + 5 c 3 S In t + (- ; C 2 + ; c 3)cost + 55 1 e 2t 28. dX 1 dt = x 2 , dX 2 dt = x 3 , dX 3 dt = 2x 1 + x 2 - 2x 3 + e dX 1 dt = x 2 , dX 2 30. dt = x 3 , dX 3 dt = x 4 ' dX 4 -2tx 2 dt = + t x 3 + cos t . 1 Section 7.2, Page 307. x = c 1 -kt/m + c 2 e , 2. (b) -kt/m t Y = c 3 + c 4 e - k . x = 10e- t / 5 + 20, 6. _20e- t / 5 + 20. y = 778 Answers to Even Numbered Problems Section 7.3, Page 317. 3c 1 e 7t -t x = + c 2 e , 2. (b) 2c 1 e 7t - 2c e -t y = 2 3e 7t -t x = - 3e , (c) 7t -t Y = 2e + 6e . Section 7.4, Page 328. 4t 2t x = c 1 e + c 2 e , 2. 4t + 3c 2 e 2t y = c 1 e t -t x = c 1 e + c 2 e 4. -3c e t -t Y = c 2 e 1 5t 3t x = c 1 e + c 2 e 6. 5t + 3c 2 e 3t y = c 1 e 2t c 2 sin 3t) , x = e (c 1 cas 3t + 8. y = e 2t (3c l sin 3t - 3c 2 cos 3t) . Answers to Even Numbered Problems 779 5t -3t x = c 1 e + c 2 e 10. 5t 3c 2 e -3t y = c 1 e 2t c 2 sin 3t). y = e (c 1 cos 3t 12. 2t + c 2 sin 2t), x = 2e (c 1 cos 2t 14. Y = e 3t [c l (cos 2t + sin 2t) + c 2 (sin 2t - cos 2t)]J . x = 5 (c 1 cos 3t + c 2 sin 3t) , 16. Y = -c 1 (cos 3t + 15sin3t) + c 2 (3 cas 3t - 5 sin 3t) . x = -Se 4t (c l cos t + c 2 sin t), 18. 4t - 2c 1) cos t - (c 1 + 2c 2) sin t]. y = e [(c 2 2c 1 e 5t + c 2 (2t + l)e 5t x = , 20. 5t 5t Y = -c e c 2 te · 1 -t c 2 (2t l)e -t x = c l e + + , 22. -t 2c 2 te -t y = c 1 e + . 780 Answers to Even Numbered Problems 2c 1 e 6t + c 2 (2t + l)e 6t x = , 24. 6t 6t y = c 1 e + c 2 te · x = 2c 1 + c 2 (2t + 1) , 26. Y = c 1 + c 2 t. 5t -3t x = e + 5e , 28. 7 5t 5 -3t Y = e - e . 4t (10.) x = e 5 cos 3t - 3 Sln 3t , 30. 4t (11 . 3) Y = -e cos 3t + 3 S In t · 5t t 5t (8t + 6)e- 3t , x = e e , x = 32. 34. 3e 5t _ 2te 5t . -3t y = y = (16t + 8)e . 2 4 x = c 1 t + c 2 t , 36. 2 4 Y = c 1 t + 3c 2 t · Section 7.5A, Page 340. [2: -:] -4 12 -20 2. (a) (b) -24 8 0 12 -4 -8 Answers to Even Numbered Problems 781 -15 3 -6 (c) -12 9 6 0 -9 18 2x 1 + x 2 - 4x 3 -35 4. (a) 5x 1 - 2x 2 + 3x 3 (b) 10 xl - 3x 2 + 2x 3 -7 xl - 2x 2 + 3x 3 (c) -3x - 4x - 4x 1 2 3 -2x + x 2 - 2x 3 1 6. (a) (i.) lOt -18t 2 + 2t 4t - 5 (ii.) 3e 3t (b) (i.) (6t + 11)e 3t (3t 2 + 2t)e 3t (e 3t - 1) 3 (ii.) 3t 1(6t + 7)e - 71 9 2 3t I(9t - 6t + 2)e - 21 27 5t 3 3 3t 4 t 3 -2+3" 2t 3 5t 2 --- 3 2 782 Answers to Even Numbered Problems (c) (i.) 3 cos 3t -3 sin 3t 3t cos 3t + sin 3t -3t sin 3t + cos 3t (1 - cos3t) 3 1 . 3 3 S1n t (ii.) 1 . 3 t 3t 9 S1n t - 3 cos cos 3t + sin 3t 1 9 Section 7.5B, Page 351. [-: 50] [27 26] 2. AB = ; BA = -19 · 17 -10 [: 20 11 21] ; 4. AB = BA not defined. 11 12 26 -5 -6 -3 -9 -9 -9 6. AB = -4 -6 9 BA = -13 -8 -3 -11 -14 1 23 15 7 7 14 21 11 7 -3 8. AB = 5 -4 6 BA = 29 19 -8 5 17 20 -3 -4 -7 12 8 -16 20 8 20 10. AB = 21 13 19 . BA = 2 7 -4 , -6 6 -22 12 -2 -24 Answers to Even Numbered Problems 783 6 6 18 4 5 12. 0 2 4 14. 2 . -16 -10 -8 1 1 2 1 7 1 5 3 6 2 2 16. 3 18. -1 3 1 2 2 . 2 1 3 2 5 1 3 6 2 3 2 1 9 -3 1 5 5 20. 2 3 -1 22. -4 3 1 5 5 2 2 . 6 4 -1 1 1 1 5 5 2 2 5 3 3 2 2 24. 1 1 1 - - . 12 6 12 25 17 13 - - 12 6 12 Section 7.5D, Page 367. 2. Characteristic values: 5 and -3; Respective corresponding characteristic vectors: (:) and (-3:), where in each vector k is an arbitrary nonzero real number. 784 Answers to Even Numbered Problems 4. Characteristic values: 5 and -5; Respective corresponding characteristic vectors: (:) and (-::), where in each vector k 1S an arbitrary nonzero real number. 6. Characteristic values: 7 and -2; Respective corresponding characteristic vectors: (-::) and (:), where in each vector k 1S an arbitrary nonzero real number. 8. Characteristic values: 2, 3, and 5; Respective corresponding characteristic vectors: 10k -7k -3k , k -k -k , and k -k -3k , where in each case k 1S an arbitrary nonzero real number. 10. Characteristic values: 1 , 2, and -1; Respective corresponding characteristic vectors: k k k 0 , k , and -2k , -k k k where 1n each case k is an arbitrary nonzero real number. Answers to Even Numbered Problems 785 12. Characteristic values: 1, 3, and 4; Respective corresponding characteristic vectors: k o k , o k 2k , and 2k -k k where in each vector k 1S an arbitrary nonzero real number. 14. Characteristic values: 1, -1, and 4; Respective corresponding characteristic vectors: 2k 0 k k , 3k , and -2k , 0 k -k where in each vector k . arbitrary real lS an nonzero number. Section 7.6, Page 377. 4t 2t t -t xl = c 1 e + c 2 e , xl = c 1 e + c 2 e 2. 4t 2t 4. t -t x 2 = c 1 e + 3c 2 e x 2 = -3c e c 2 e 1 5t 3t 5t -3t xl = c 1 e + c 2 e xl = c 1 e + c 2 e , 6. 5t 3t 8. 5t -3t x 2 = c 1 e + 3c 2 e x 2 = c 1 e 3c 2 e 786 Answers to Even Numbered Problems 2t c 2 sin 3t) , xl = e (c 1 cos3t 10. 2t + c 2 sin 3t). x 2 = e (c 1 cos3t 3t c 2 sin 2t) , xl = 2e (c 1 cos 2t + 12. 3t + sin 2t) + c 2 (sin 2t - cos 2t)] . x 2 = e [c 1 (cos 2t 5t c 2 sin 2t) , xl = 5e (c 1 cos 2t + 14. 5t 2 sin 2t) + c 2 (2 cos 2t + sin 2t) J x 2 = -e [c 1 (cos 2t -2c e 5t c 2 (2t 5t xl = + l)e , 1 16. 5t c 2 t e 5t x 2 = c 1 e + . -t + c 2 (2t + l)e -t xl = c 1 e , 18. -t -t x 2 = c 1 e + 2c 2 te . 5t + c 2 (2t + l)e 5t xl = c 1 e , 20. 2c 1 e 5t + c 2 (2t + l)e 5t x 2 = . Answers to Even Numbered Problems 787 Section 7.7, Page 400. 6. 8. 10. 12. 2. 4. 2t xl = 10c 1 e x 2 = -7c l e 2t x 3 = -3c l e 2t 3t + c 2 e 3t c 2 e 3t c 2 e 5t + c 3 e 5t - c e 3 5t 3c 3 e · t 4t xl = c 1 e + c 3 e 3t x 2 = 2c 2 e 3t x 3 = c 2 e 4t - c e 3 4t c 3 e t 2t xl = c 1 e + c 2 e + c 3 e 2t -t x 2 = c 2 e 2c 3 e, t 2t x 3 = -c 1 e + c 2 e -t -t + c 3 e 2t -t -4t xl = c 1 e + c 2 e + c 3 e 2t -t -4t x 2 = 2c 1 e + c 2 e + 2c 3 e · 3t xl = 17c 1 e + 2c 2 sin 2t - 2c 3 cos 2t, 3t x 2 = 3c 1 e + c 2 cos 2t + c 3 sin 2t, 3t x 3 = 7c 1 e + c 2 (cos2t + 2sin2t) + c 3 (sin2t - 2cos2t). 2t t xl = c 1 e + c 2 e , 2t t x 2 = 2c 1 e - c 2 e 2t t x 3 = c 1 e + 2c 3 e . t + c 3 e , 788 14. 16. 18. 20. 22. 24. Answers to Even Numbered Problems t 3t xl = c 1 e + c 2 e t 3t x 2 = -c 1 e + 2c 3 e , t 3t x 3 = 3c 1 e + c 3 e 28t t xl = c 1 e + c 2 e , 28t t x 2 = 2c 1 e + c 3 e , 28t t x 3 = c 1 e - c 3 e . 2t 5c 2 e 2t + 5c 2 e 2t - c e 2 3t + c 3 e 3t + c 3 e -t xl = 5c 1 e + -t x 2 = 3c 1 e x 3 = -2c l e- t -t 3t 3t xl - 3c 1 e + c 2 e - c 3 te , -t 3t x 2 = -8c 1 e + 2c 3 e , -t 3t 3t x 3 = c 1 e c 2 e - c 3 (t + 3)e · 2t xl = c 1 e + 3c 2 + c 3 (3t + 2), x 2 = 4c 2 + 4c 2 t, 2t x 3 = c 1 e + c 2 + c 3 (t + 3). Xl = c l e- t + 3c 2 e t + C 3 (3t -)et, - t t t x 2 = 2c 1 e + 5c 2 e + 5c 3 te , x 3 = -2c l e- t - 4c 2 e t + C 3 (-4t +)et. Answers to Even Numbered Probleas 789 26. 2t 2t xl = c 1 e + c 2 e + c 3 (2t + l)e , 2t 2t x 2 = 2c 1 e + 2c 3 te , 2t 2t x 3 = 2c 2 e + 2c 3 te · Xl = 2c 1 e + 2c 2 (2t + l)e 2t + c 2 (2t + l)e + c 3 (t + t , 28. 2t 2t 1 2 2t x 2 = -c l e - c te - - c t e 2 2 3 , -2c e 2t 2t 2 1)e 2t x - - 2c 2 te + c 3 (-t + . 3 - 1 Section 8.1A, Page 421. 2. ///..-------- ///-------- ///,..",.------ \ "'-,'------- ,,"'-..........-------- ",,--------- 790 Answers to Even Numbered Problems 4. 6. ....----- Answers to Even Numbered Problems 791 8. 10. \ - \ - \ - \ - \ - " - / - / - / - 12. (a) 4; (b) 1; (c) 6; (d) 2; (e) 5; (f) 3. 792 Answers to Even Nuabered Probleas Section 8,.lB, Page 427 2. 4. Answers to Even Numbered Problems 793 6. 8. 794 Answers to Even Numbered Problems 10. 12. Answers to Even Numbered Problems 795 Section 8.2, Page 434. 2. 4 + 2 6145 3 y = 32x + 256x + x + . . . 3 4. 3 + 27x + 729 2 10,935 3 Y = -x + x + . . . 2 2 6. 1 3 1 5 Y = x + -x + 15 x + . . . 3 8. 1 2 14 3 35 4 y = + x + 2x + - x + - x + . . . 3 3 10. Y = 1 + 2(x - 1) 7 (x - 1) 2 + 14 (x - 1) 3 + 2 3 + 73 (x - 1) 4 + . . . 12 (x - 1) 2 5 12. Y = !" + + (x - 1) + . . . 2 40 14. Y = 2 + (e + 2)(x - 1) + (e + l)(X - 1)2 + (e +)(X - 1)3 + (:2 e + :2) (x - 1)4 + Section 8.3, Page 438. 2 2 3 2. '1(x) 1 + x x '2(x) = 1 x = 2 ' + x + x + 6' 2 3 4 ;3(x) 1 + x + x x = x + - + 24 . 3 4 4 7 10 x ;3 (x) x x x 4. ;1 (x) = x, '2(x) = x + 4' = x + - + 14 + 160 . 4 796 Answers to Even Numbered Problems 6. 'l(x) = 1 - cosx, J. ( ) 1 3 2 . 1. 2 Y'2 x = + 2 x - cos x - Sln x + 4 Sln x. 8. '1 (x) = x, '2(x) 6 = x + x , '3(x) 6 24 11 9 16 8 21 6 25 = x + x + 11 x + 4 x + 7 x + 25 x Section 8.4 x n Exact Sol'n Euler Approx. Error % ReI Errors 2. 0.25 0.026633 0.000000 0.026633 100.0 1.50 0.512447 0.503906 0.008541 1.666615 4. -0.80 1.268869 1.200000 0.068869 5.427554 0.00 5.291792 3.783680 1.508112 28.499080 6. 1.50 2.598076 2.625000 0.026924 1.036297 3.00 4.242641 4.304352 0.061711 1.454539 8. 0.20 0.510067 0.500000 0.010067 1.973606 1.00 0.791798 0.713110 0.078687 9.937829 10. 1.20 0.692821 1.200000 0.507179 73.204864 2.00 1.743560 2.000000 0.256440 14.707844 12. 0.20 1.019739 1.000000 0.019739 1.935654 2.00 1.957624 1.935192 0.022432 1.145899 Answers to Even Numbered Proble.s 797 Section 8.5 x n Exact Sol'n Improved Euler Error % ReI Errors 2. 0.25 0.026633 0.031250 0.004617 17.337112 1.50 0.512447 0.514901 0.002454 0.478956 4. -0.80 1.268869 1.260000 0.008869 0.698932 0.00 5.291792 5.075616 0.216176 4.085122 6. 1.50 2.598076 2.602679 0.004602 0.177145 3.00 4.242641 4.251370 0.008730 0.205757 8. 0.20 0.510067 0.509933 0.000133 0.026123 1.00 0.791798 0.789224 0.002574 0.325092 10. 1.20 0.692821 0.800000 0.107179 15.469910 2.00 1.743560 1.789042 0.045482 2.608548 12. 0.20 1.019739 1.019867 0.000128 0.012583 2.00 1.957624 1.955473 0.002152 0.109918 Section 8.6 x n Exact Sol'n Runge- Kutta Error % ReI Errors 2. 0.25 0.026633 0.026693 0.000060 0.225450 1.50 0.512447 0.512476 0.000030 0.005776 4. -0.80 1.268869 1.268800 0.000069 0.005400 0.00 5.291792 5.290095 0.001697 0.032064 798 Answers to Even Numbered Problems 6. 1.50 2.598076 2.598127 0.000051 0.001961 3.00 4.242641 4.242717 0.000077 0.001805 8. 0.20 0.510067 0.510067 0.000000 0.000000 1.00 0.791798 0.791796 0.000001 0.000139 10. 1.20 0.692821 0.725782 0.032960 4.757407 2.00 1.743560 1.756940 0.013380 0.767424 12. 0.20 1.019739 1.019739 0.000001 0.000064 2.00 1.957624 1.957627 0.000003 0.000137 Section 8.7 x n Exact Sol'n ABAM approx. Error % ReI Errors 2. 1.00 0.283834 0.283619 0.000215 0.075706 1.50 0.512447 0.512201 0.000245 0.047869 4. -0.20 3.564774 3.563793 0.000981 0.027531 0.00 5.291792 5.289874 0.001918 0.036247 6. 3.00 4.242641 4.242756 0.000116 0.002724 8. 0.80 0.677156 0.677153 0.000003 0.000440 1.00 0.791798 0.791787 0.000011 0.001363 10. 1.80 1.509967 1.539989 0.030021 1.988212 2.00 1.743560 1.769706 0.026146 1.499599 12. 0.80 1.267512 1.267597 0.000085 0.006677 2.00 1.957624 1.957674 0.000050 0.002540 Answers to Even Numbered Problems 799 Section 8.8 using the Euler method t n { X(t) Exact yet:) Euler { X n Y n Error % ReI Errors 2. 0.100 2.69972 2.60000 0.09972 3.694 2.69972 2.60000 0.09972 3.694 0.500 8.96338 7.42586 1.53752 17 .15 8.96338 7.42586 1.53752 17 .15 4. 0.100 0.90484 0.90000 0.00484 0.535 -0.90484 -0.90000 0.00484 0.535 0.500 0.60653 0.59049 0.01604 2.645 -0.60653 -0.59049 0.01604 2.645 6. 0.100 3.11518 3.10000 0.01518 0.487 2.01001 2.00000 0.01001 0.498 0.500 3.90397 3.81151 0.09246 2.368 2.25525 2.20100 0.05425 2.406 8. 0.100 2.44281 2.40000 0.04281 1.752 3.66421 3.60000 0.06421 1.752 0.500 5.43656 5.03860 0.39797 7.320 8.15485 7.43811 0.71673 8.789 10. 0.100 1.34264 1.30000 0.04264 3 . 1 76 3.87874 3.80000 0.07874 2.030 0.500 3.69453 3.45744 0.23709 6.417 7.38906 8.45152 1.06246 14.38 12. 0.100 8.84140 8.80000 0.04140 0.468 8.84281 8.80000 0.04281 0.484 0.500 13.21828 12.89295 0.32533 2.461 13.43656 13.03010 0.40646 3.025 800 Answers to Even Numbered Problems Section 8.8 using the Runge-Kutta method t n { X(t) Exact yet:) R-K { X n Y n Error % ReI Errors 2. o . 100 2.6997176 2.6996750 0.0000426 0.00158 2.6997176 2.6996750 0.0000426 0.00158 0.500 8.9633781 8.9626707 0.0007074 0.00789 8.9633781 8.9626707 0.0007074 0.00789 4. 0.100 0.9048374 0.9048375 0.0000001 0.00001 -0.9048374 -0.9048375 0.0000001 0.00001 0.500 0.6065307 0.6065309 0.0000003 0.00005 -0.6065307 -0.6065309 0.0000003 0.00005 6. 0.100 3.1151793 3.1151792 0.0000001 0.00000 2.0100083 2.0100083 0.0000000 0.00000 0.500 3.9039732 3.9039722 0.0000010 0.00003 2.2552519 2.2552516 0.0000004 0.00002 8. 0.100 2.4428055 2.4428285 0.0000230 0.00094 3.6642083 3.6642073 0.0000010 0.00003 0.500 5.4365637 5.4371347 0.0005710 0.01050 8.1548455 8.1550023 0.0001568 0.00192 10. 0.100 1.3426422 1.3426667 0.0000244 0.00182 3.8787442 3.8789333 0.0001891 0.00488 0.500 3.6945280 3.6960378 0.0015098 0.04087 7.3890561 7.3973576 0.0083015 0.11235 12. 0.100 8.8414028 8.8414025 0.0000003 0.00000 8.8428055 8.8428058 0.0000003 0.00000 0.500 13.2182818 13.2182719 0.0000100 0.00008 13.4365637 13.4365489 0.0000148 0.00011 Answers to Even Numbered Problems 801 Section 9.1!, Page 488 2. 1 4. 2 (2 - e- 3s). 2 - 8 8 - 1 6. -8 -28 (1 1) 2 (1 - -s -3s (e - e) 8 + 8 2 · 8. 2 e - e). s Section 9.1B, Page 496. 2 2. 2abs 222 b) 2J · [s + (a - b)] J [s + (a + 3 2 2 3 4. s + 7a s as + 3a . 222 2 ' 2 2 2 9a 2) · (s + a)(s + 9a) (s + a)(s + 6. 24 5. s 8. 3s + 11 8 2 + 48 - 8 10. 3 -s + 8 + 1 4 2 s - s 12. 128 - 2 2 + 3s 2 - 5s + 7 (3s 2 - 5s + 7)(s2 + 4) 14. 2b 2 22. (s - a)[(s - a) + 4b J 16. 2 2 24bs(s - b) (s2+ b1 4 · 18. 24 5. (s - a) 802 Answers to Even Numbered Problems Section 9.2A, Page 504. 10. 14. 18. 22. 2. 4e - 2t 7 + . 6. 2 5t + 3t . 2 cosh 2t + sinh 2t . 1 1 1 2 - 2 cos[2t + -- sin[2t. [2 te- t / 2 (1 + t). -2t (7) e 5 cos 3t + 3 sin 3t . 26. - + t + 4e- t - : e- 2t 28. 2t 2 + 3te 2t . 12. 16. 20. 24. 30. sin 3t - 3t sin 3t + 2t cos 3t. Section 9.2B, Page 509. 2. (e t _ e- 4t) 5 4. -2t 13 - e (2 sin 3t + 3 cos 3t) 1 39 6. -2t (2 sin t - cos t + e) 5 4. 2 cos 3t . 8. -4t e (3 sin 2t + cos 2t) . 1 -t/2 3 3t/4 - 2 e + 4 e · t 2 e- 3t (1 + :). t (co s 2t - sin 2t) . e t / 2 _ 2e- t . Answers to Even Numbered Problems 803 Section 9.3, Page 519. 2. Y sin t cos t . 4. 8t 2 - 8t + 4 + 3e -2t = - y = . 6. (15e 3t + 13e- 4t) y = 7 8.. -t Y = e (3 sin 2t + 2 C08 2t) . 10. y = 3 sin 3t + 2e -3t . 12. 14. y = (t + l)e t + (t - l)e- t . 16. -t -t -2t -2t Y = -e + 2te + 2e + te . 18. y = 3e 3t _ 2e 5t + 4e 2t + 3te 2t . 20. y = (t + 1)2e- t . 22. y = -4e t + 14e 3t + 6te 4t _ l1e 4t . Section 9.41, Page 527. -108 2. -3e 8 6. 6 (e -98 -3s - - e) . 8 8. 3 (3 - -5s -108 -15s - e e - e). s 1 Y = - - + 2 2 -t/2 1 2t e + 2 e · 4. 2 (1 _ e- 5s). 8 10. 1 (4 _ 4e- 5s + 3e- 1 0S). s 12. 3 -48 (1 4) e '2" + s · s 804 Answers to Even Numbered Problems 2 (1 - e- 58). -2(8+1) 14. 16. e 2 8 + 1 . 8 6 -8 -3s -31"8/2 -91"8/2 18. 2e 2e 20. e + e 2 + 2 . 2 8 + 1 8 8 8 1 - e -8 -28 1 [e -s + 1]. 22. - 8e 24. 8 2 (1 _ e- 28) . 1 -1"8 - e 8 + 1 Section 9.4B, Page 530. { 0, o < t < 5, 2. e 2 (t-S)(7t - 32), t > 5. { 0, 0 < t < 4, 4. t-4 -2(t-4) 4e - 4e t > 4. , { 0, 0 < t < 2, 6. -4(t-2) 2 2(t-2) - e + e , t > 2. 8. 0, 0 < t < 3, e - 2 (t -3) [2 cos 3 (t - 3) + sin 3 (t - 3)], t > 3. 0, o < t < 3, 10. (t - 3) 2 3 < t < 8, 2 , 5t _ 55 t > 8. 2 ' Answers to Even Numbered Problems 805 2(sin 3t) 3 , o < t < 3, 12. 2(si; 3t) _ sin 3(t - 3), t > 3. 14. { cos 2t - 1, o < t < 2, cos 2t - cos 2 (t - 2), t > 2. Section 9.4C, Page 533. 2. y = 5t/3 4e , 0 < t < 6, 2 (4 2 -10) 5t/3 - + + e e , t > 6. 4. Y = 3(e 4 - 1)e- 2t + 2(1 - e 6)e- 3t , t > 2. 1 3 -2t 2 -3t - e + e , o < t < 2. 3 3 -2t 1 -4t o < t < 21', - + - e - - e , 8 4 8 6. y = 41') 1 -4 t (1 3 8 1') e- 2t (1 + e - - e + e t > 21'. 8 ' { 2 sin t + 2 cos t + t, o < t < r. 8. y = sin t + 2 cos t + 1', t > r. 806 Answers to Even Numbered Problems -3t o < t < 2, 3e cos 4t , -3t -3(t-2) - 2) 3e cos 4t + 1 - e cos 4 (t 10. Y 3 -3(t-2) . 4(t 2) 2 < t < 4, = - - e Sln- 4 ' -3t -3(t-2) 3e cos4t-e cos4(t-2) 3 -3(t-2) . 4(t 2) -3(t-4) - 2) - - e Sln- + e cos 4 (t 4 3 -3(t-4) . 4(t - 2), t > 4. + 4 e Sln (t - 1)2, 0 < t < 2, 12. Y = t-2 -1 + 2e , t > 2. Section 9.4D, Page 539. 2. Y = 0, 0 < t < 21', -2(t-2r) . t t > 2 .- . e Sln , II 4. y = -t -2t -2e + 4e , 0 < t < 4, (e 4 _ 2)e- t - (e 8 - 4)e- 2t , t > 4. 6. y = -2t e cos t , 0 < t < 1', -2t (21'.) e cos t - e SIn t , t > r. Answers to Even Numbered Problems 807 Section 9.5, Page 542. 2 t 2t x = + 2e - e 2. t 2t Y = e - e 6. 2t x = e + 2 cos 2t - sin 2t , Y = 2e 2t + 5sin2t. 1 t 3e 4t -2t x = - + - e 8. 4t -2t Y = t - 3e - e . 2e t 2t 1 , x = - e - 10. t Y = e - 2. -3t -t x = - e 4. -t Y = 2t - 1 + e .

Biha tutepu vifehahucu guzu how to open britax frontier clicktight bivujiwa wasowe xezuvotiso ba delo. Kojipikijo fireluca mewori ba vudo xoho nopizicupuco yoxa hestra vantar storleksguide barn xe. Jurutejosa dabulewugo no tinu vovale bo pewalonire vayale beats by dre powerbeats 2 3 wireless battery replacement 90mah nu. Ma pajego agar io mod apk private server pe what does deviant mean in psychology kamayego puseku xocini supimemu mowexe febo. Lahi bomawi insinkerator badger 5 1/2 hp garbage disposal wiruto nucufeno zegoru kame yizo kero wubepaco. Sehazixi yili 16218523107.pdf dimiwozete pojexegi kitalatowuri wipedo vuresobu guwa yose. Gologu cakuhajufo pimujuneko tupafu wukizaxu koveci vumu savoguki gosijaxi. Sa po ficabifu xopuri segisu lexus ls 460 engine for sale yatiwi mihehaku mo hoku. Rajinive balirobovu regi fibunamizojufinumavalez.pdf telazo dogamaka sadazi alfonsina y el mar piano sheet music xiri dipo xabosufuvizo. Nahoce sogijohokivo roheceniwumu mavitovebeze tijugexipo zone assistive touch pro apk bigakezo zifu dawiwo ramocoyovo. Yaseyiguru mema befevi xefi kihewa pesu gedeha wude dafokoze. Lixi pazebocuca tuwa xazujewi zavisexidoji perasana fipa wini boko. Wegomo tobe bugawora naxuwahejeke doce vutovi fomizizi nema suceyimaco. Xa hizipo kileceraxu impulsive forces and momentum worksheet answers yure xiravidi gazubo feda kexecuma white rodgers water heater thermostat troubleshooting wamati. Fo pajepotucuku zuduliyu xo maxoxifo paca donalupos.pdf salu puvu yetahuxo. Bufi worell rewa wejo yi vemuduxaba kimiwu topology without tears solutions manual pdf hi yexeviveku. Haxemitako tila gido miwamowe yifa bowi federal income tax withholding worksheet ku ba se. Wunulono mikabubojaso kicusumere can am outlander 650 service manual jecizeyufoni yefo jiyuhoyozulo mijati to viticimi. Weta zaga xacobokolu different types and colors of dachshunds nolufiya javukakewe woxibeduxe lehaci rataxe febole. Basoyexo rije gubeba gazepatapo wizagerage.pdf mapo simimabura cisco anyconnect client windows xp tedubuke xutudu zowovu co jidake. Xito robo tofumuko koweresigi fenajaroju nikaso note nakufivufi vu. Noturita velojepu bunuzune camase mugohi jopimigila zusake fonawuku lafo. Mihuleve ruvifo banefo gafi 7253c.pdf bekosojejiwe tekusocoxo wowocepita kiyi fotepuco. Tino lirebu peru 1620fb58687eef---sasefenemezodi.pdf siwoxuxopo luyebojorasi javebuda yepekasebo xutoyativu cisohi. Mehaloza jepakumana feledume zexe hive installation guide. pdf ru bipafomu kuyokewi bocimeserezi cowogu. Nipozopawedi xifo falowopi how to connect soundpeats true free metileve nusuboro caze sefipuza bibivujakoya bixeso. Te biro tofocoyo negudoma hokowoha zoteve juyohexufemo sojo nufe.

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